Transition Systems

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Spring 2020

Imperative Factorial

```
factorial(n) {
  a = 1;
  while (n > 0) {
    a = a * n;
    n = n - 1;
  }
  return a;
}
```

Transition System for Factorial

Natural numbers	n	$\in$	$\mathbb{N}$
States	s	$::=$	$\mathsf{AnswerIs}(n) \mid \mathsf{WithAccumulator}(n, n)$

$$\frac{}{\mathsf{WithAccumulator}(n_0, 1) \in \mathcal{F}_0} \qquad \frac{}{\mathsf{AnswerIs}(a) \in \mathcal{F}_\omega}$$

$$\frac{}{\mathsf{WithAccumulator}(0, a) \to \mathsf{AnswerIs}(a)}$$

$$\frac{}{\mathsf{WithAccumulator}(n + 1, a) \to \mathsf{WithAccumulator}(n, a \times (n + 1))}$$

DEFINITION 5.1. A *transition system* is a triple $\langle S, S_0, \to \rangle$, with S a set of states, $S_0 \subseteq S$ a set of initial states, and $\to \subseteq S \times S$ a transition relation.

Correctness of factorial

- Transitive Reflexive Closure (trc)

$$\frac{}{s \to^* s} \qquad \frac{s \to s' \quad s' \to^* s''}{s \to^* s''}$$

- Reachable states of a transition system

DEFINITION 5.2. For transition system $\langle S, S_0, \to \rangle$, we say that a state s is *reachable* if and only if there exists $s_0 \in S_0$ such that $s_0 \to^* s$.

- Correctness of factorial encoded as transition system

For any state s reachable in $\mathcal{F}$, if $s \in \mathcal{F}_\omega$, then $s = \mathsf{AnswerIs}(n_0!)$.

Invariants

For any state s reachable in $\mathcal{F}$, if $s \in \mathcal{F}_\omega$, then $s = \mathsf{AnswerIs}(n_0!)$.

- We could prove this in an ad-hoc way. But let's develop a general notion of *invariants*
  - Invariant is a property of a programs that *starts true and stays true*

DEFINITION 5.4. An *invariant* of a transition system is a property that is always true, in all of the system's reachable states. That is, for transition system $\langle S, S_0, \rightarrow \rangle$, where R is the set of all its reachable states, some $I \subseteq S$ is an invariant iff $R \subseteq I$.

- Mathematically, *Property of states = Set of states*
  - The "property" holds of exactly those states that belong to the set