

1 SAL: Multi-modal Verification of Replicated Data Types 56

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23 Abstract 72

24 Designing correct replicated data types (RDTs) is challenging 73
25 because replicas evolve independently and must be merged 74
26 while preserving application intent. A promising approach 75
27 is correct-by-construction development in a proof-oriented 76
28 programming language such as F^* , Dafny and Lean, where 77
29 desired correctness guarantees are specified and checked as 78
30 the RDTs are implemented. Recent work Neem [19] proposes 79
31 the use of replication-aware linearizability (RA linearizability) 80
32 [20] as the correctness condition for state-based CRDTs 81
33 and mergeable replicated data types (MRDTs), with automation 82
34 in the SMT-aided, proof-oriented programming language 83
35 F^* . However, SMT-centric workflows can be opaque 84
36 when automation fails to discharge a verification condition 85
37 (VC), and they enlarge the trusted computing base (TCB). 86

38 We present SAL, a multi-modal workflow to design and 87
39 verify state-based CRDTs and MRDTs in Lean. SAL combines 88
40 (i) kernel-checkable automation with proof reconstruction, 89
41 (ii) SMT-aided automation when needed, and (iii) interactive 90
42 theorem proving for remaining proof obligations. When 91
43 automated verification fails, we leverage Lean’s property-based 92
44 testing to automatically generate and visualize counterexamples, 93
45 helping developers debug incorrect specifications 94
46 or implementations. We report on our experience verifying 95
47 a suite of 13 CRDTs and MRDTs with SAL: 69% of verification 96
48 conditions are discharged by kernel-verified automation 97
49 without SMT, and counterexamples automatically expose 98
50 subtle bugs such as the well-known enable-wins flag anomaly. 99
51 The codebase for SAL is open-sourced, and is available 100
52 at <https://github.com/fplaunchpad/sal>. 101

53 **CCS Concepts:** • Software and its engineering → Software 102
54 verification and validation; • Computing methodologies → Distributed 103
55 algorithms. 104

56 **Keywords:** CRDT, Verification, Lean, Multi-Modal Proofs, 105
57 Counterexample Generation 106

58 1 Introduction 107

59 Local-first collaboration tools allow users to continue working 108
60 while offline, synchronizing in the background when 109
61 connectivity returns [10]. This programming model requires 110
62 replicated data types whose states can evolve independently 111
63 at each replica and later be merged without violating application 112
64 intent. Conflict-Free Replicated Data Types (CRDTs) [18] 113
65 and Mergeable Replicated Data Types (MRDTs) [8] are widely 114
66

67 used to implement such replicated state. However, designing 68 correct RDTs is subtle. Even well-known designs such 69 as Replicated Growable Arrays (RGA) [9] have had serious 70 anomalies discovered after publication¹. 71

72 Replication-aware (RA) linearizability [20] provides a principled 73 correctness condition for CRDTs, relating replica executions 74 to a sequential explanation of updates. Prior work, 75 Neem [19], showed that RA linearizability can be extended to 76 MRDTs and also be reduced to a set of 24 verification conditions 77 (VCs) amenable to automation. Neem implements this 78 approach in F^* , an SMT-aided, proof-oriented programming 79 language. Developers implement RDTs in F^* and leverage 80 SMT automation to discharge the RA-linearizability VCs 81 automatically. In practice, however, SMT-centric verification 82 workflows can be difficult to iterate on. When automation 83 fails, developers often get little actionable information 84 beyond an unproved VC, and debugging incorrect implementations 85 remains manual and time-consuming. 86

87 This debugging experience contrasts sharply with everyday 88 software development, where programmers iterate 89 quickly by writing code, running tests, inspecting failing 90 traces, and refining implementations. Understanding why 91 a particular VC fails requires a laborious “proof-debugging” 92 workflow: developers add intermediate assertions (`assert`) 93 or assumptions (`admit`) and progressively push them deeper 94 to localize the failing reasoning step. When the root cause 95 is an unexpected interaction between definitions and the 96 SMT encoding, additional tuning (e.g., controlling unfolding 97 or solver limits) may be needed. Moreover, SMT-aided 98 proofs can be brittle: small changes may cause previously 99 discharged VCs to fail. Relying on an external SMT solver 100 also increases the trusted computing base. 101

102 We present SAL, a verification workflow for RA-linearizability 103 in Lean that addresses these issues by making failures 104 actionable. At the core of SAL is a staged Lean tactic that 105 prioritizes proof reconstruction (e.g., simplifying goals and 106 invoking `grind` [12]) and falls back to SMT-aided automation 107 (`lean-blaster` [7]) only when necessary. When a VC is false, 108 SAL uses property-based testing (`Plausible` [13]) to automatically 109 synthesize small counterexamples and a `ProofWidgets` 110 [1]-based visualizer to render the corresponding execution 111 trace, enabling a test-like debugging loop for RDT 112 verification. In our evaluation over a suite of CRDTs and 113

1¹<https://martin.kleppmann.com/2019/03/25/papoc-interleaving-anomalies.html#errata>

111 MRDTs, SAL discharges most VCs without SMT and automatically
 112 redisCOVERS subtle bugs such as the well-known
 113 enable-wins flag anomaly (Table 2).

114 This paper makes the following contributions:

- 115 1. A Lean formalization of RA-linearizability VCs for a
 116 suite of state-based CRDTs and MRDTs.
- 117 2. A counterexample-generation and visualization work-
 118 flow for failing VCs, based on property-based testing
 119 in Lean.
- 120 3. SAL, a custom tactic that attempts proof reconstruction-
 121 based automation first and falls back to SMT-aided
 122 automation, with interactive proving as a last resort.
- 123 4. An evaluation across a suite of CRDTs and MRDTs,
 124 including cases where counterexamples expose subtle
 125 bugs (Table 2).

2 Background

126 We assume familiarity with replicated data types and focus
 127 on the specific models and specifications used in this paper.
 128 State-based CRDTs reconcile replicas via a deterministic
 129 two-way merge $\mu(v_1, v_2)$; a common sufficient condition for
 130 convergence is that the merge function μ is the join of a
 131 semilattice.

132 Mergeable Replicated Data Types (MRDTs) [8] avoid em-
 133 bedding causal metadata in the state of the RDT. Instead,
 134 they assume a versioned storage model (e.g., Git-like histo-
 135 ries) that can provide the lowest common ancestor (LCA)
 136 for a three-way merge. This interface often yields compact
 137 implementations: for example, a counter MRDT can be $O(1)$,
 138 whereas state-based counter CRDTs require $\Omega(n)$ space in
 139 the number of replicas [2].

140 Formally, an MRDT implementation for a data type τ [19]
 141 is a tuple $\mathcal{D}_\tau = \langle \Sigma, \sigma_0, \text{do}, \text{merge}, \text{rc} \rangle$, where:

- 142 • Σ is the set of states, $\sigma_0 \in \Sigma$ is the initial state.
- 143 • $\text{do} : \Sigma \times \mathcal{T} \times \mathcal{R} \times \mathcal{O}_\tau \rightarrow \Sigma$ implements update operations
 144 parameterized by timestamp in \mathcal{T} , replica id in \mathcal{R} and
 145 operations in \mathcal{O}_τ .
- 146 • $\text{merge} : \Sigma \times \Sigma \times \Sigma \rightarrow \Sigma$ is a three-way merge function.
- 147 • $\text{rc} \subseteq \mathcal{O}_\tau \times \mathcal{O}_\tau$ is the conflict resolution policy to be
 148 followed for concurrent *conflicting* update operations.
 149 The relation is interpreted as a partial order where
 150 $(o_1, o_2) \in \text{rc}$ means that o_1 is ordered before o_2 .

151 For example, the increment-only counter MRDT is defined
 152 as follows:

- 153 • $\Sigma = \mathbb{N}$ with $\sigma_0 = 0$
- 154 • $\mathcal{O} = \{\text{inc}\}$
- 155 • $\text{do}(\sigma, _, _, \text{inc}) = \sigma + 1$
- 156 • $\text{merge}(\sigma_{lca}, \sigma_1, \sigma_2) = \sigma_{lca} + (\sigma_{lca} - \sigma_1) + (\sigma_{lca} - \sigma_2)$ – the
 157 merged state is the sum of the states of the LCA and the difference between the LCA and the two versions.
- 158 • $\text{rc} = \emptyset$ – no conflicting operations.

159 An observed-removed set (OR-set) MRDT [8] is defined as
 160 follows:

- 161 • $\Sigma = \mathcal{P}(\mathcal{T} \times E)$ where E is the set of elements with
 $\sigma_0 = \emptyset$
- 162 • $\mathcal{O} = \{\text{add}_e, \text{rem}_e | e \in E\}$
- 163 • $\text{do}(\sigma, t, _, \text{add}_e) = \sigma \cup \{(e, t)\}$
- 164 • $\text{do}(\sigma, _, _, \text{rem}_e) = \sigma \setminus \{(e, i) | (e, i) \in \sigma\}$
- 165 • $\text{merge}(\sigma_{lca}, \sigma_1, \sigma_2) = (\sigma_{lca} \cap \sigma_1 \cap \sigma_2) \cup (\sigma_1 \setminus \sigma_{lca}) \cup$
 $(\sigma_2 \setminus \sigma_{lca})$ – the merged state contains elements com-
 166 mon to all three versions (unchanged elements remain;
 167 deleted elements are not included), as well as elements
 168 added in either version since the LCA.
- 169 • $\text{rc} = \{(\text{rem}_e, \text{add}_e) | e \in E\}$ – remove is ordered before
 170 add for the same element, and hence, adds win over
 171 concurrent removes.

172 We verify RDT correctness using replication-aware (RA)
 173 linearizability [20], which makes merge semantics explicit
 174 while retaining a linearizability-style reading. Neem [19]
 175 reduces RA linearizability for MRDTs and state-based CRDTs
 176 to a finite set of verification conditions (VCs) over do, merge,
 177 and rc. SAL focuses on discharging these VCs effectively and
 178 on producing actionable feedback when automation fails.

179 Lean 4 [16] is a theorem prover and functional program-
 180 ming language with a small trusted kernel; proofs can be
 181 checked as proof terms by the kernel. Lean's metaprogram-
 182 ming support enables custom tactics and domain-specific
 183 automation, which is useful for VC-heavy developments like
 184 ours.

185 Lean offers several automation techniques with different
 186 tradeoffs. We rely on the following:

- 187 • dsimp and aesop : simplification and proof search for
 188 routine goals.
- 189 • grind [12]: SMT-style automation with proof recon-
 190 struction, producing kernel-checkable proof terms.
- 191 • lean-blaster [7]: an SMT backend (Z3) that is effec-
 192 tive on many VCs (notably those with lambdas), but
 193 without proof reconstruction; it therefore enlarges the
 194 trusted computing base.

195 SAL stages automation accordingly: we first attempt re-
 196 constructible automation and only fall back to SMT when
 197 needed.

3 SAL framework

198 In this section, we describe the SAL multi-modal verification
 199 framework for RA-linearizability in Lean. We start with the
 200 challenge of designing data structures conducive to auto-
 201 mated verification in Lean.

3.1 Data structures for automated verification

210 Many RDTs rely fundamentally on sets and maps. For ex-
 211 ample, the OR-set MRDT maintains a set of timestamped
 212 elements; map-based MRDTs and CRDTs require reasoning
 213 about extensional equality of mappings. However, Lean's
 214

standard library provides data structures optimized for different purposes: the mathematical Set type uses propositions ($\alpha \rightarrow \text{Prop}$) suitable for manual proofs, while computational maps like RBMap and HashMap emphasize efficient iteration but complicate extensional reasoning.

For automated verification, we use decidable representations that tools like `grind` can reason about effectively. We therefore implement custom set and map interfaces inspired by F*'s verification-oriented designs. Our sets use boolean-valued membership functions rather than propositions, since they typically contain datatypes where equality is decidable.

```
232 abbrev set (a:Type) [DecidableEq a] := a → Bool
```

This representation is decidable by construction: since membership returns a boolean, automation can directly compute and compare set operations without requiring proof objects to reason about decidability. We require `DecidableEq` for element types, ensuring all operations remain computable. This is a natural condition which is satisfied across the RDTs we verify with this framework.

Similarly, our map interface makes the domain explicit to enable extensional reasoning:

```
243 structure map (key:Type) [DecidableEq key]
244   (value:Type) where
245     mappings: key → value
246     domain: set key
```

Two maps are equal when their domains and mappings agree. Since the mapping is restricted to the domain key which supports decidable equality, the map supports extensional equality.

Critically, we annotate all definitions and lemmas with `@[simp, grind]` attributes and provide `grind_pattern` hints. This builds a domain-specific rewrite database: when `grind` encounters goals involving set membership, union, or map selection, it automatically applies the appropriate lemmas without manual guidance. These annotations trade the generality of Lean's standard library for automation-friendliness, enabling `grind` to discharge most VCs involving sets and maps without SMT assistance (Table 2). This is analogous to SMT patterns in F*.

3.2 Counterexample generation in Lean

When a VC fails during verification, understanding why it failed is critical for debugging. In SMT-centric workflows like F*, a failed VC provides little actionable feedback: developers must manually construct execution traces, add intermediate assertions, and progressively narrow down the source of the failure. This process is labor-intensive and requires significant expertise. In contrast, SAL leverages Lean's property-based testing framework, `Plausible` [13], to automatically generate concrete counterexamples when VCs fail, transforming opaque proof failures into tangible test cases that developers can inspect and debug.

```
276 abbrev concrete_st := Int × Bool
277 inductive app_op_t : Type where
278   | Enable
279   | Disable
280 abbrev op_t := N × N × app_op_t
281   /-timestamp, rid, operation-
282 def do_ (s:concrete_st) (o: op_t) : concrete_st
283   := match o with
284   | (_, _, .Enable) => (Prod.fst s + 1, true)
285   | (_, _, .Disable) => (Prod.fst s, false)
286 def merge_flag (l a b: concrete_st) :=
287   if Prod.snd a && Prod.snd b then true
288   else if not (Prod.snd a) && not (Prod.snd b) then
289     false
290   else if Prod.snd a then Prod.fst a > Prod.fst l
291   else Prod.fst b > Prod.fst l
292 def merge (l a b: concrete_st) : concrete_st
293   := (Prod.fst a + Prod.fst b - Prod.fst l,
294     merge_flag l a b)
```

Figure 1. Buggy enable-wins flag MRDT implementation

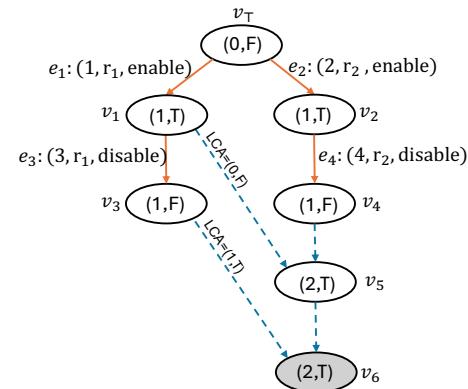


Figure 2. An enable-wins flag execution: both replicas see a disable at the end, yet merging produces (2, true) at v_6 , incorrectly reporting the flag as enabled.

We demonstrate this approach using the enable-wins flag MRDT, a shared boolean flag that represents a disabled or enabled state. The desired specification is that in the case of concurrent enable and disable operations, the enable wins. This is expressed as $\text{disable} \xrightarrow{\text{rc}} \text{enable}$, meaning that when we read a replica's state, the flag should be true if there exists an enable operation that is not causally preceded by a disable.

Consider the enable-wins flag implementation in Figure 1, which tracks the number of concurrent enables using a counter and uses this counter to determine the flag's state after merging. This implementation contains a subtle bug. The bug manifests in executions where enable operations are followed by disables on each replica, yet the merged

331 state incorrectly reports the flag as enabled. Figure 2 shows
 332 such an execution: when merging v_3 and v_5 (with LCA v_1),
 333 the counter value of v_5 exceeds v_1 , causing `merge_flag` to
 334 compute the new flag to be true in v_6 . However, all enable
 335 events in this execution are subsequently disabled on their
 336 respective replicas, violating the enable-wins specification!
 337 When verifying the RA-linearizability VCs, one of the VCs
 338 fails. However, finding such counterexamples manually is
 339 challenging, particularly for more complex RDTs with larger
 340 state spaces.

341 To automate counterexample discovery, we leverage Plau-
 342 sible [13], a property-based testing tool for Lean inspired by
 343 QuickCheck [3]. Plausible requires that VCs be expressed
 344 as decidable propositions – that is, properties that can be
 345 algorithmically evaluated to true or false. Once decidability
 346 is established, Plausible generates random test inputs and
 347 checks whether they satisfy the VC. When a violation is
 348 found, Plausible reports the failing input, such as:

349 `postcondition violated for input`
 350 `(disable, (enable, (enable, (disable, (0, false)))))`

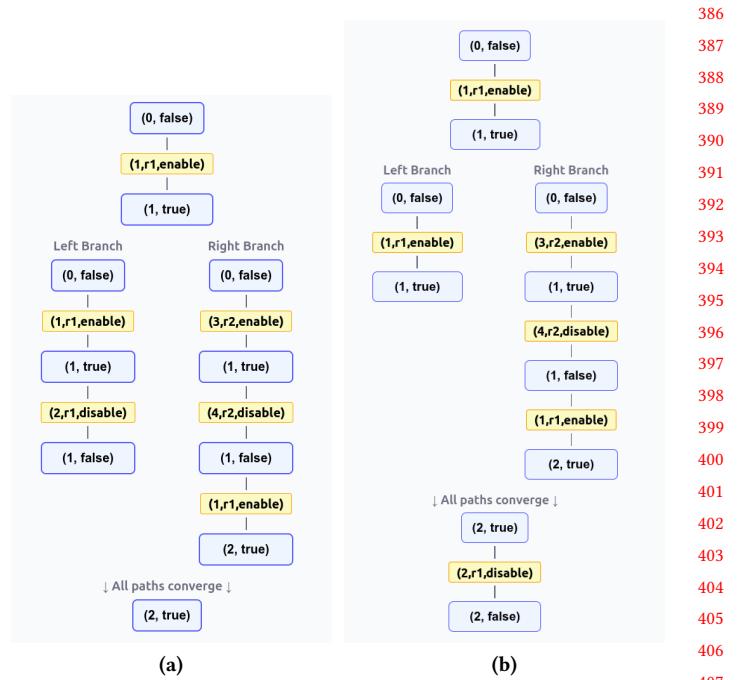
352 While this output identifies a counterexample, diagnosing
 353 the cause of the failure is difficult. To address this, we imple-
 354 ment an execution trace visualizer using Lean's ProofWid-
 355 getts framework [1]. The visualizer instruments the `do` and
 356 `merge` operations to record intermediate states and opera-
 357 tions, producing a step-by-step execution trace.

358 SAL, like Neem [19], includes VCs that check whether
 359 concurrent executions can be RA-linearized to the same final
 360 state. This is achieved using *bottom-up linearization*, where
 361 we peel off events in a bottom-up manner to construct a
 362 linearization order. In the failed VC, we attempt to prove:

$$364 \quad \begin{array}{c} v_1 \quad \quad v_3 \quad \quad v_5 \\ \mu(\overbrace{e_1(0, \text{false}), e_3(e_1(0, \text{false})), e_1(e_4(e_2(0, \text{false})))}^{\text{LHS}}) = \\ e_3(\mu(\overbrace{e_1(0, \text{false}), e_1(0, \text{false})}^{\text{v}_1}, \overbrace{e_1(e_4(e_2(0, \text{false})))}^{\text{v}_5})) \end{array}$$

370 where μ is the merge operation, $e_1 \dots e_4$ are events, and each
 371 tuple $(\text{Int} \times \text{Bool})$ constitutes a state. The state at v_6 in Fig-
 372 ure 2 is obtained by merging v_3 and v_5 , with v_1 as LCA. This
 373 corresponds to the LHS of the VC. This state should match
 374 the state obtained by first merging v_1 and v_5 , with v_1 as LCA,
 375 and then applying e_3 , through bottom-up linearization.

376 Figure 3 illustrates the visualization for the enable-wins
 377 flag that violates this VC. The states and operations are
 378 shown in blue and yellow boxes, respectively, and corre-
 379 spond to `concrete_st` and `op_t`, respectively, from Fig-
 380 ure 1. The LCA trace and state is shown at the top. In both
 381 cases, the LCA state is v_1 . The left panel (a) shows the execu-
 382 tion trace for the LHS, which evaluates to $(2, \text{true})$, and the
 383 right panel (b) shows the RHS, which evaluates to $(2, \text{false})$.
 384 By comparing these traces, developers can quickly identify



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Figure 3. Visualization for the failed VC for the buggy enable-wins flag in Figure 1.

412 that the bug arises from the counter-based merge logic fail-
 413 ing to account for subsequent disables. This visualization
 414 transforms the abstract VC failure into a concrete debugging
 415 scenario, analogous to examining a failing unit test trace in
 416 conventional software development.

417 The combination of automatic counterexample genera-
 418 tion and interactive visualization significantly accelerates
 419 the debugging workflow. Instead of manually inspecting
 420 failed VCs and constructing hypothetical execution traces,
 421 developers receive concrete, visualizable counterexamples
 422 automatically. This approach also complements the multi-
 423 modal proof strategy: when `grind` or `lean-blaster` fail to
 424 discharge a VC, Plausible can quickly determine whether the
 425 failure stems from an incorrect implementation (producing a
 426 counterexample) or requires manual interactive proof (when
 427 no counterexample exists).

3.3 Visualizing functional sets

428 While the enable-wins flag uses simple concrete types (inte-
 429 gers and booleans), many RDTs are set-based. Our custom
 430 sets (Section 3.1), designed as functional predicates $a \rightarrow \text{Bool}$
 431 for verification, pose a visualization challenge: they are ab-
 432 stract, infinite by nature, and do not support iteration or
 433 enumeration. Yet both counterexample debugging and gen-
 434 eral trace inspection require displaying actual set contents
 435 as concrete element lists.

436 To bridge this gap, we implement a universe tracking
 437 mechanism. During execution, we maintain a finite HashSet

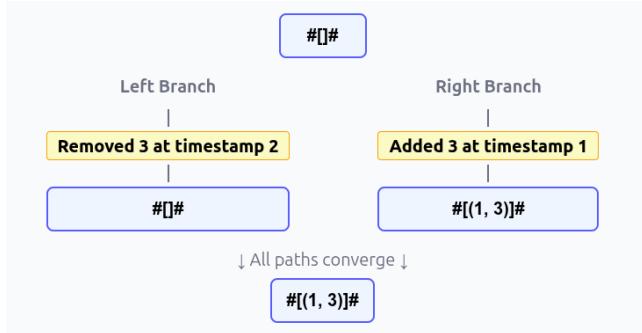


Figure 4. OR-set execution visualized using ProofWidgets

of all elements that have been added or removed. When visualizing the set state, we only check membership for elements in this finite universe, which suffices to characterize the set’s observable behavior. The implementation augments our abstract sets with a concrete universe:

```

structure set_with_universe (α: Type) [ToString α]
  [DecidableEq α] [Hashable α] where
  _set : set α
  _universe : HashSet α

```

Figure 4 demonstrates this approach on an OR-set execution with concurrent add and remove operations on element 3. The visualization shows concrete set states using the notation $\#[(1, 3)]\#$, where the tuple contains the timestamp and element. The left branch removes element 3 (resulting in the empty set $\#[]\#$), while the right branch adds element 3 (resulting in $\#[(1, 3)]\#$). Both branches converge to $\#[(1, 3)]\#$, confirming that adds win over concurrent removes as specified. Operations are displayed in yellow boxes, and users provide operation labels via format strings.

This mechanism works uniformly for both correct executions and counterexamples, enabling developers to inspect set-based RDT behavior regardless of whether they are debugging a failed VC or validating a correct implementation.

3.4 Multi-modal proofs using the SAL tactic

The SAL tactic orchestrates a staged proof strategy that adapts to VC complexity. As described in Section 2, Lean provides multiple automation techniques with different trade-offs between power, trust, and performance. The SAL tactic attempts these approaches sequentially, prioritizing proof reconstruction before falling back to more powerful but less trustworthy methods.

The tactic proceeds in the following stages:

1. `dsimp + grind` (DG) – This combination applies simplification followed by `grind`’s SMT-style automation with proof reconstruction, producing kernel-checkable proof terms while maintaining the smallest TCB. We exclude `aesop` [14] due to prohibitively high verification times on RDT VCs.

2. `lean-blaster` (LB) – When DG fails, `lean-blaster` encodes the goal to Z3. While more powerful for VCs with higher-order functions and lambdas, this sacrifices proof reconstruction and enlarges the TCB. We select it over Lean-SMT [15] and Lean-Auto [17] for its superior support for higher-order functions.

If both automated stages fail and no counterexample was generated (Section 3.2), the VC requires interactive proving. Developers must then manually construct proofs using Lean’s tactic language. To prevent runaway automation, the SAL tactic incorporates a heartbeat-based timeout mechanism that bounds the execution time of each stage. When a timeout expires, control returns to the user, allowing them to either increase the limit or proceed directly to interactive proving.

4 Evaluation

Table 1 compares F^* and Lean across several dimensions relevant to RDT verification. Lean’s rich tactic system enables the multi-modal workflow central to SAL, while built-in counterexample generation (Plausible) and proof reconstruction support actionable debugging and reduced TCB. However, Lean’s standard library data structures are designed for mathematical reasoning rather than automation, necessitating the custom sets and maps described in Section 3.1.

Table 2 shows the results of verifying 13 RDTs (312 VCs in total) using SAL. Across all benchmarks, **215 VCs (68.9%) are discharged by `dsimp+grind`**, a lightweight tactic with proof reconstruction verified by Lean’s kernel. An additional 87 VCs (27.9%) require `lean-blaster` (SMT-based), and only 9 VCs (3%) fall back to interactive proving. This demonstrates that the majority of RA-linearizability VCs can be verified without enlarging the TCB through SMT solvers.

We observe notable differences between CRDTs and MRDTs. MRDTs generally require less SMT than CRDTs due to their simpler three-way merge; for example, PN-counter MRDT needs no `lean-blaster` while PN-counter CRDT requires it for 2 VCs plus 6 interactive proofs. The 9 interactive proofs are concentrated in map-based RDTs, reflecting that Lean’s map automation is less mature than its set automation.

The enable-wins flag MRDT demonstrates the value of counterexample generation. This implementation contains a known bug from prior work [19]; when the corresponding VC fails, Plausible automatically generates a concrete counterexample, which our ProofWidgets-based visualizer renders for inspection (Figure 3). This workflow transforms opaque VC failures into actionable debugging scenarios.

5 Related work

Multi-modal verification. Loom [5] is a recent work on multi-modal verification for data structures in Lean, using Dijkstra Monads to model effectful operations on mutable arrays and counters. SAL adapts this multi-modal approach

Table 1. Comparing F* and Lean for verifying RA Linearizability

Criterion	F*	Lean
Automation	Direct SMT solving via Z3. Automation often works out of the box at scale.	Multiple tactic-based tools (aesop, grind, lean-blaster), less effective on large proof goals.
Multi-Modal Proofs	Limited tactic system; most proofs require SMT solver.	Rich tactic system allows combining automated and interactive proving.
Counterexamples	No counterexample generation; reports only success or failure.	Plausible generates counterexamples for decidable properties.
Data Structures	Rich set and map libraries designed for verification.	Mathematical data structures; custom sets and maps needed for automation (Section 3.1).
Trustworthiness	No proof reconstruction; relies on trusting Z3.	Proof reconstruction for most tools; kernel verifies proofs. Uncertified proofs marked sorry.

Table 2. Number of VCs discharged by the SAL tactic (total VCs = 24 per RDT). DG refers to dsimp + grind, and LB refers to lean – blaster. #Contains known bug; counterexample automatically generated (Section 3.2).

RDT	DG	LB	Fallback to ITP
Increment-only counter MRDT	24	0	0
PN-counter MRDT	24	0	0
OR-set MRDT	3	21	0
Enable-wins flag MRDT #	9	14	0
Efficient OR-set MRDT	2	22	0
Grows-only set MRDT	24	0	0
Grows-only map MRDT	22	0	2
Replicated growable array MRDT	15	9	0
Multi-valued register MRDT	24	0	0
Increment-only counter CRDT	24	0	0
PN-counter CRDT	16	2	6
Multi-valued register CRDT	24	0	0
OR-set CRDT	4	19	1

to RDT verification in Lean, but leverages conventional tactics rather than monadic effects since RDTs are monotonic and non-mutable. Unlike Loom’s focus on sequential data structures, SAL targets the unique challenges of replicated systems with merge operations and causal reasoning.

CRDT verification. Several approaches verify CRDT correctness. Isabelle/HOL has been used to verify strong eventual consistency [6], but requires substantial manual proof effort. Neem [19] automates RA-linearizability verification in F* using SMT solvers, achieving high automation but at the cost of an enlarged TCB and opaque failures. Verifx [4] provides a specialized language for RDTs with built-in verification, trading generality for domain-specific automation. SAL distinguishes itself through kernel-verified automation (69% of VCs discharged by proof reconstruction), automated

counterexample generation when verification fails, and interactive visualization of execution traces—capabilities absent in prior CRDT verification tools.

Counterexample generation. Property-based testing tools like QuickCheck [3], QuickChick [11] (Coq), and Plausible (Lean) generate test inputs for executable properties. SAL leverages Plausible for decidable VCs, but extends it with domain-specific visualization for RDT execution traces, including handling of functional set representations through universe tracking (Section 3.3). This bridges property testing and formal verification, transforming failed VCs into debuggable counterexamples.

6 Conclusion

We present SAL, a multi-modal verification framework for replicated data types in Lean that addresses key limitations of SMT-centric verification workflows. By staging automation—prioritizing kernel-verified proof reconstruction via `grind`, falling back to SMT-based `lean-blaster`, and supporting interactive proving for complex obligations—SAL achieves 69% kernel-verified automation across 13 CRDTs and MRDTs while minimizing the trusted computing base. When automation fails, automated counterexample generation via Plausible and interactive visualization through ProofWidgets transform opaque proof failures into actionable debugging scenarios, as demonstrated by rediscovering the enable-wins flag bug.

Our experience reveals patterns in RDT verification: MRDTs generally require less SMT than CRDTs due to simpler three-way merge, and map-based reasoning remains more challenging than set-based reasoning for current Lean automation. Future work includes extending our custom data structure library to improve map automation, developing optimizations for the staged tactic based on VC patterns, and evaluating SAL on larger-scale RDT developments.

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