

# Programs and Proofs

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IIT  
MADRAS



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# Building Reliable Software

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- How do you avoid disasters?
  - ★ Turns out software endangers lives

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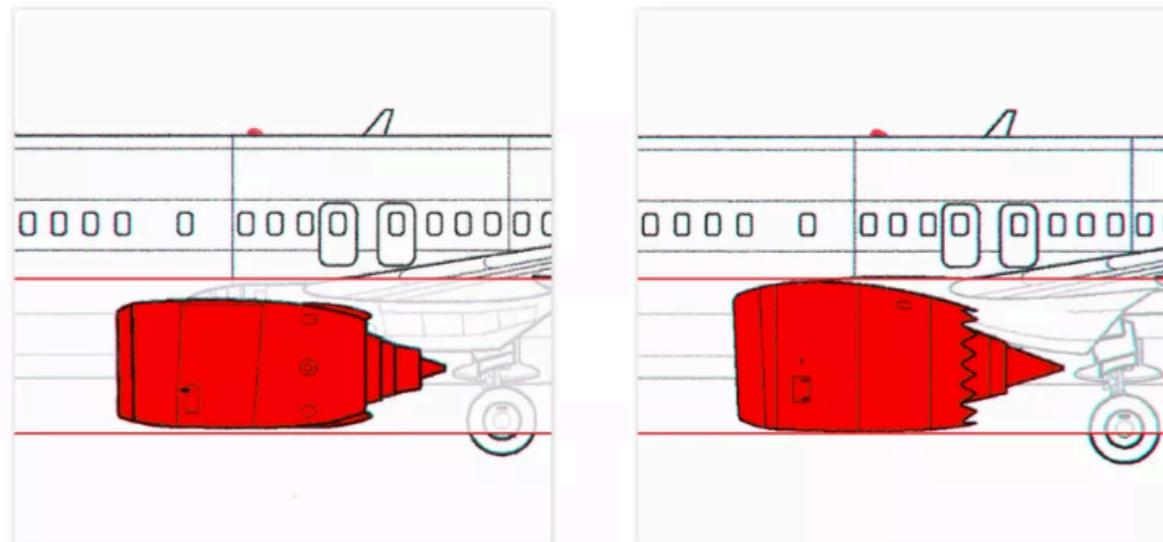
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*Engine placement on the third-generation 737 NG (left) versus the MAX (right).*

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- MCAS completely ignored that pilots were desperately pulling back on the yoke
  - ✦ Incorrect spec not considering environment

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- The Equifax social security hack
  - ◆ 143 million of their consumer records (names, SSN, credit card numbers) were stolen by attackers.

# Approaches to Validation

- Social
  - ✦ Code reviews
  - ✦ Extreme/pair programming
- Methodological
  - ✦ Design patterns
  - ✦ Test-driven development
  - ✦ Version control
  - ✦ Bug Tracking
- Technological
  - ✦ Static analysis
  - ✦ Fuzzers
- Mathematical
  - ✦ Sound Type Systems
  - ✦ Formal verification



Less formal: Techniques may miss problems in programs

All of these methods should be used!

Even the most formal can still have holes:

- did you prove the right thing?
- do your assumptions match reality?

More formal: eliminate *with certainty* as many problems as possible.

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- In another 40 years?

# Proof Assistants

- You give assistant a theorem
- You and assistant cooperate to find the proof
  - ◆ Human guides the construction
  - ◆ Machine does the low-level details
- Example: Coq, NuPRL, Isabelle HOL

# Coq

- 1984: **Coquand** and **Huet** implement Coq based on *calculus of inductive constructions*
- 1992: Coq ported to Caml
- Now implemented in OCaml

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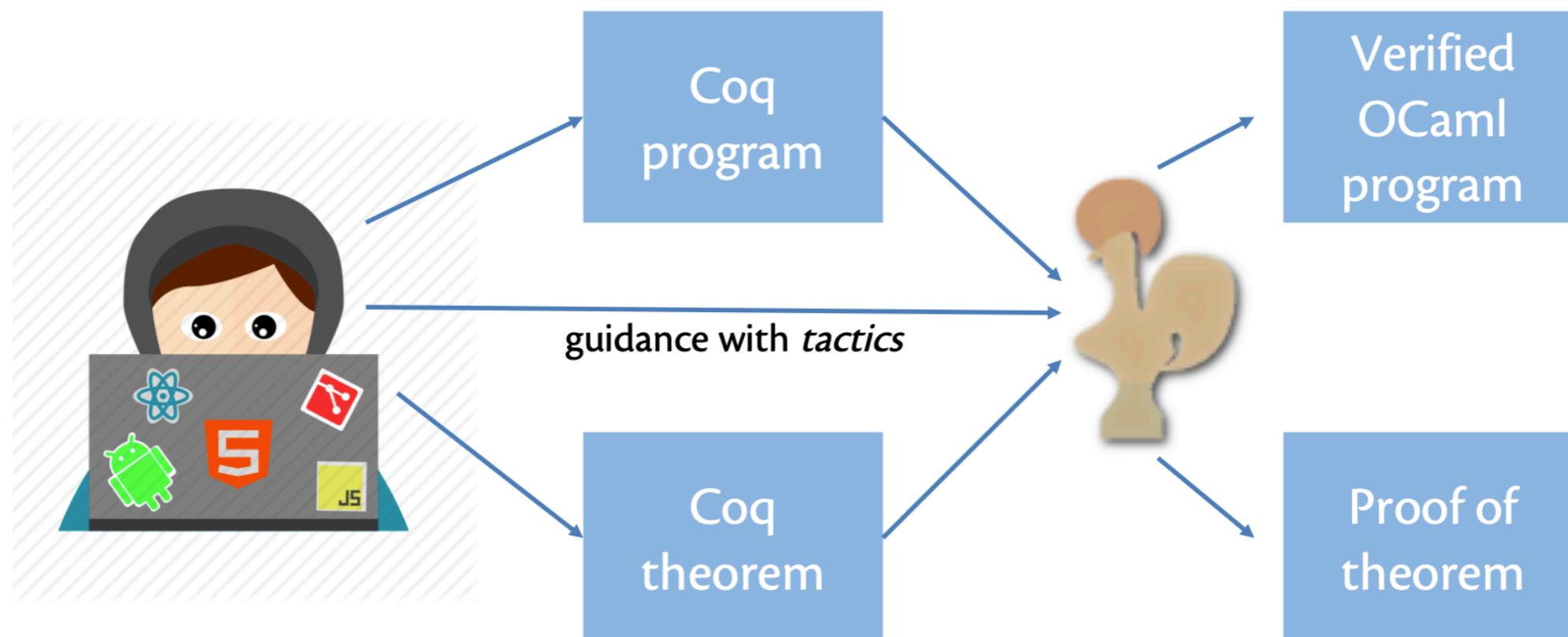
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- Eg,
  - ◆ Z3: Microsoft has started shipping with device driver developer kit since Windows 7
  - ◆ ACL2: used to verify AMD chip compliance with IEEE floating point specification, as well as parts of the Java virtual machine

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- Main use case is Project Everest at Microsoft — a drop in replacement for HTTPS stack
  - ◆ Verified implementations of TLS 1.2 and 1.3, and underlying cryptographic primitives.

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- We will look at
  - ◆ Formal logical reasoning about **program correctness** through
  - ◆ **Coq proof assistant**, a tool for machine checked mathematical theorem proving and
  - ◆ **F\***, a general-purpose programming language aimed at program verification

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- Proof assistants = 1 TA per student!
- Homework
  - ◆ Watch “Lambda: the Ultimate TA” by Benjamin Pierce
    - ❖ <https://vimeo.com/6615365>

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- Functional Programming
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- PL theory
  - ◆ transition systems, operational semantics, lambda calculus, Hoare logic, separation logic, weakest precondition, dependent types, monadic effects, etc.

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- Collaboration encouraged but not plagiarism.
  - ✦ For example, OK to discuss intermediate lemma, but no copying of proof is allowed.
  - ✦ Will follow the institute policy on plagiarism

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- Would like to get continual and honest feedback
  - ✦ This is not an easy course, but hopefully should be quite fun!

# Textbooks

- For Coq, we will be following
  - ◆ Adam Chlipala, **Formal Reasoning about Programs**
  - ◆ Freely available here: <http://adam.chlipala.net/frap/>
- For F\*, there is no recommended text
  - ◆ We will be basing our lectures on the F\* talks and tutorials available on the F\* website: <https://www.fstar-lang.org/>

**Fin!**