Convergence is Not Enough

Automatically Verifying Replicated Data Types

KC Sivaramakrishnan
Joint work with Pranav Ramesh, Vimala Soundarapandian, Kartik Nagar

SERI 2026
16th Jul 2026

FP Launchpad

IIT MADRAS

Tarides

Collaborative Applications

Collaborative Applications

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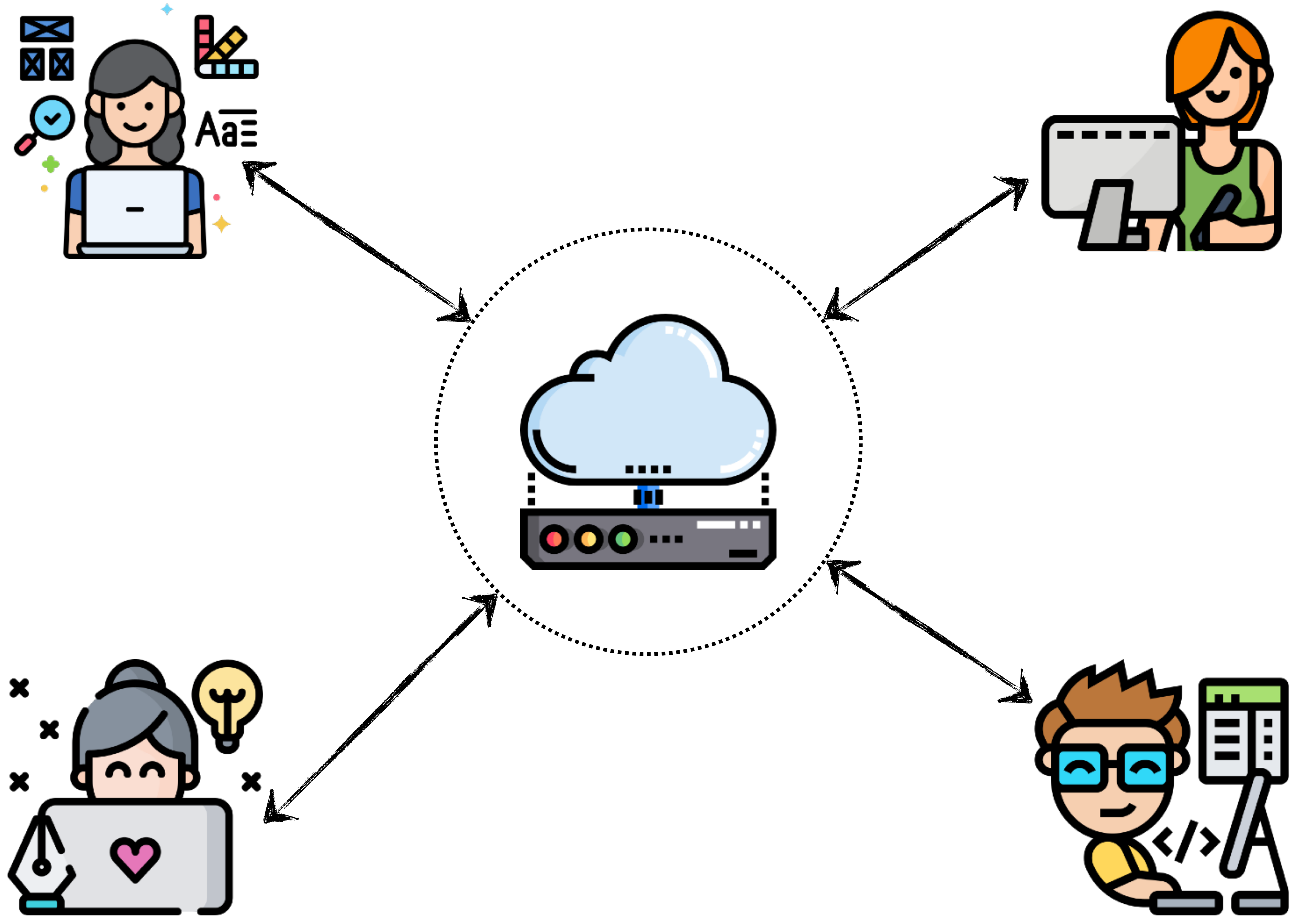

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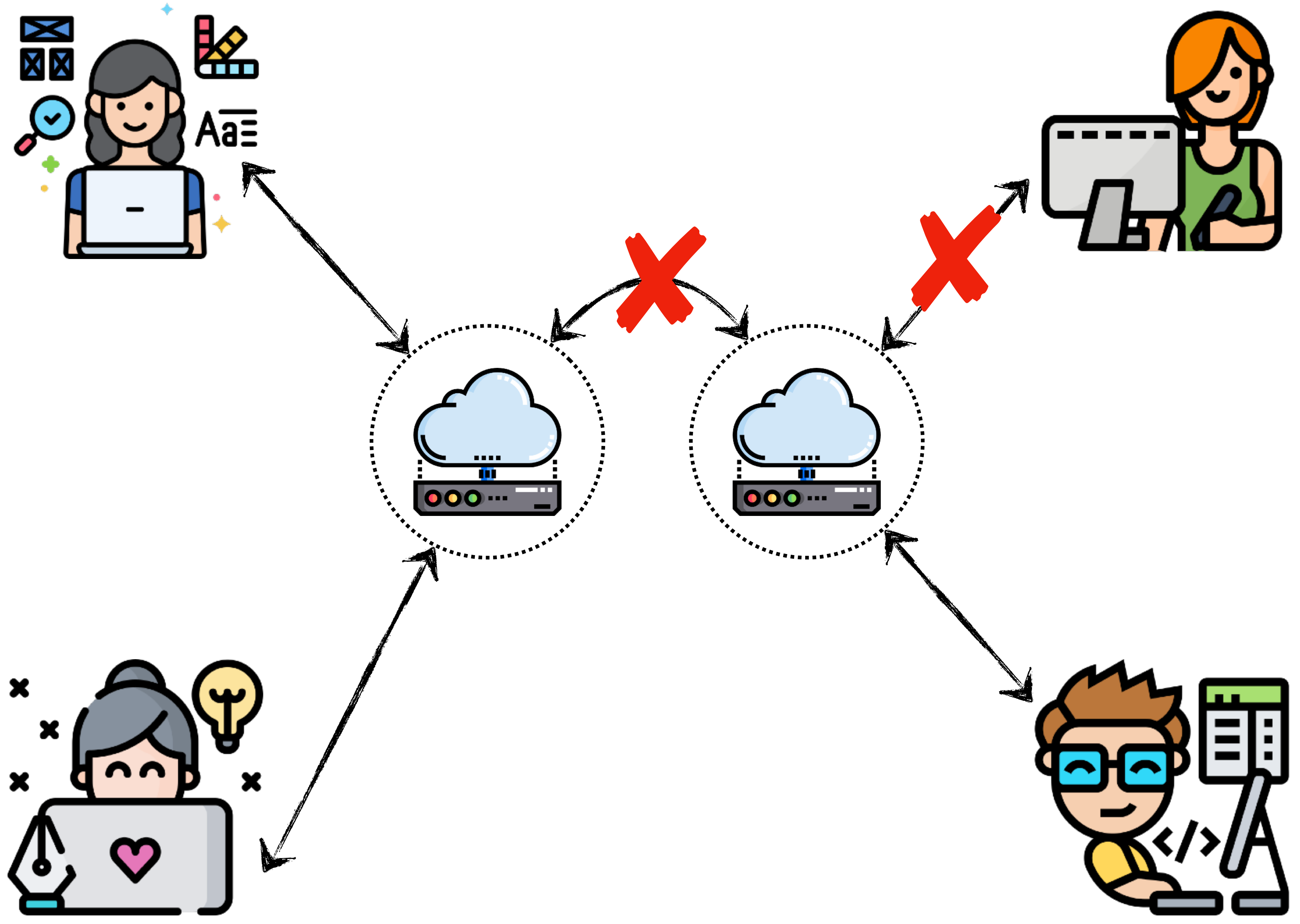
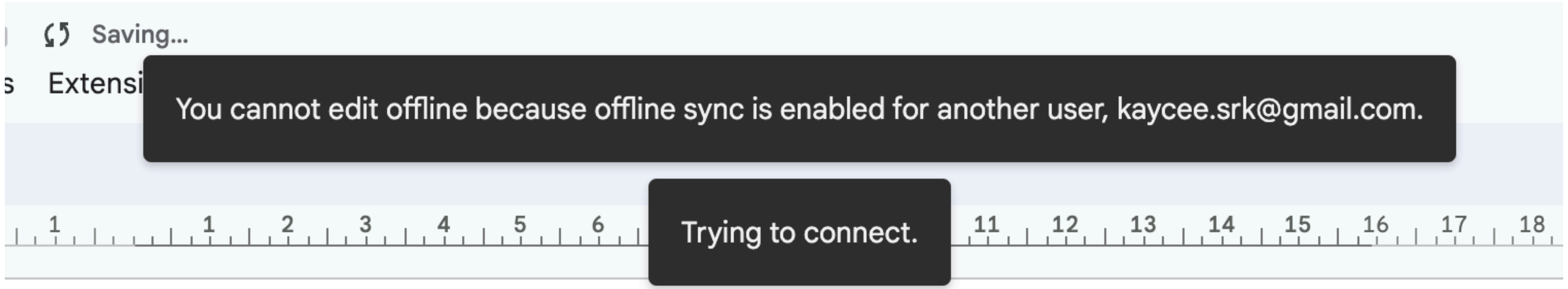


Network Partitions

- Centralised Apps provide limited support for offline editing



Enabling offline sync for one account prevents other accounts from working offline

Local-first software

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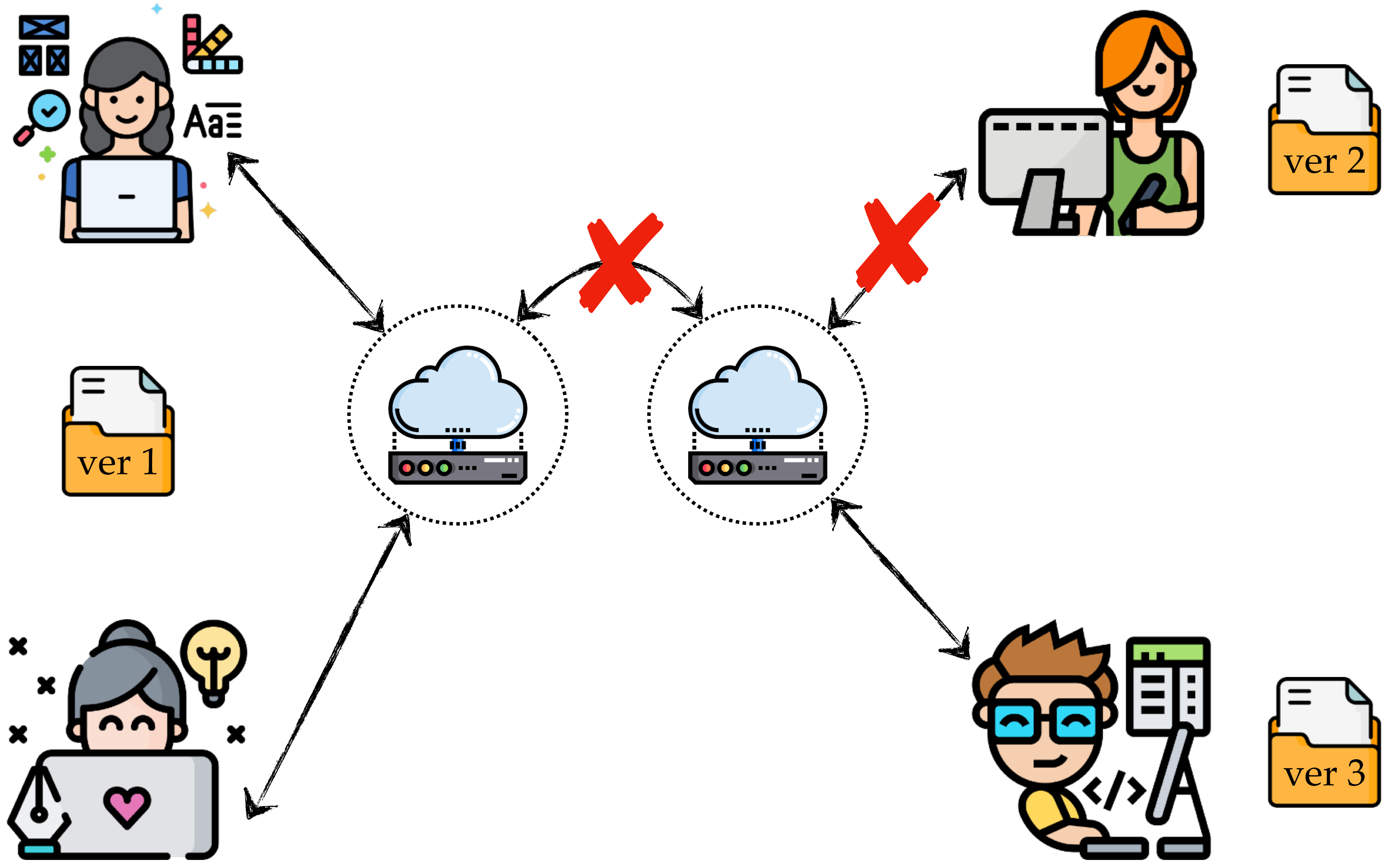

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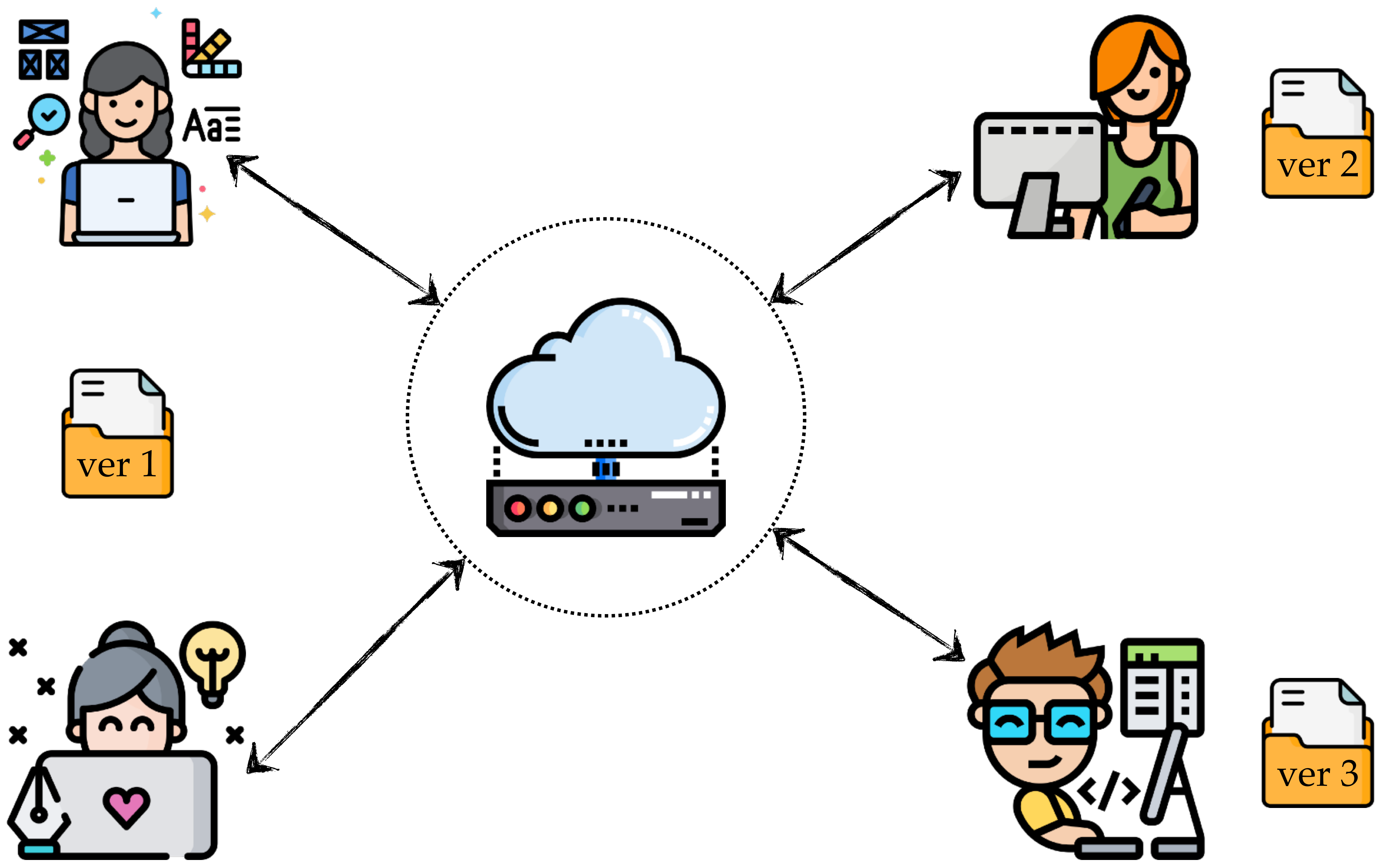

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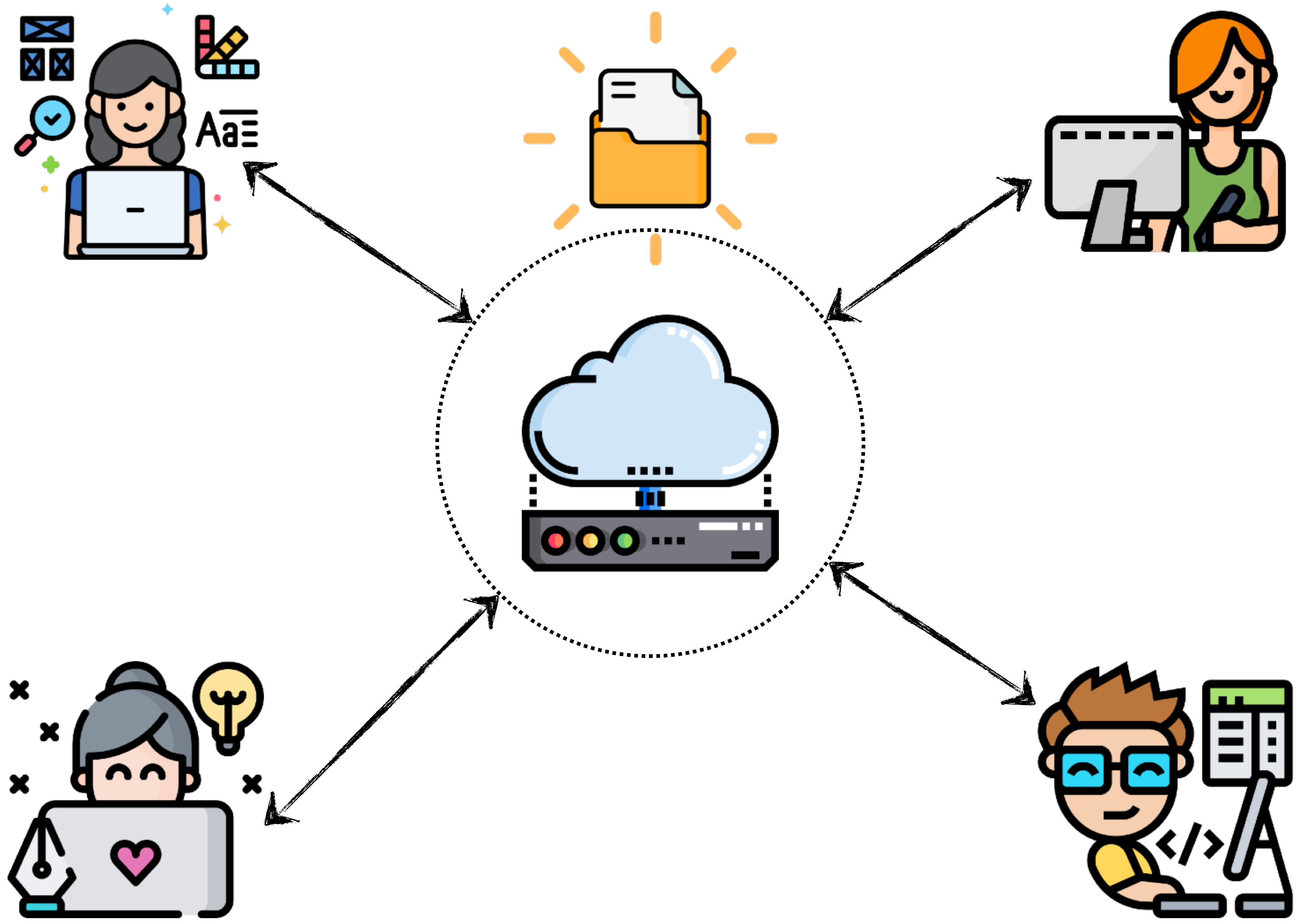

How do we build such applications?

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Embed the notion of replication into the data types

Convergent Replicated Data Types (CRDTs)

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- Known RDTs
    - Counters, Sets, Maps, Sequences/Lists, Trees, Graphs, JSON
- In practice
    - Collaborative editing — YATA/Yjs, Automerge
    - Figma (CRDT-ish), Zed Editor, …

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- MRDTs = Sequential data types + 3-way merge function à la Git

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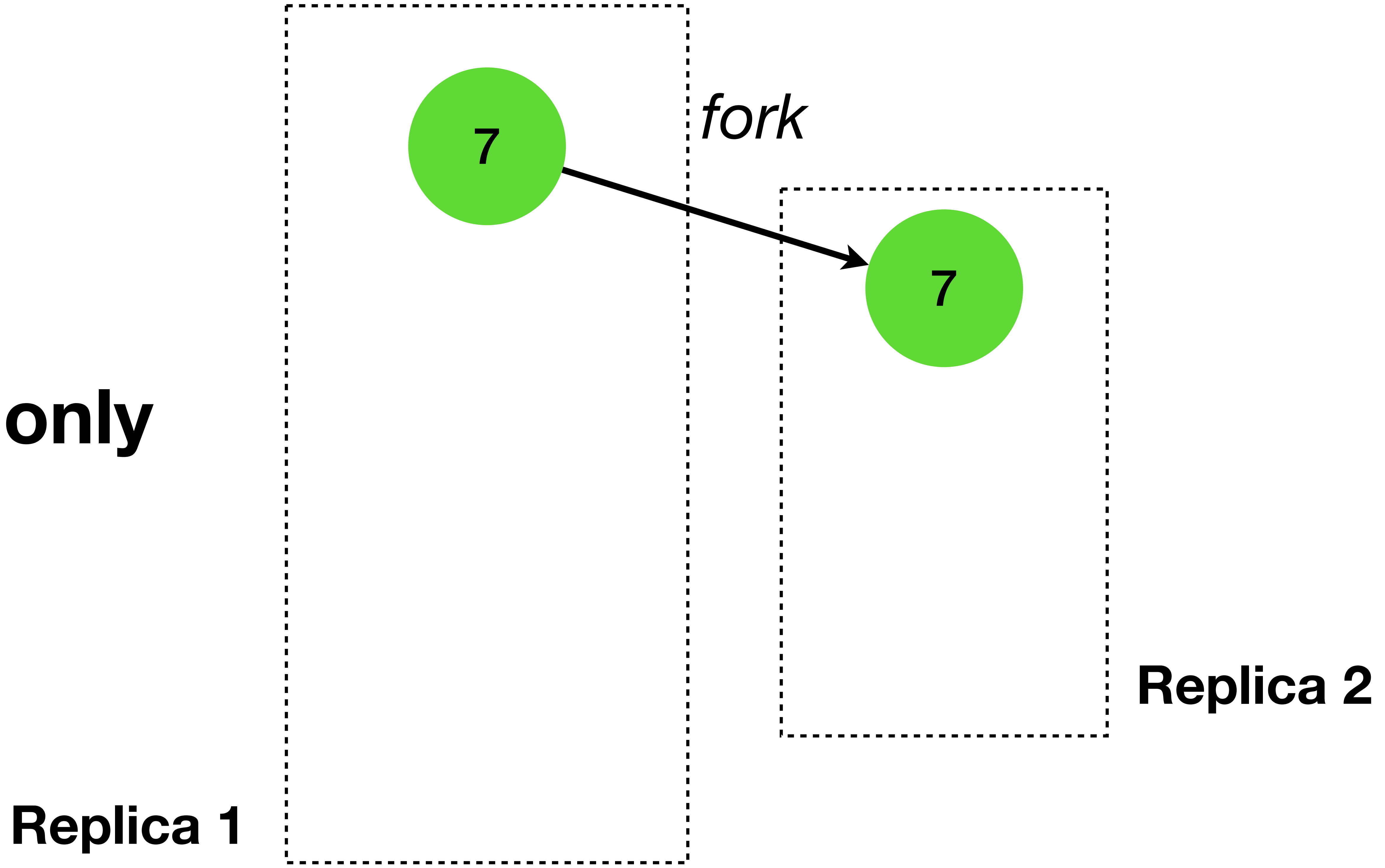

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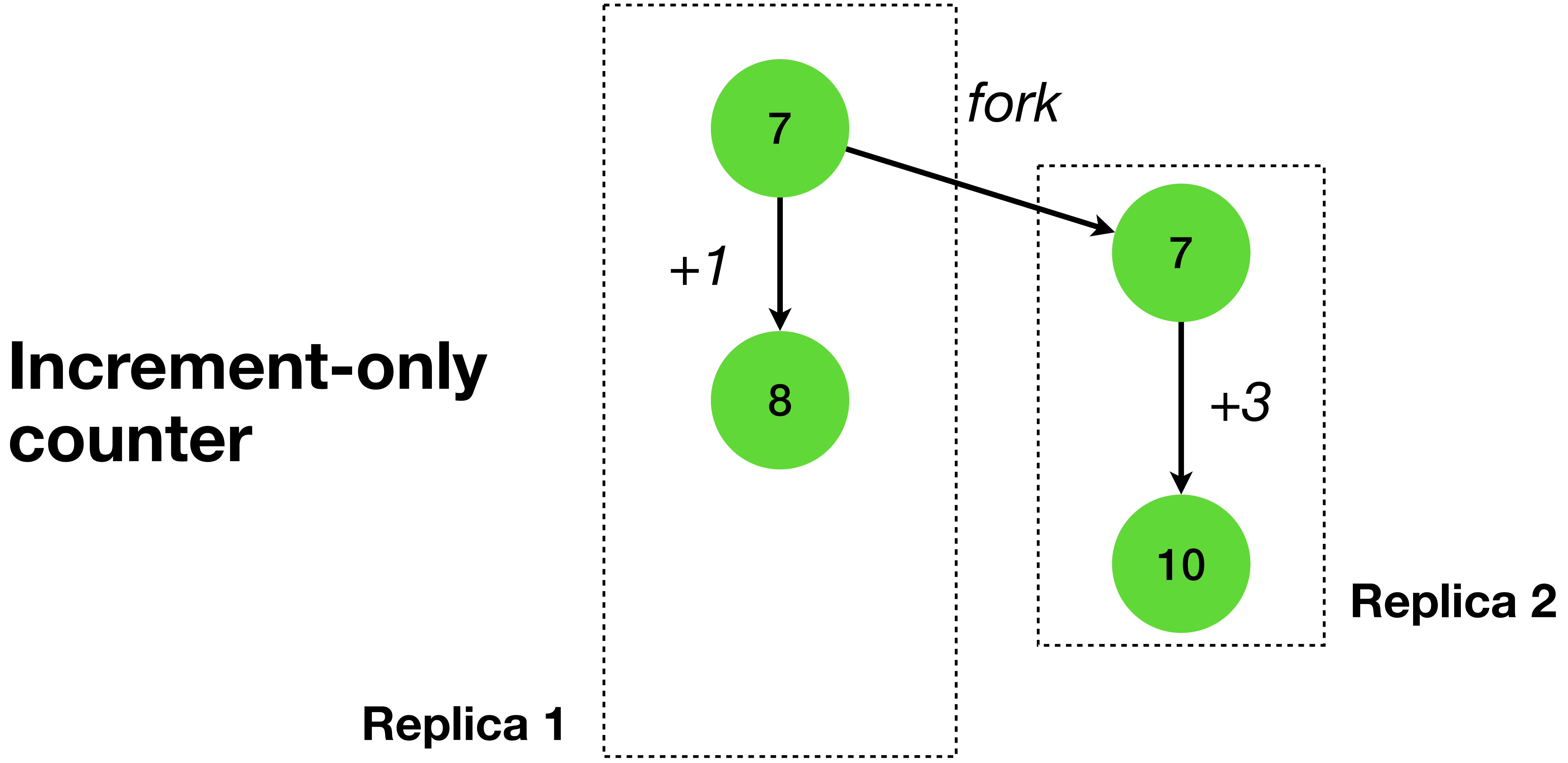

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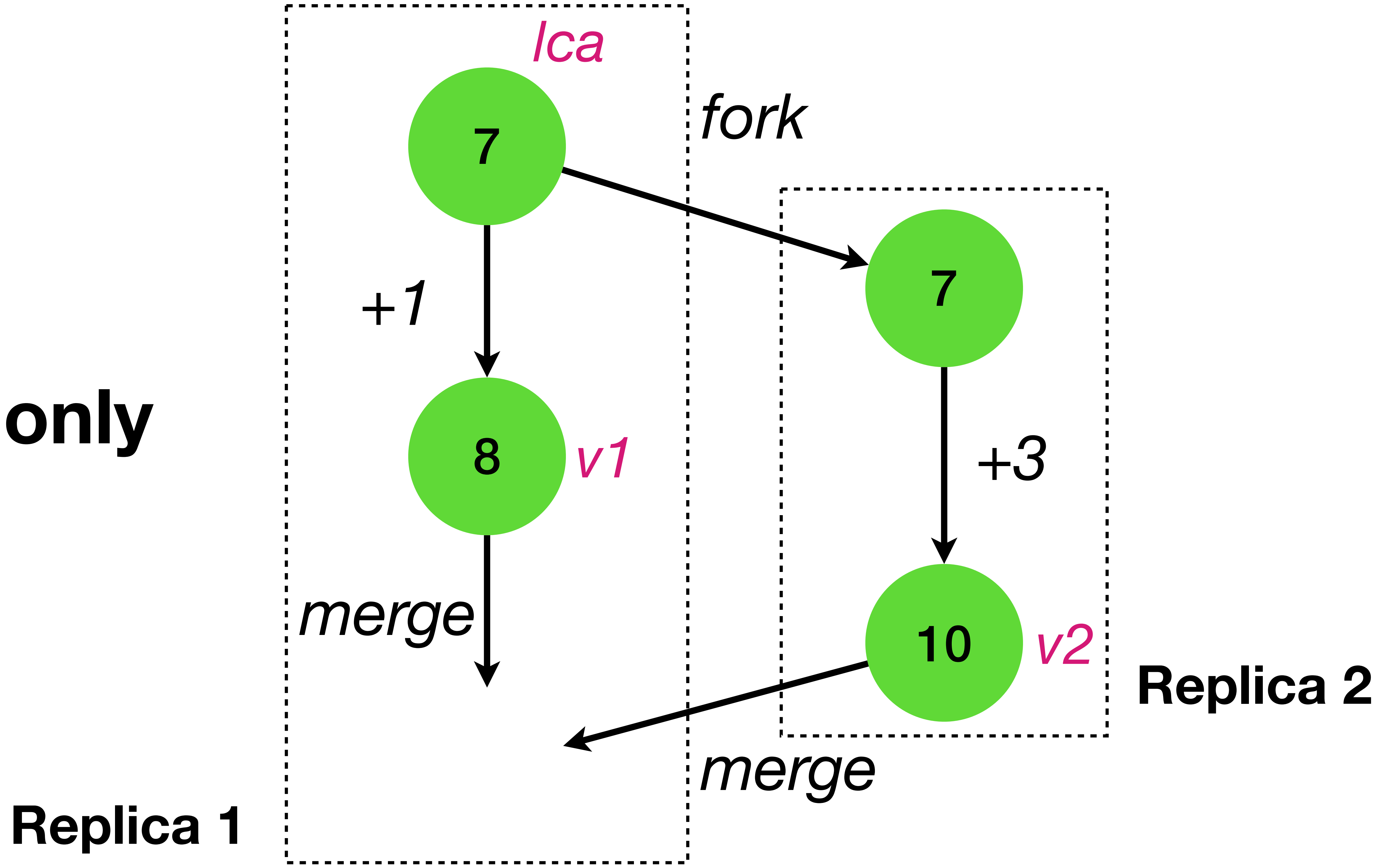


```
let merge lca v1 v2 =
  lca + (v1 - lca) + (v2 - lca)
```

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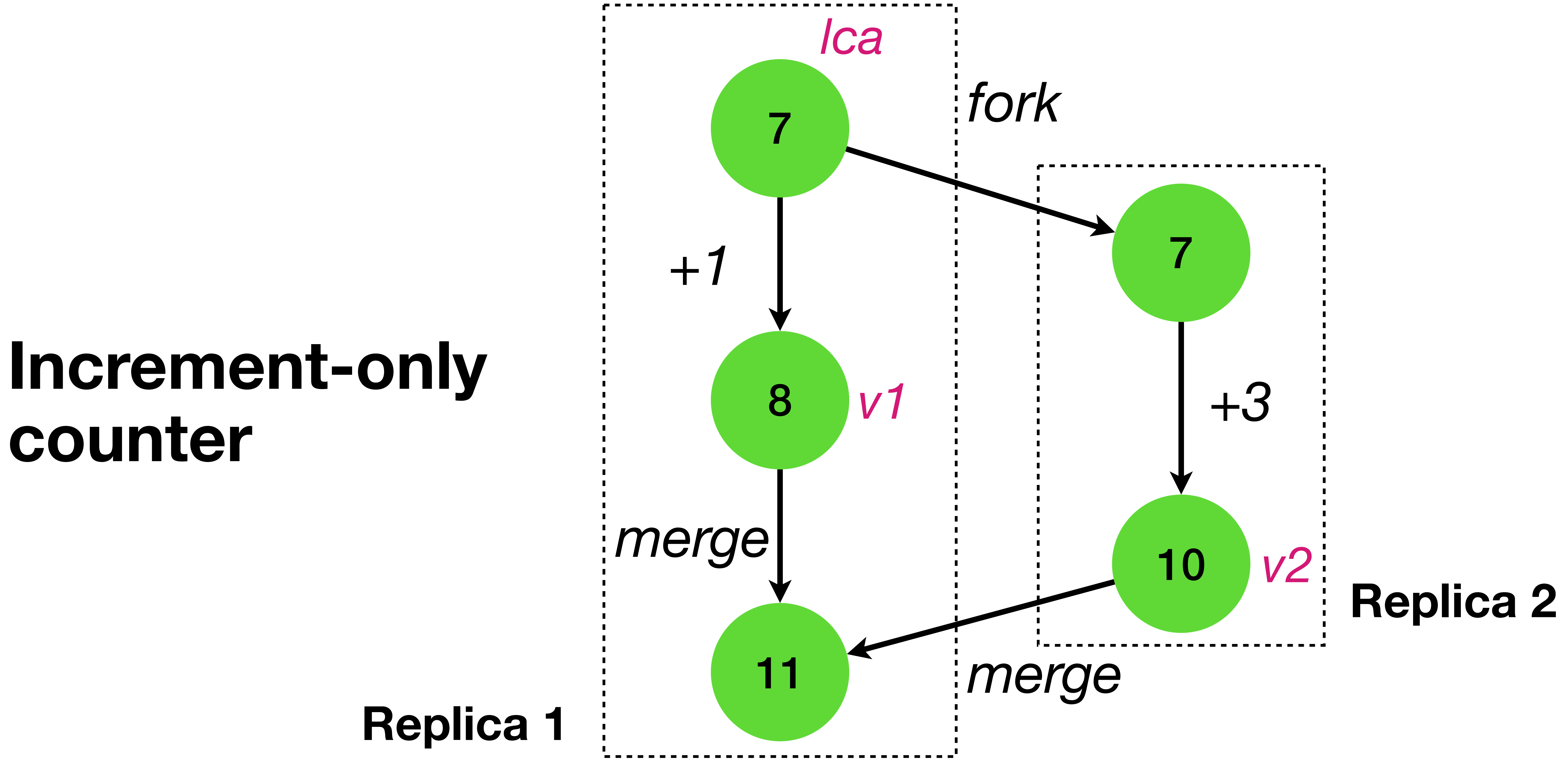


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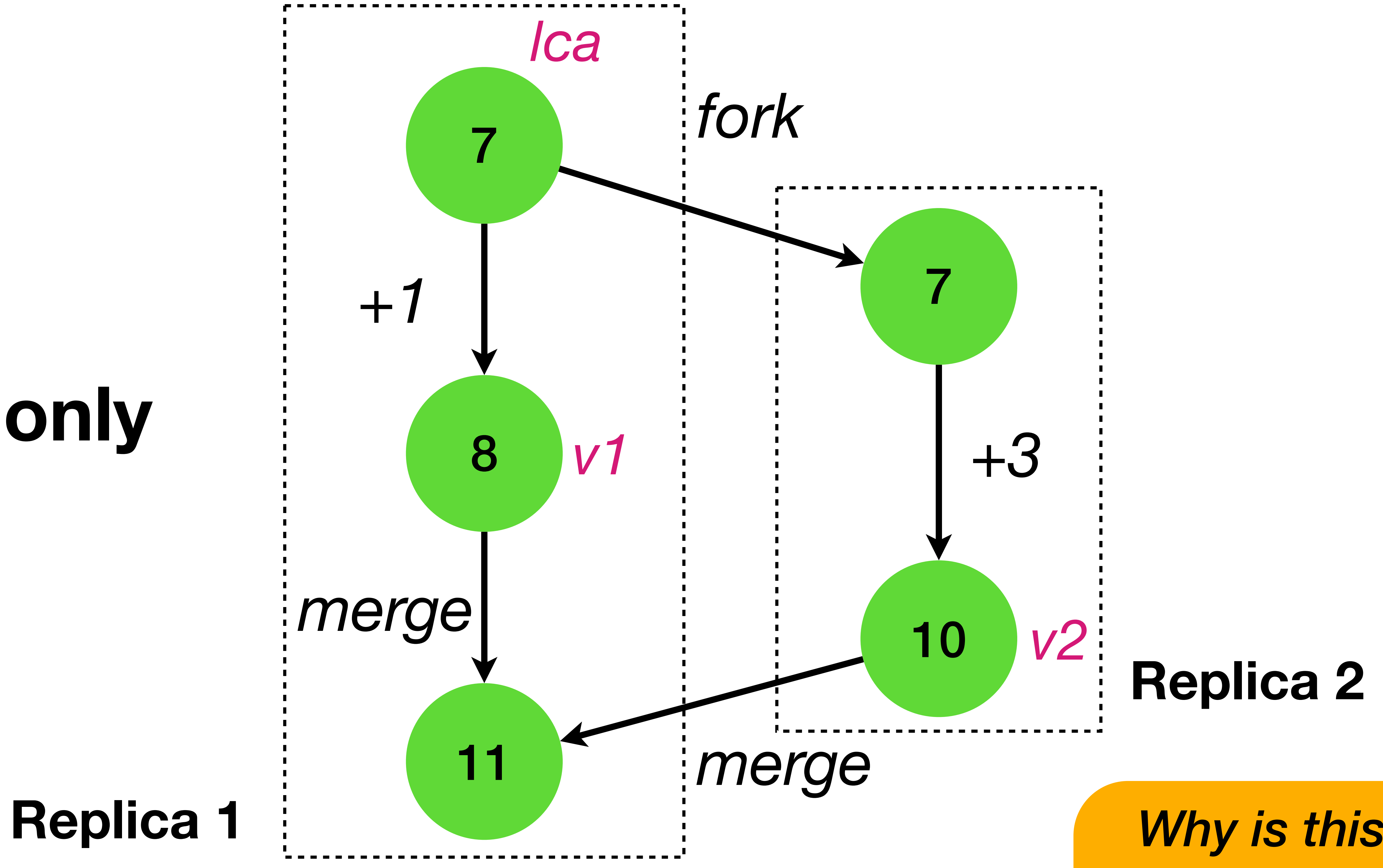


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Why is this correct?

How do we automatically verify it?

OR Set MRDT (aka Add-Wins Set)

{ (a,1), (b,2) }

OR Set MRDT (aka Add-Wins Set)

{ (a,1), (b,2) }

- Associate ***timestamp (id)*** with the element

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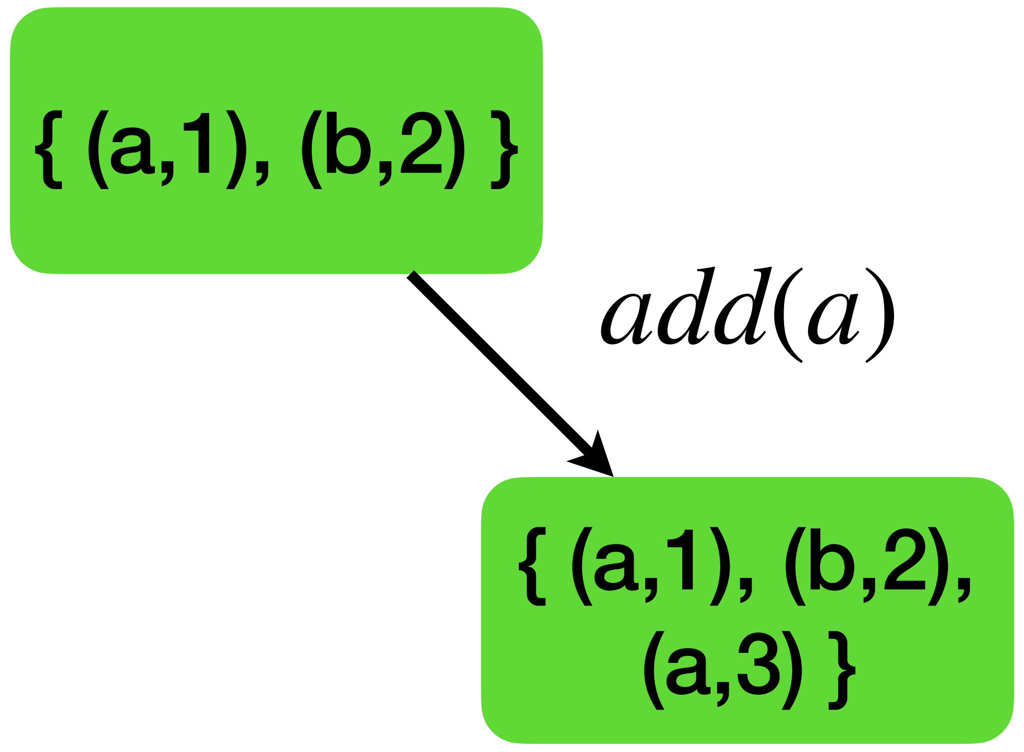

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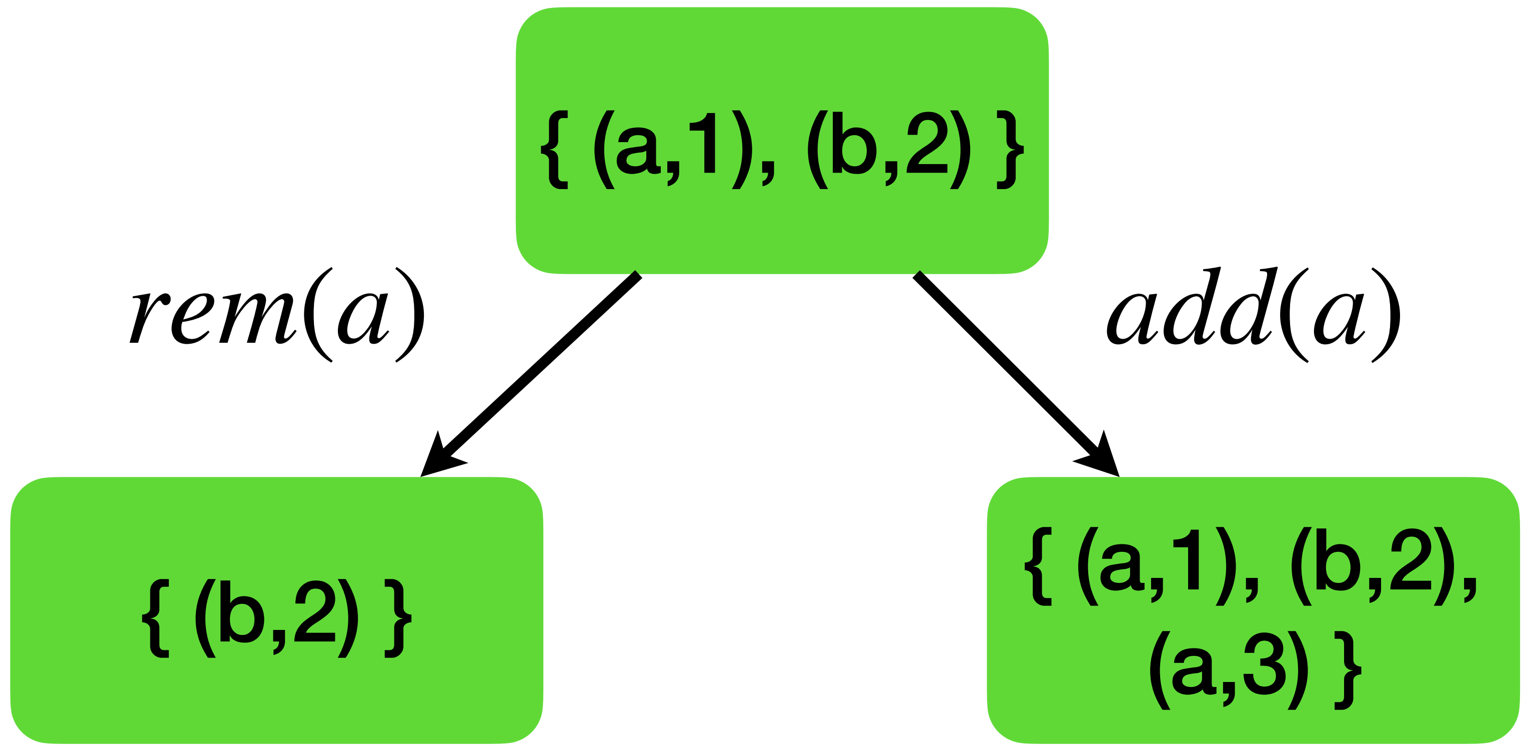

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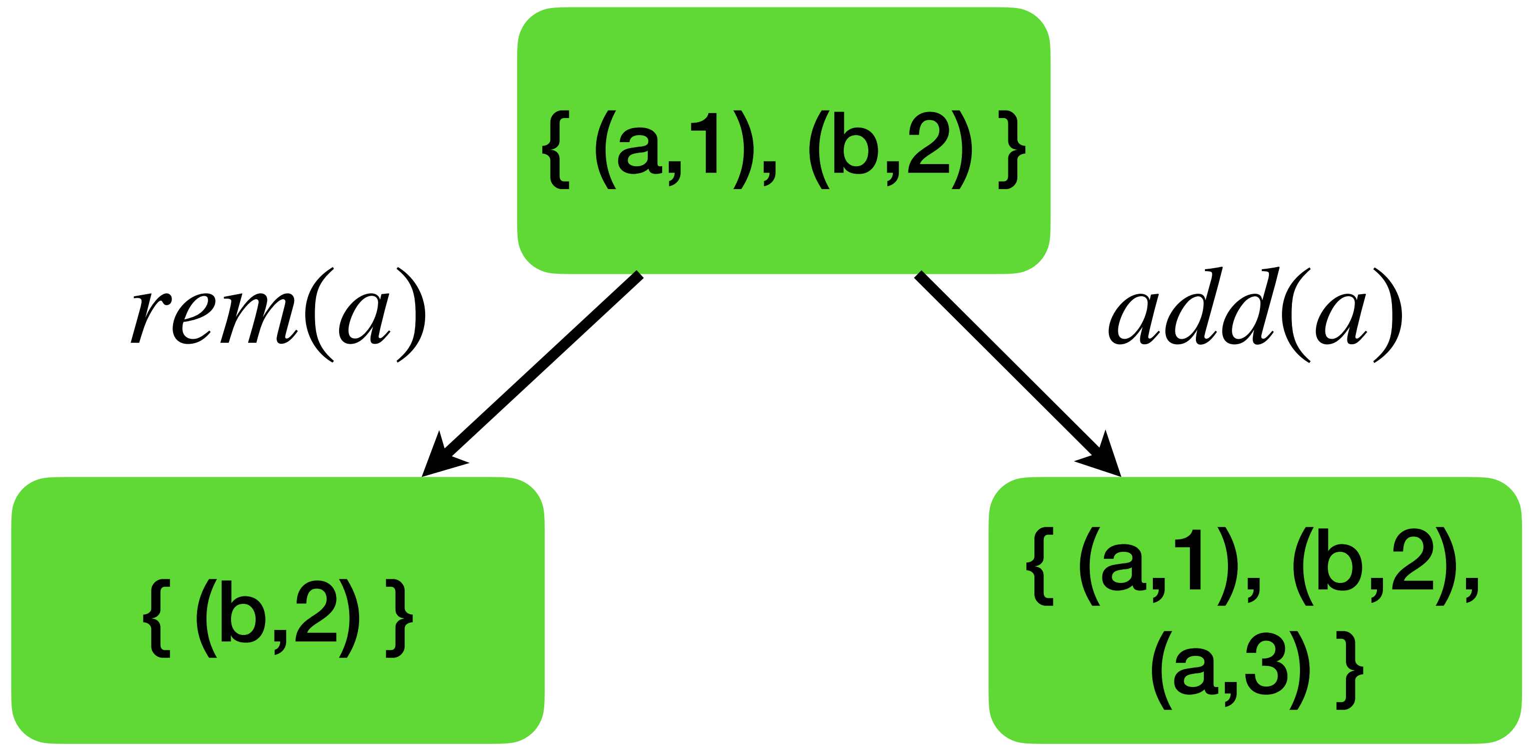

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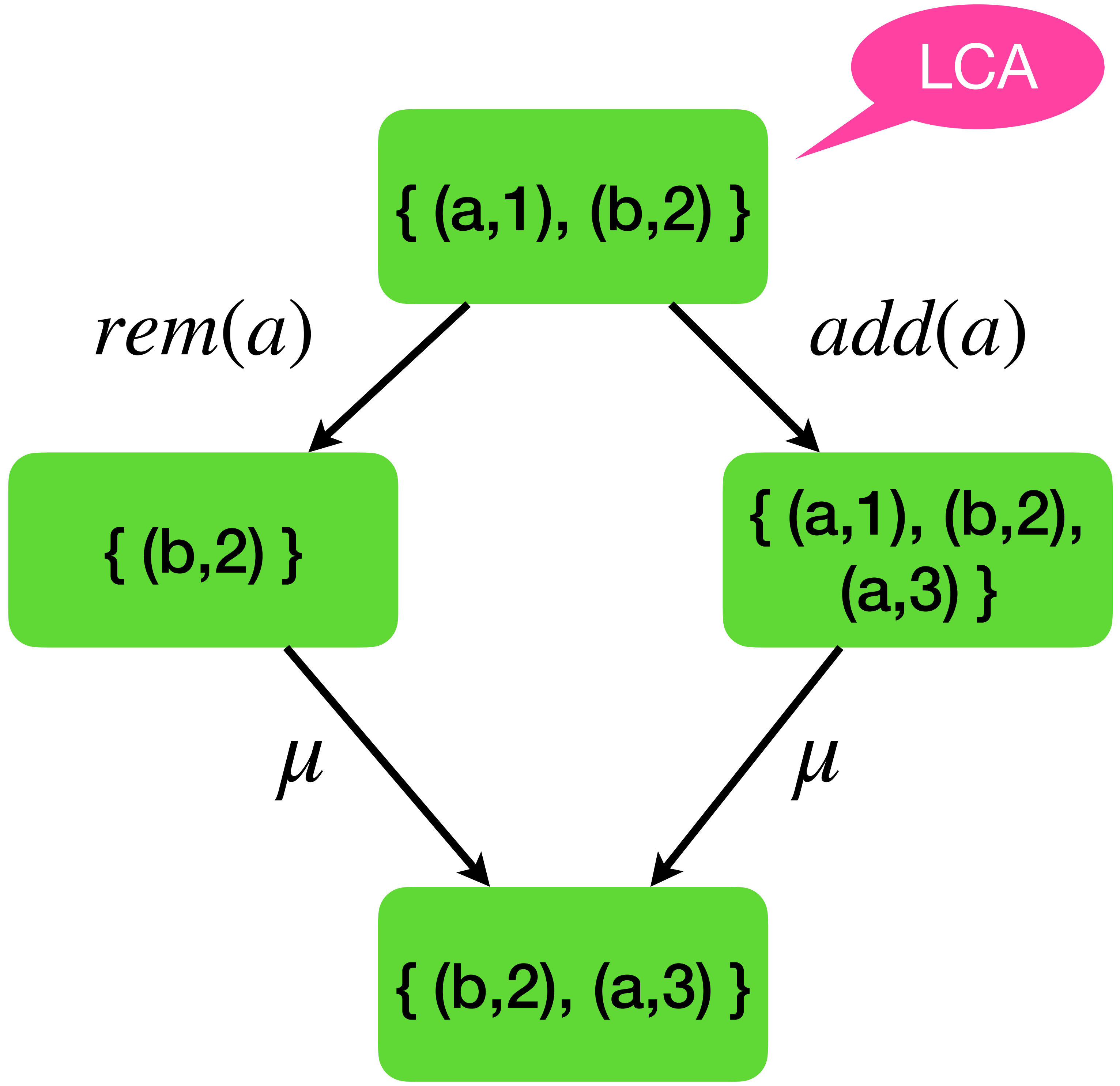

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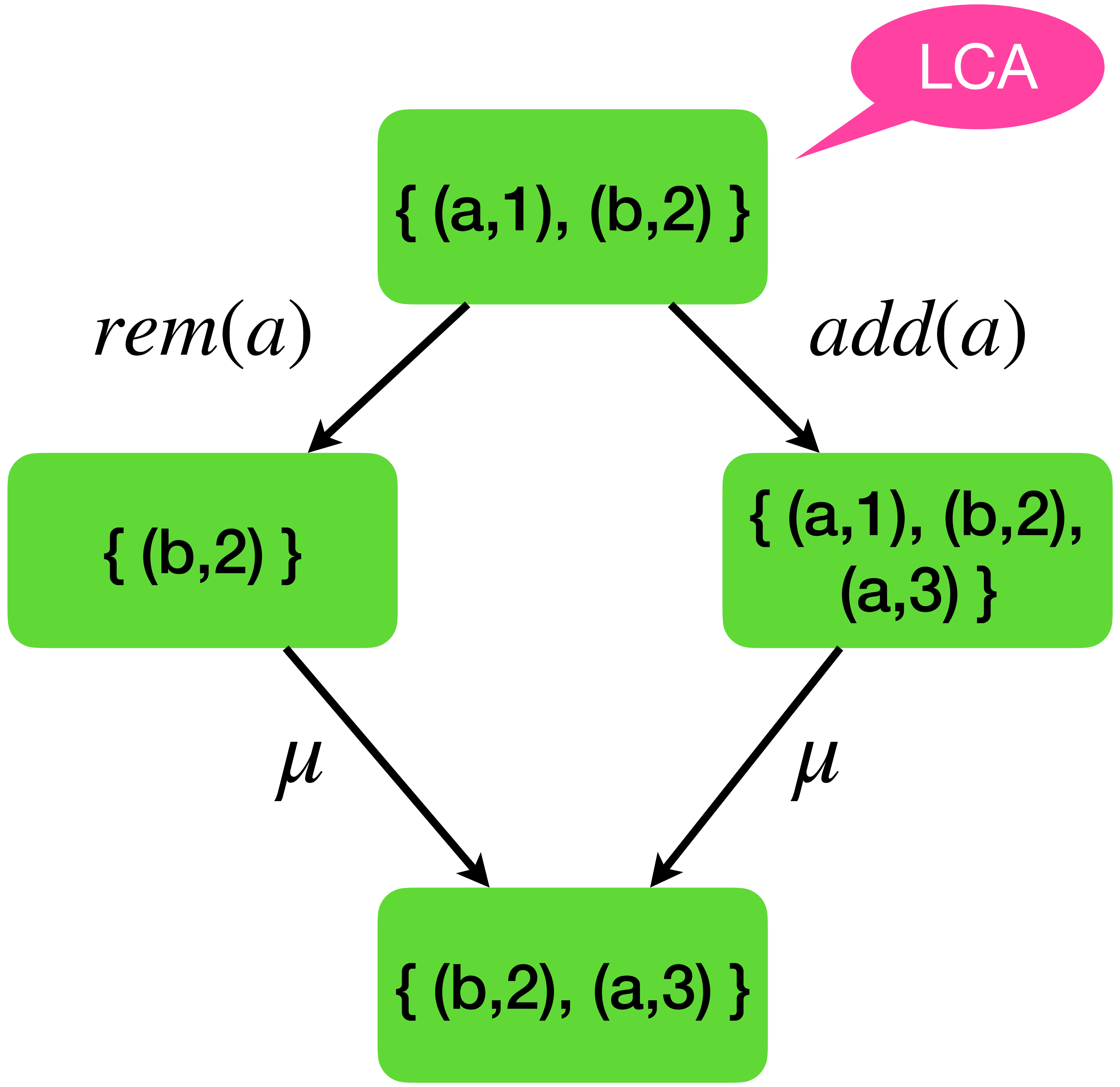

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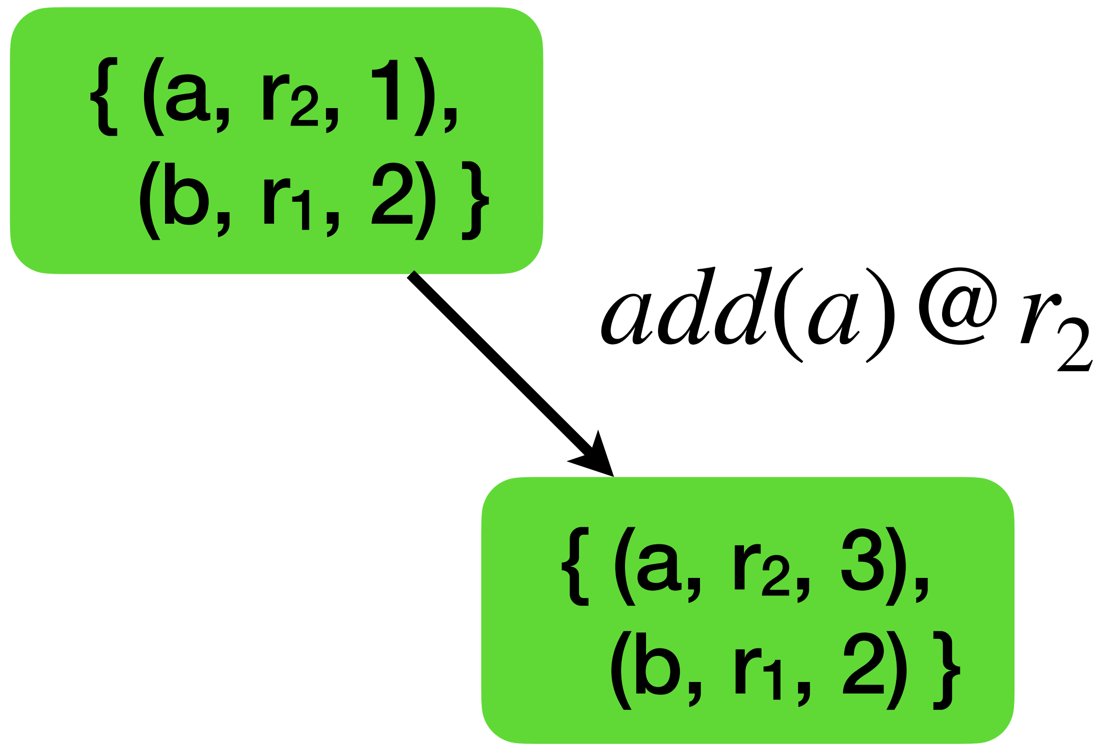
- Space usage $O(|\text{adds} \cap \text{live}|)$
  - Duplicates — same element with different ids

Efficient OR set MRDT

$\{ (a, r_2, 1),$
$(b, r_1, 2) \}$

- Elements tagged with ***replica id*** and ***timestamp***

Efficient OR set MRDT



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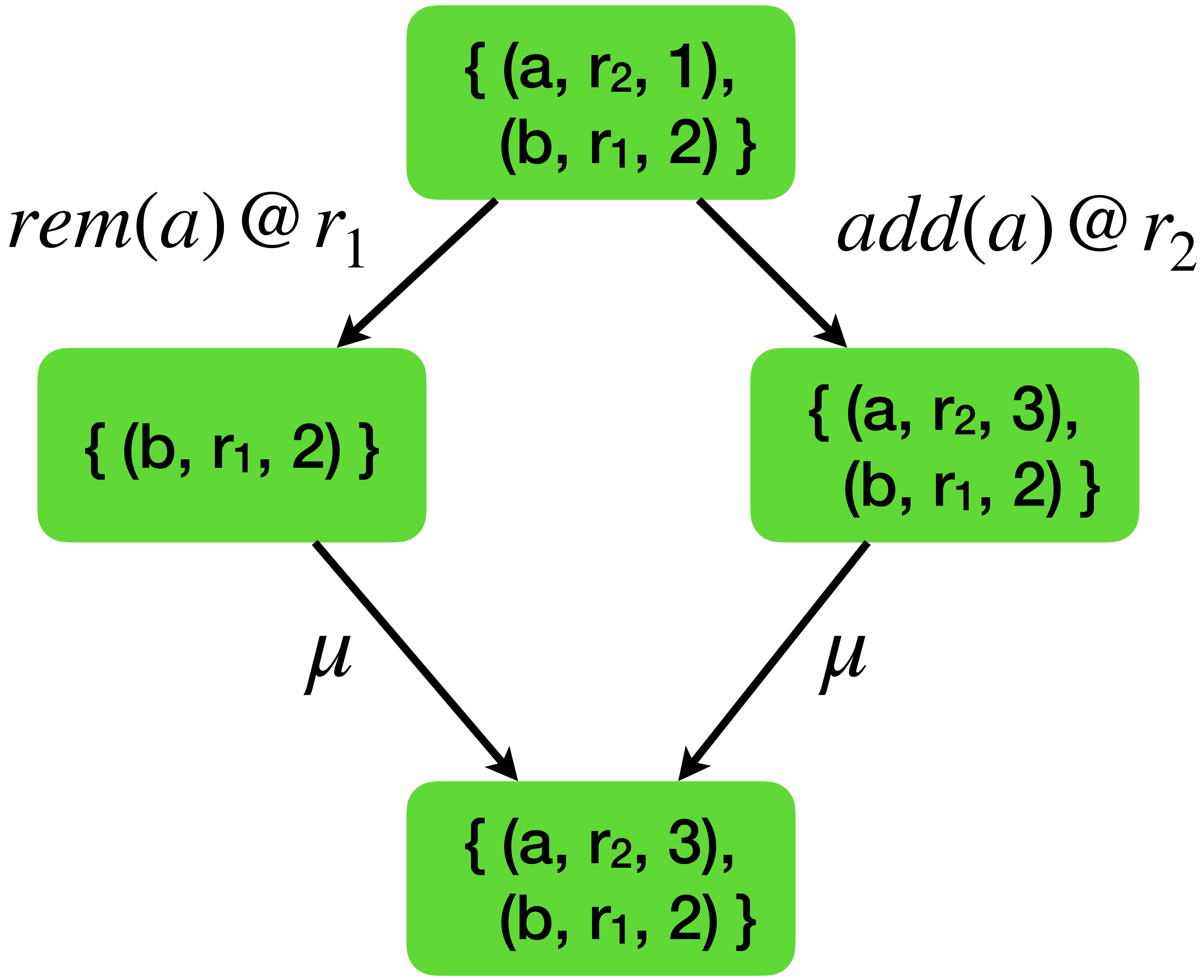

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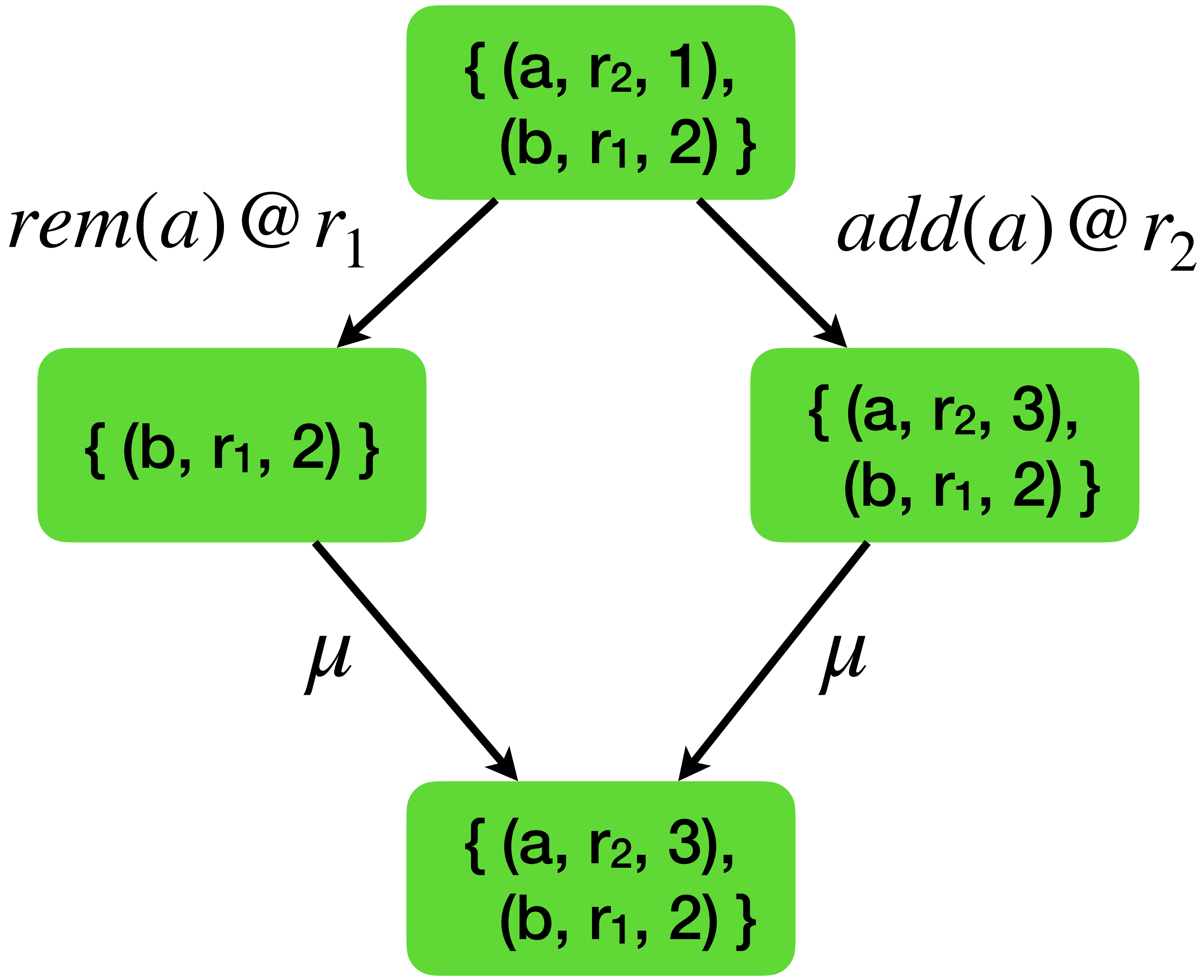

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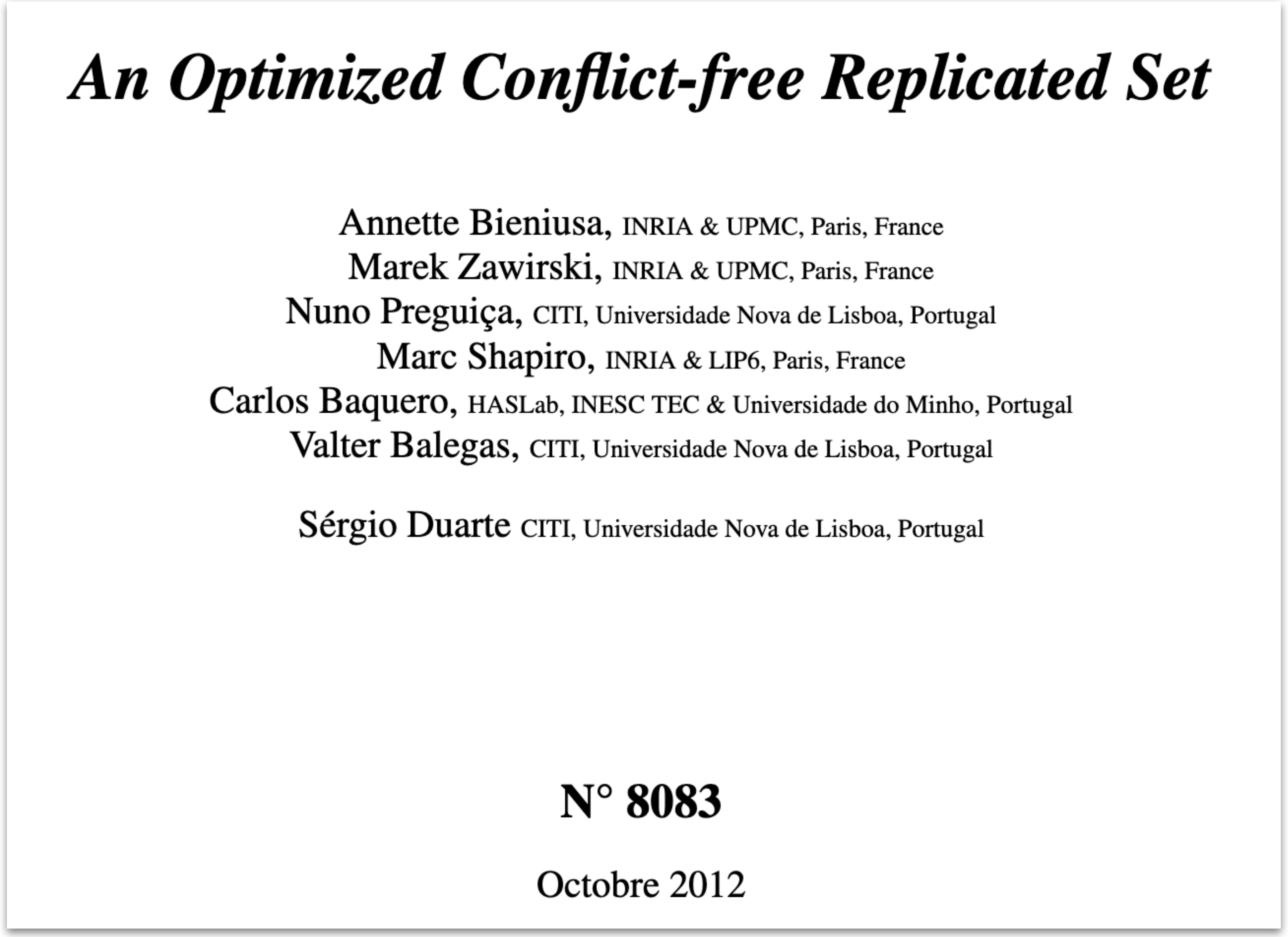
- Space usage $O(|\text{live}|.n)$
    - Duplicates — same element from different replicas

Not the first efficient OR set



An Optimized Conflict-free Replicated Set

Annette Bieniusa, INRIA & UPMC, Paris, France
Marek Zawirski, INRIA & UPMC, Paris, France
Nuno Preguiça, CITI, Universidade Nova de Lisboa, Portugal
Marc Shapiro, INRIA & LIP6, Paris, France
Carlos Baquero, HASLab, INESC TEC & Universidade do Minho, Portugal
Valter Balegas, CITI, Universidade Nova de Lisboa, Portugal

Sérgio Duarte CITI, Universidade Nova de Lisboa, Portugal

N° 8083

Octobre 2012

- Operation-based CRDT
- Maintains causality info in a version vector
- Space complexity $O(|\text{live}|.n + n)$

Not the first efficient OR set

RESEARCH-ARTICLE | FREE ACCESS

Riak DT map: a composable, convergent replicated dictionary

Authors: Russell Brown, Sean Cribbs, Christopher Meiklejohn, Sam Elliott | Authors Info & Claims

PaPEC '14: Proceedings of the First Workshop on Principles and Practice of Eventual Consistency • Article No.: 1, Page 1
https://doi.org/10.1145/2596631.2596633

Published: 13 April 2014 Publication History

Abstract

Conflict-Free Replicated Data-Types (CRDTs) [6] provide greater safety properties to eventually-consistent distributed systems without requiring synchronization. CRDTs ensure that concurrent, uncoordinated updates have deterministic outcomes via the properties of bounded join-semilattices.

- State-based CRDT in Riak

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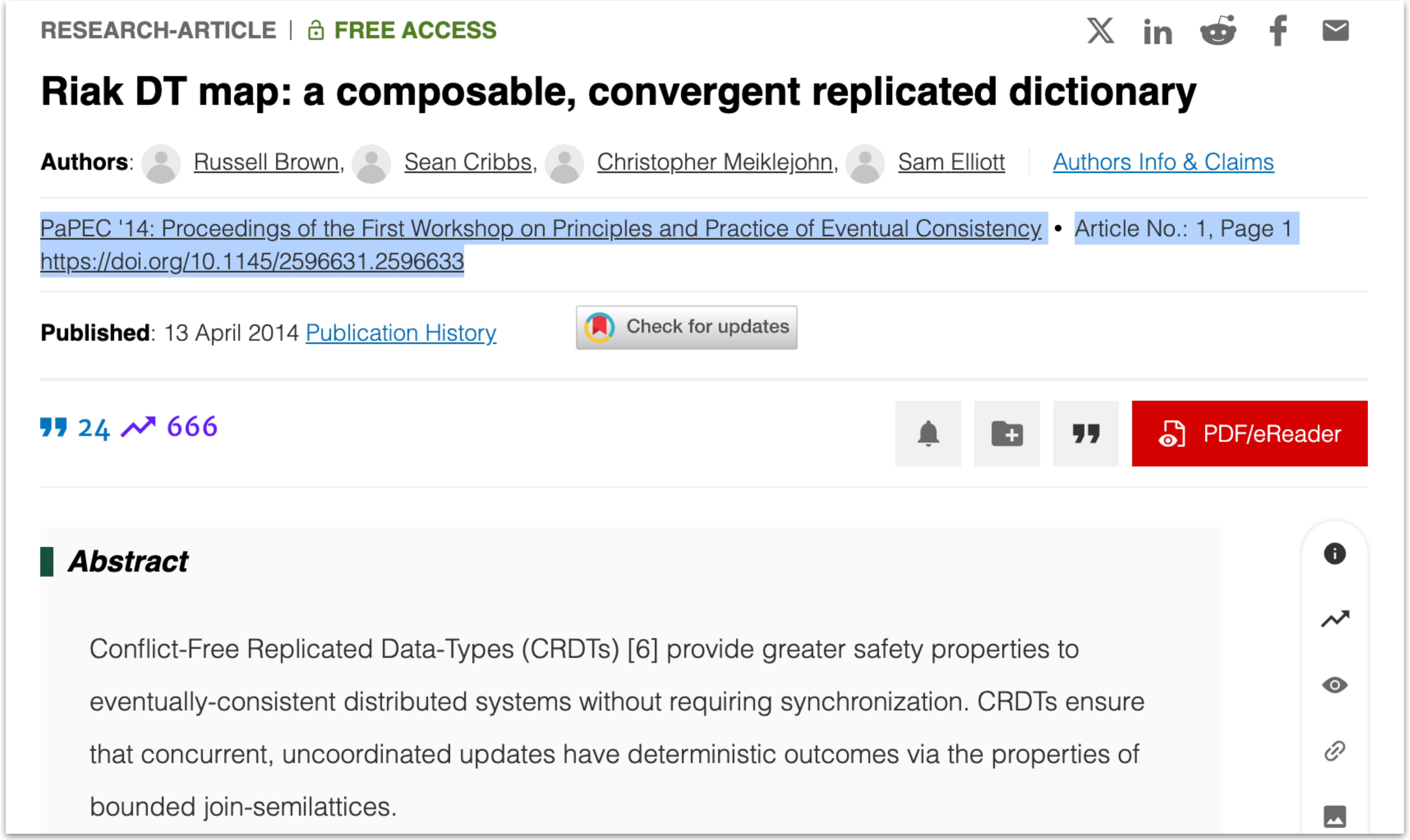
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Check for updates

24 666

PDF/eReader

Abstract

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- State-based CRDT in Riak
- ***And buggy!***

Buggy Riak DT Map

Unsound Optimizations

Implementing CRDTs can be quite challenging. Not only does the developer who is implementing CRDTs must ensure that updates to lattices are *inflationary* – ensuring the data structure is always moving up the lattice with respect to each change – but also that the merge function must be both *deterministic* and compute the *least-upper-bound.* We have even got this wrong a few times ourselves.

https://christophermeiklejohn.com/erlang/lasp/2019/03/08/monotonicity.html

Buggy Riak DT Map

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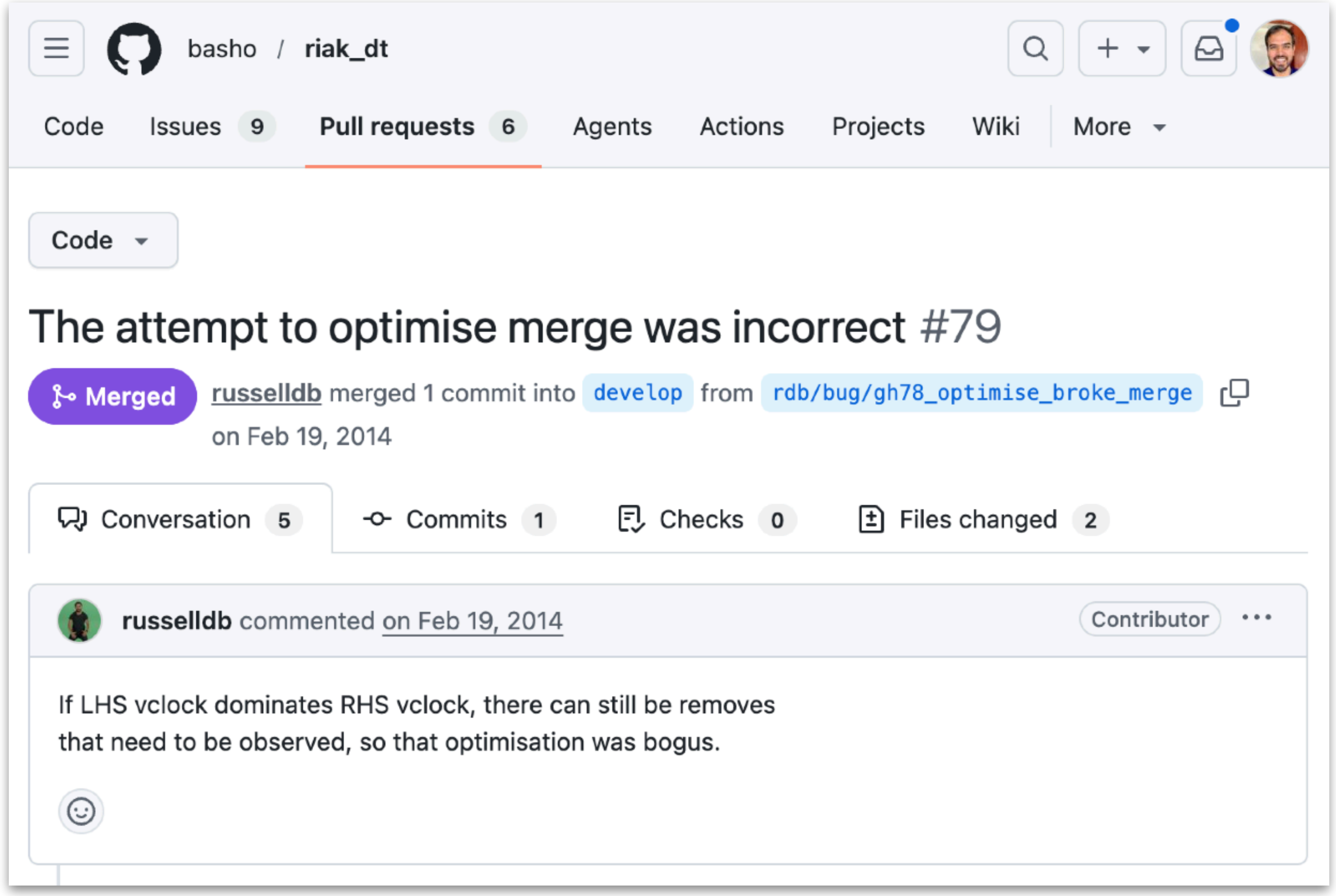

Not an isolated incident

Martin Kleppmann

Student Projects About/Contact Supporters

Interleaving anomalies in collaborative text editors

Martin Kleppmann, Victor B. F. Gomes, Dominic P. Mulligan, and Alastair R. Beresford

6th Workshop on Principles and Practice of Consistency for Distributed Data (PaPoC), Dresden, Germany, March 2019.

- Paper (open access)
- Paper (version of record)
- doi:10.1145/3301419.3323972
- Slides

Errata

A few years after publication, we learned about some serious flaws in this paper:

- The definition of non-interleaving in Section 2.1 cannot be guaranteed by any algorithm.
- The algorithm presented in Section 3.1 does not guarantee convergence.

Please see the Fugue paper for more details.

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- The goal is to ***verify*** the correctness of RDTs

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  - Use SMT-solvers, proof-oriented PLs, proof-assistants

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RESEARCH-ARTICLE | OPEN ACCESS

Certified mergeable replicated data types

Peepul

Authors: Vimala Soundarapandian, Adharsh Kamath, Kartik Nagar, KC Sivaramakrishnan | Authors Info & Claims

PLDI 2022: Proceedings of the 43rd ACM SIGPLAN International Conference on Programming Language Design and Implementation
Pages 332 - 347 • https://doi.org/10.1145/3519939.3523735

RESEARCH-ARTICLE | OPEN ACCESS |

Automatically Verifying Replication-Aware Linearizability

Authors: Vimala Soundarapandian, Kartik Nagar, Aseem Rastogi, KC Sivaramakrishnan | Authors Info & Claims

Proceedings of the ACM on Programming Languages, Volume 9, Issue OOPSLA1
Article No.: 111, Pages 871 - 897 • https://doi.org/10.1145/3720452

Published: 09 April 2025 Publication History | Check for updates

Neem

SAL: Multi-modal Verification of Replicated Data Types

Pranav Ramesh, Indian Institute of Technology, Madras, Chennai, India, cs22b015@smail.iitm.ac.in
Vimala Soundarapandian, Indian Institute of Technology, Madras, Chennai, India, cs19d750@smail.iitm.ac.in
KC Sivaramakrishnan, Indian Institute of Technology, Madras, Chennai, India, kcsrk@cse.iitm.ac.in

Abstract

Designing correct replicated data types (RDTs) is challenging because replicas evolve independently and must be merged while preserving application intent. A promising approach is correct-by-construction development in a proof-oriented programming language such as F*, Dafny and Lean, where

and Mergeable Replicated Data Types (MRDTs) [9] are widely used to implement such replicated state. However, designing correct RDTs is subtle. Even well-known designs such as Replicated Growable Arrays (RGA) [10] have had serious anomalies discovered after publication[1].

Replication-aware (RA) linearizability [21] provides a principled correctness condition for CRDTs, relating replica exe-

Convergence alone is insufficient

Merge function μ

$\mu(a, b) = \mu(b, a)$	Commutativity
$\mu(a, a) = a$	Idempotence
$\mu(\mu(a, b), c) = \mu(a, \mu(b, c))$	Associativity

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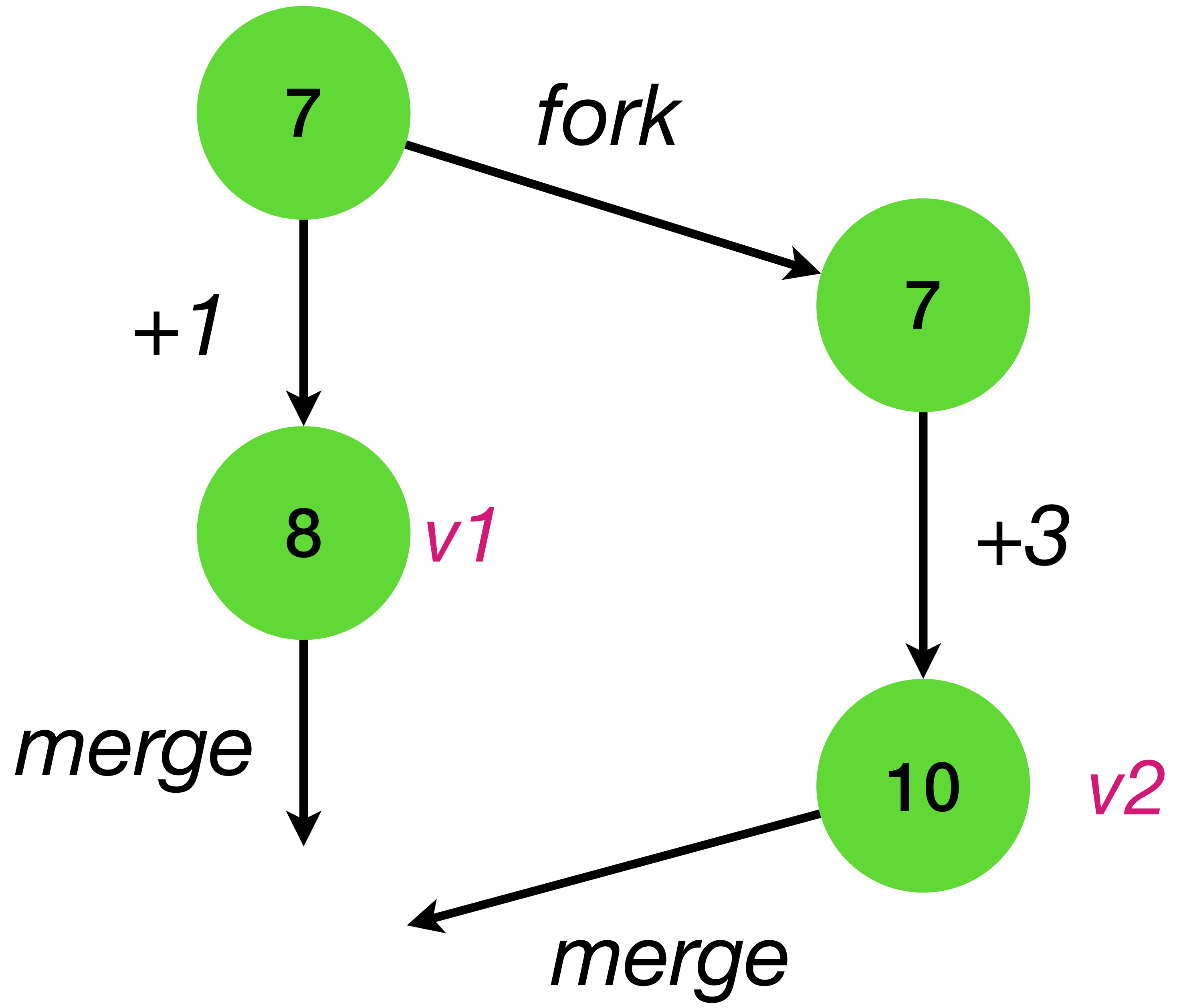

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Satisfies algebraic properties

```
let merge v1 v2 = max v1 v2
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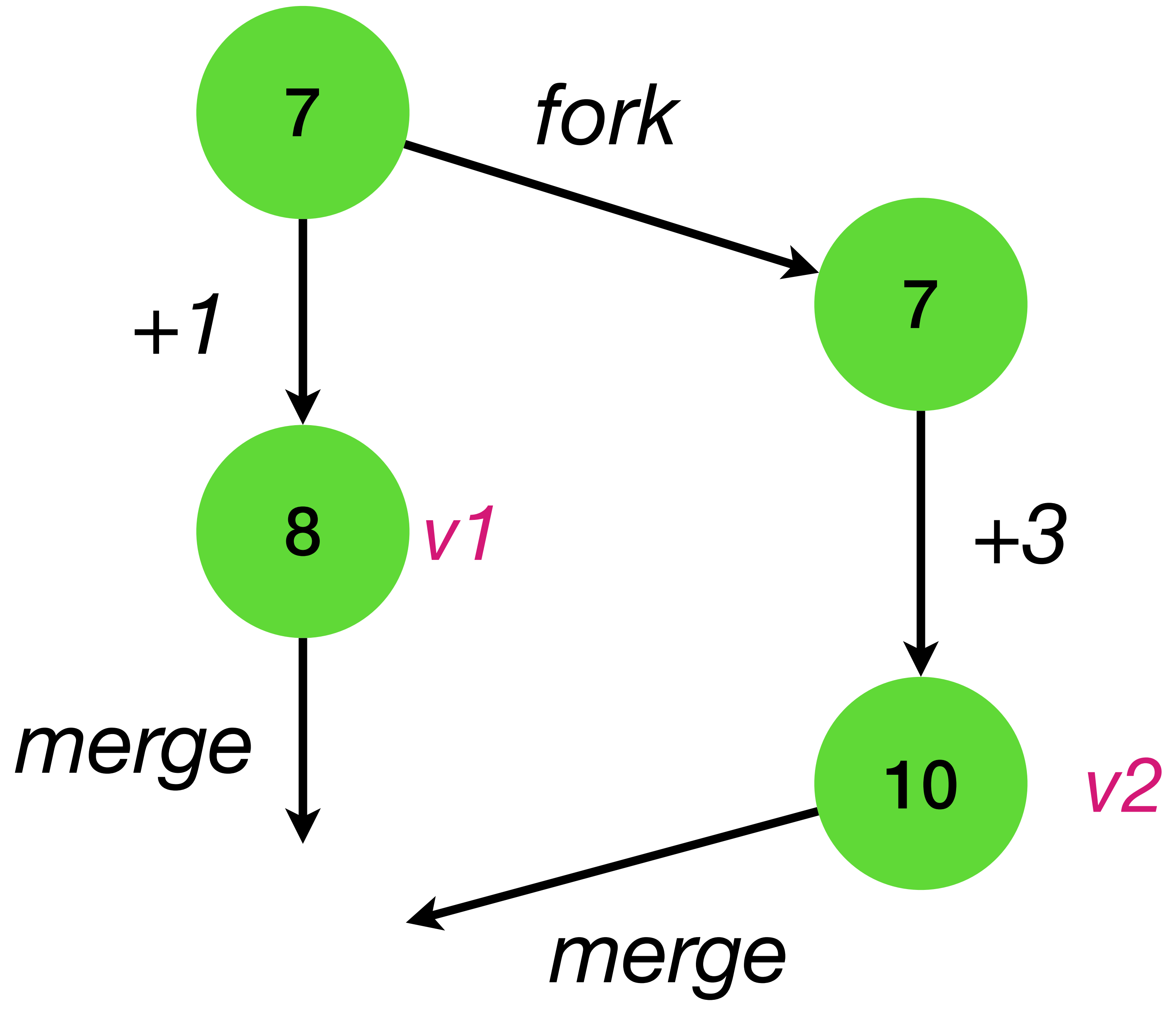

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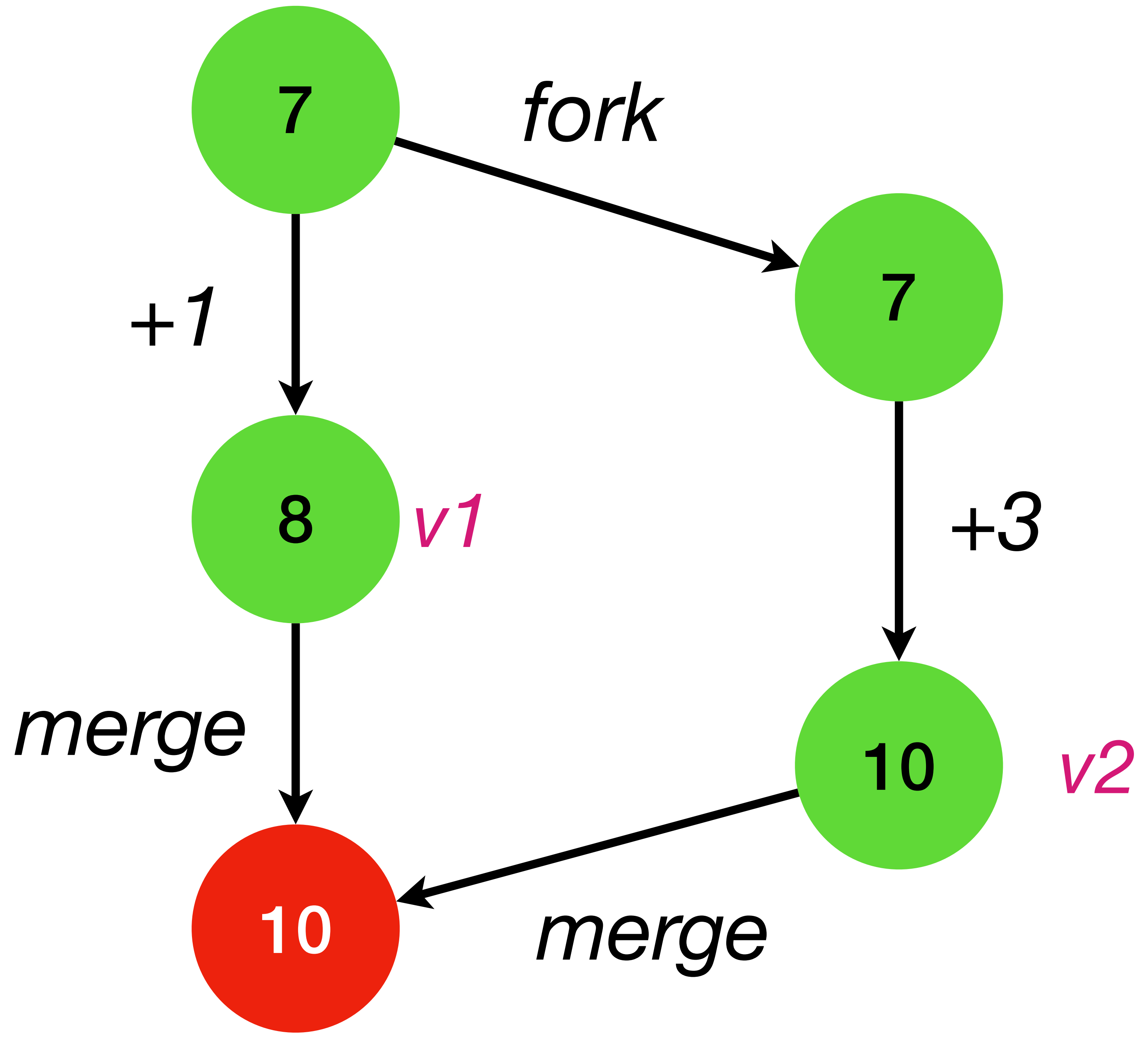
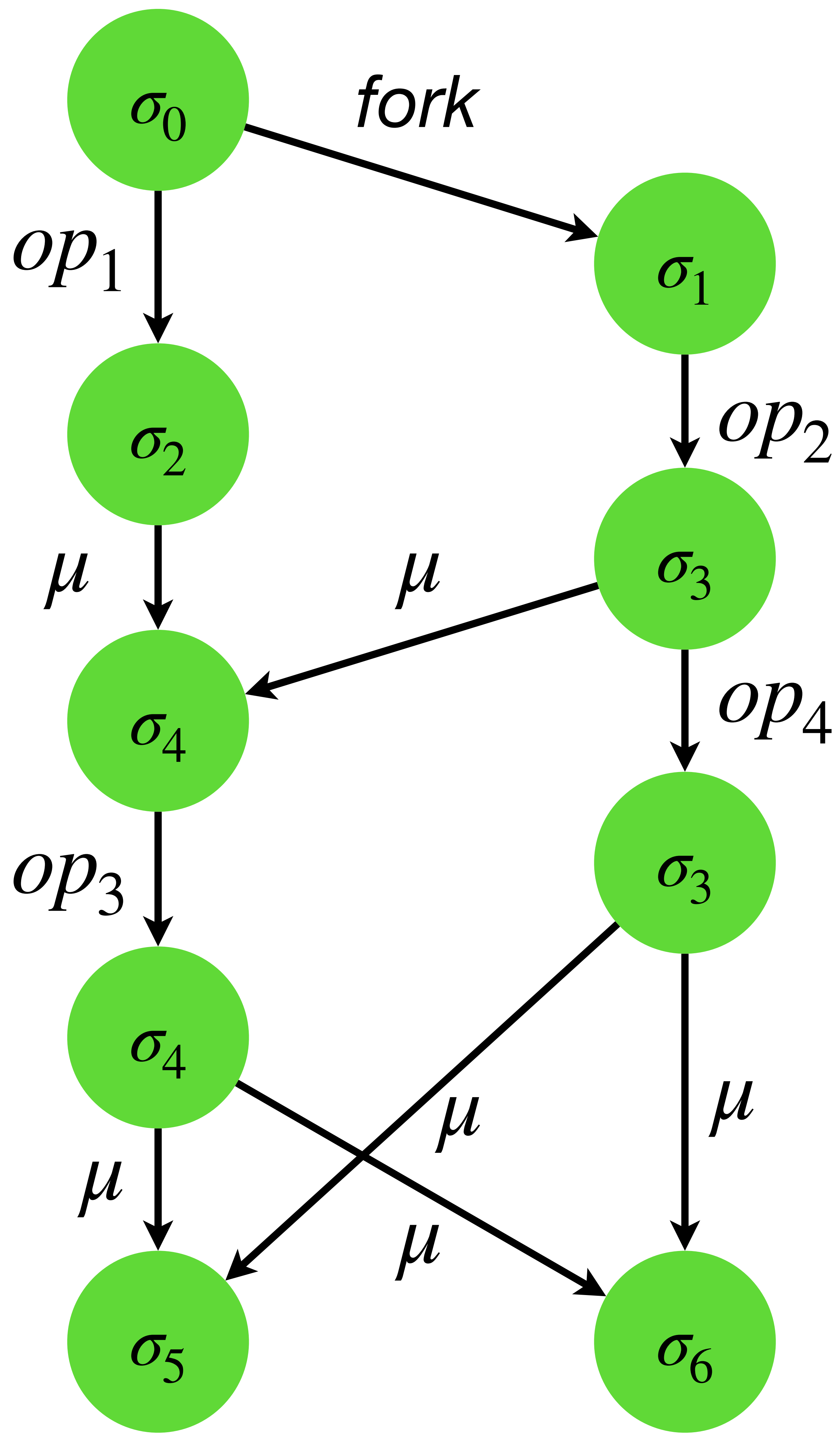
Intent is not captured



Is there a better spec?



$$\sigma_5 = \sigma_6 = \text{linearization}(\{op_1, op_2, op_3, op_4\})\ \sigma_0$$

Replication-aware Linearizability

RESEARCH-ARTICLE

Replication-aware linearizability

Authors: Chao Wang, Constantin Enea, Suha Orhun Mutluergil, Gustavo Petri | Authors Info & Clai

PLDI 2019: Proceedings of the 40th ACM SIGPLAN Conference on Programming Language Design and Implementation
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- Replica states should be a ***linearisation*** of observed ***update*** operations
  - Linearisation total order lo compatible with partially-ordered visibility relation vis
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- Replica states should be a ***linearisation*** of observed ***update*** operations
  - Linearisation total order lo compatible with partially-ordered visibility relation vis
  - No real-time ordering requirement unlike traditional linearizability
- Payoff
  - If a replicated object is RA-linearizable, reason about it using ***sequential semantics***

Using RA-linearizability for verification

`add(a);`	`add(a);`
`rem(a);`	
`X = read();`	`Y = read();`

$$a \in X \implies a \in Y$$

Using RA-linearizability for verification

```
add(a);          ||  add(a);
rem(a);          ||
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- Since Add-wins set is RA-linearizable, you can use ***totally ordered trace*** and the ***sequential reasoning***

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`add(a);`	`rem(a);`	`add(a);`	`X = read();`	`Y = read()`
{a}	{}	{a}	X = {a}	Y = {a}

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- Let's prove by contradiction
  - Make $a \in X$ true and $a \in Y$ false

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- Presented a proof methodology to show that a CRDT is linearisable
- Not automated or mechanised

Automatic verification of RDTs

- Definition of RA-linearizability for MRDTs
- A novel induction scheme for MRDTs and state-based CRDTs to ***automatically*** verify RA-linearizability
- Implemented in F* and ***Lean***

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Automatically Verifying Replication-Aware Linearizability

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Neem

SAL: Multi-modal Verification of Replicated Data Types

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Abstract

Designing correct replicated data types (RDTs) is challenging because replicas evolve independently and must be merged while preserving application intent. A promising approach is correct-by-construction development in a proof-oriented programming language such as F*, Dafny and Lean, where

and Mergeable Replicated Data Types (MRDTs) [9] are widely used to implement such replicated state. However, designing correct RDTs is subtle. Even well-known designs such as Replicated Growable Arrays (RGA) [10] have had serious anomalies discovered after publication[1].

Replication-aware (RA) linearizability [21] provides a principled correctness condition for CRDTs, relating replica exe-

Specifying RDTs in Neem

- Not all operations commute
  - **Add-wins set** — `add(a)` and `rem(a)` do not commute
  - Specify ordering using the **Conflict Resolution** relation $rc = \{(rem_a, add_a) \,|\, a \in \mathbb{E}\}$
  - Linearization order lo must be compatible with rc for concurrent events

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  - Linearization order lo must be compatible with rc for concurrent events
- Neem developers provide
  - MRDT = Sequential Data Type + 3-way merge
  - Conflict Resolution rc relation

Add-wins Set

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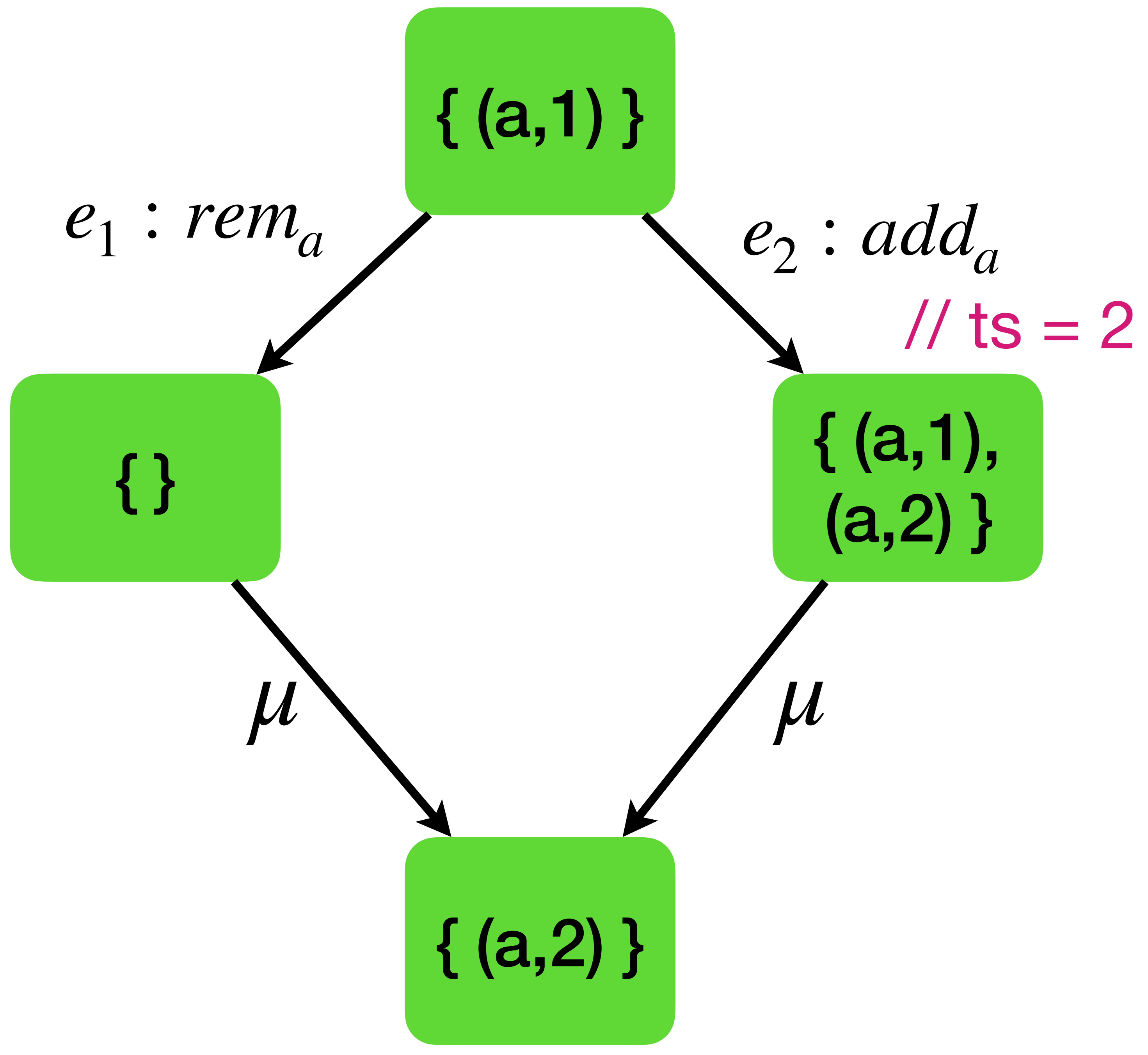

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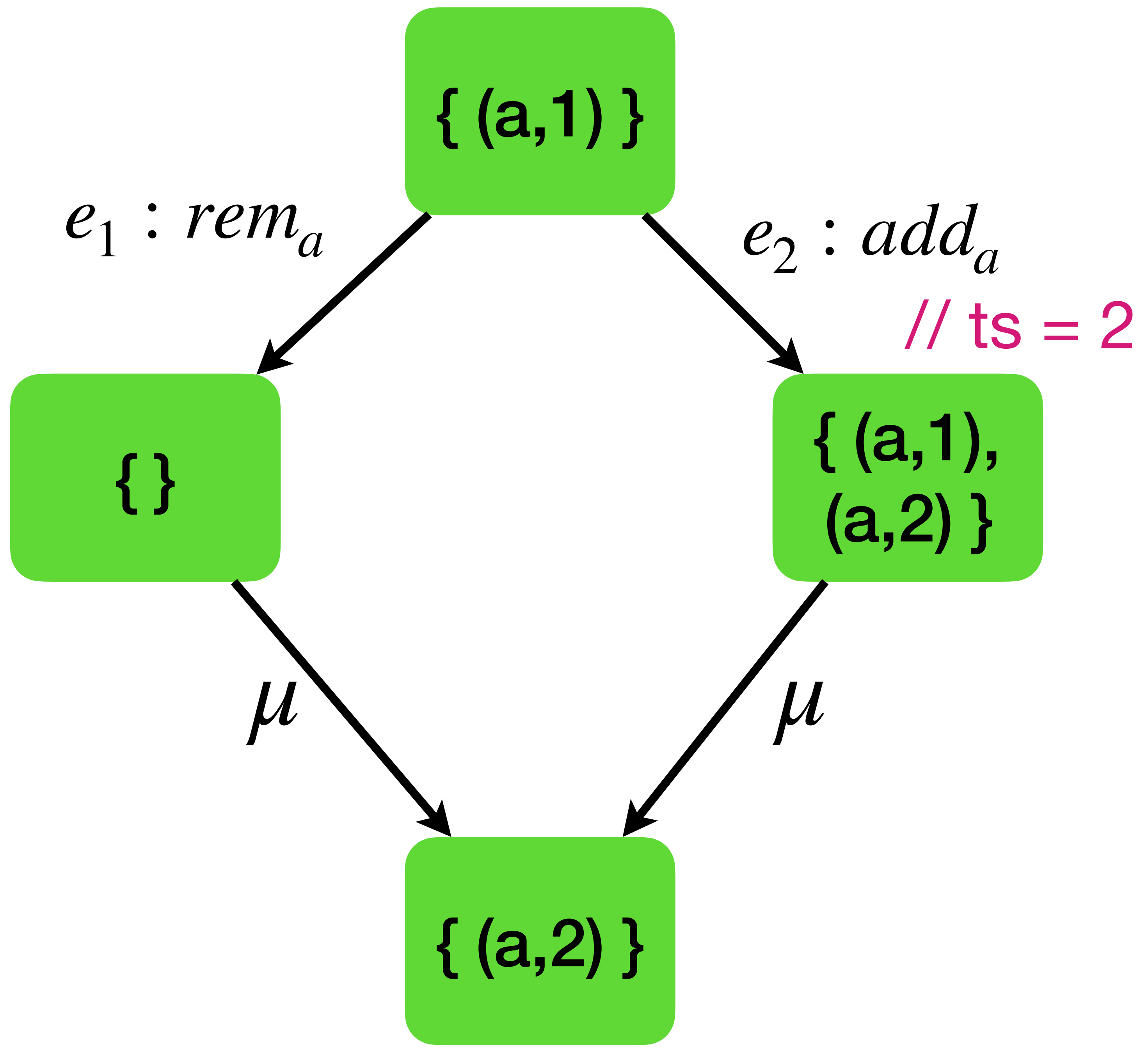

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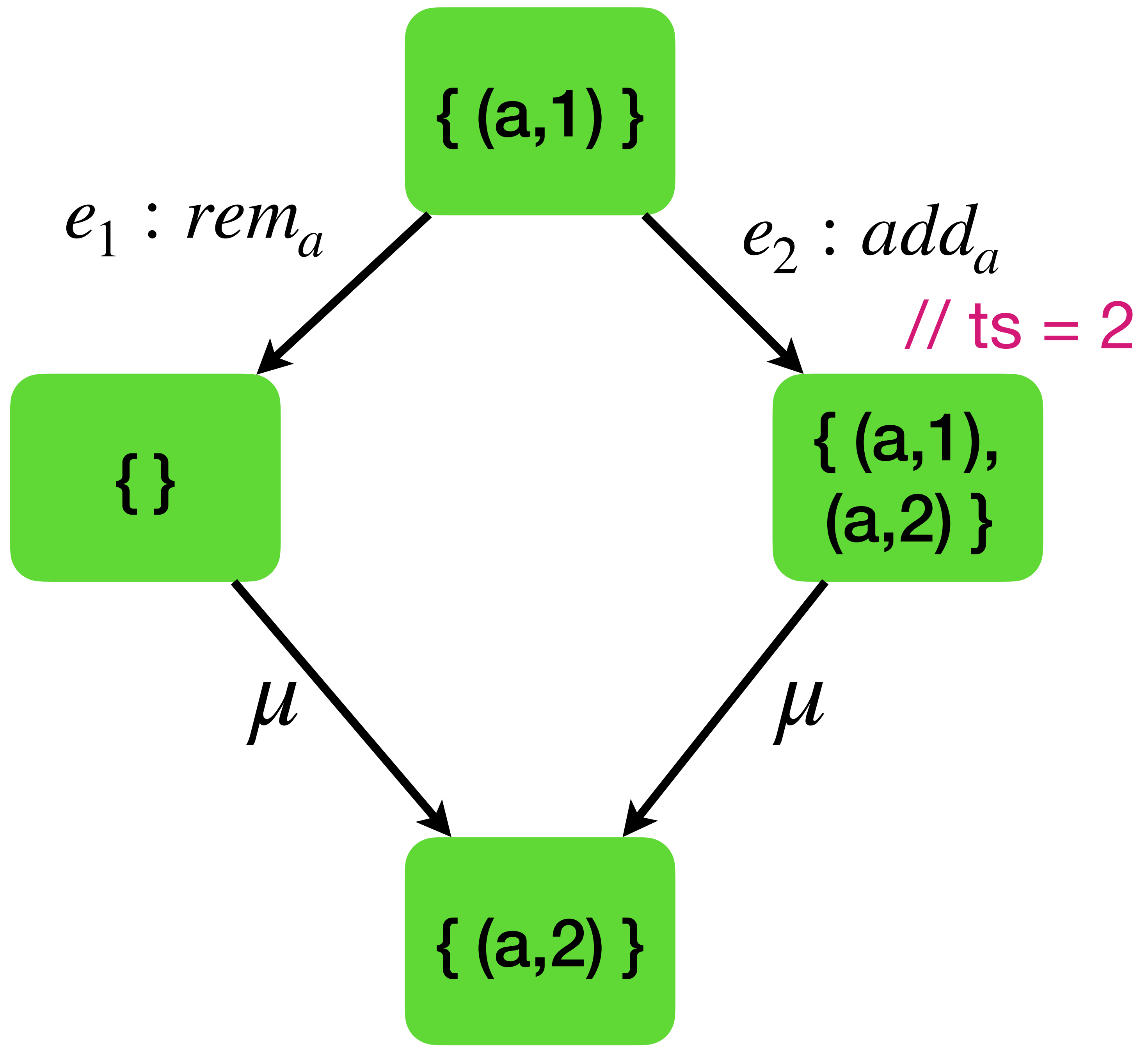

$$rc = \{(rem_a, add_a) \,|\, a \in \mathbb{E}\}$$

$$\{(a,2)\} = e_2(e_1\{(a,1)\}) = add_a(rem_a\{(a,1)\})$$

Linearization order

$$\forall e_1, e_2 \in \mathcal{E}_C.\ e_1 \xrightarrow{\mathsf{lo_C}} e_2 \Leftrightarrow (e_1 \xrightarrow{\mathsf{vis(C)}} e_2 \wedge \neg e_1 \rightleftarrows e_2)$$

$$\vee\ (e_1 \mid\mid_C e_2 \wedge e_1 \xrightarrow{\mathsf{rc}} e_2 \wedge \neg(\exists e_3 \in \mathcal{E}.\ e_2 \xrightarrow{\mathsf{vis(C)}} e_3 \wedge \neg e_2 \rightleftarrows e_3))$$

Linearization order

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$$\lor \left(e_1 \,||_C\, e_2 \land e_1 \xrightarrow{\text{rc}} e_2 \land \neg(\exists e_3 \in \mathcal{E}.\ e_2 \xrightarrow{\text{vis(C)}} e_3 \land \neg e_2 \rightleftarrows e_3)\right)$$

lo must preserve visibility unless the events commute

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{ }

$e_1 : add_a$ (with $ts = 1$)

{ (a,1) }

$e_2 : add_b$ (with $ts = 2$)

{ (a,1), (b,2) }

$$e_1(e_2(\varnothing)) = e_2(e_1(\varnothing)) = \{(a,1), (b,2)\}$$

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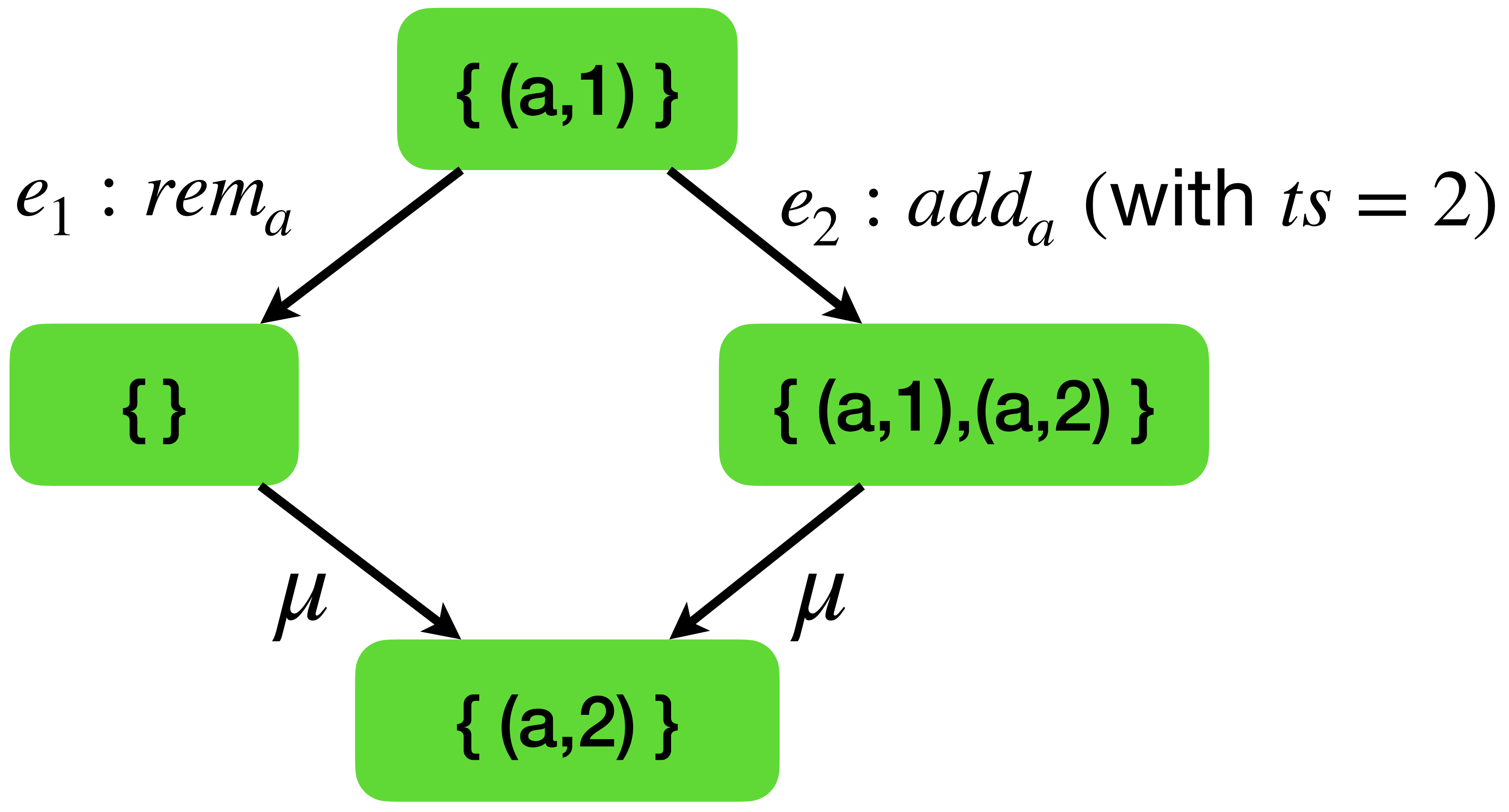

$$\{(a,2)\} = e_2(e_1\{(a,1)\})$$

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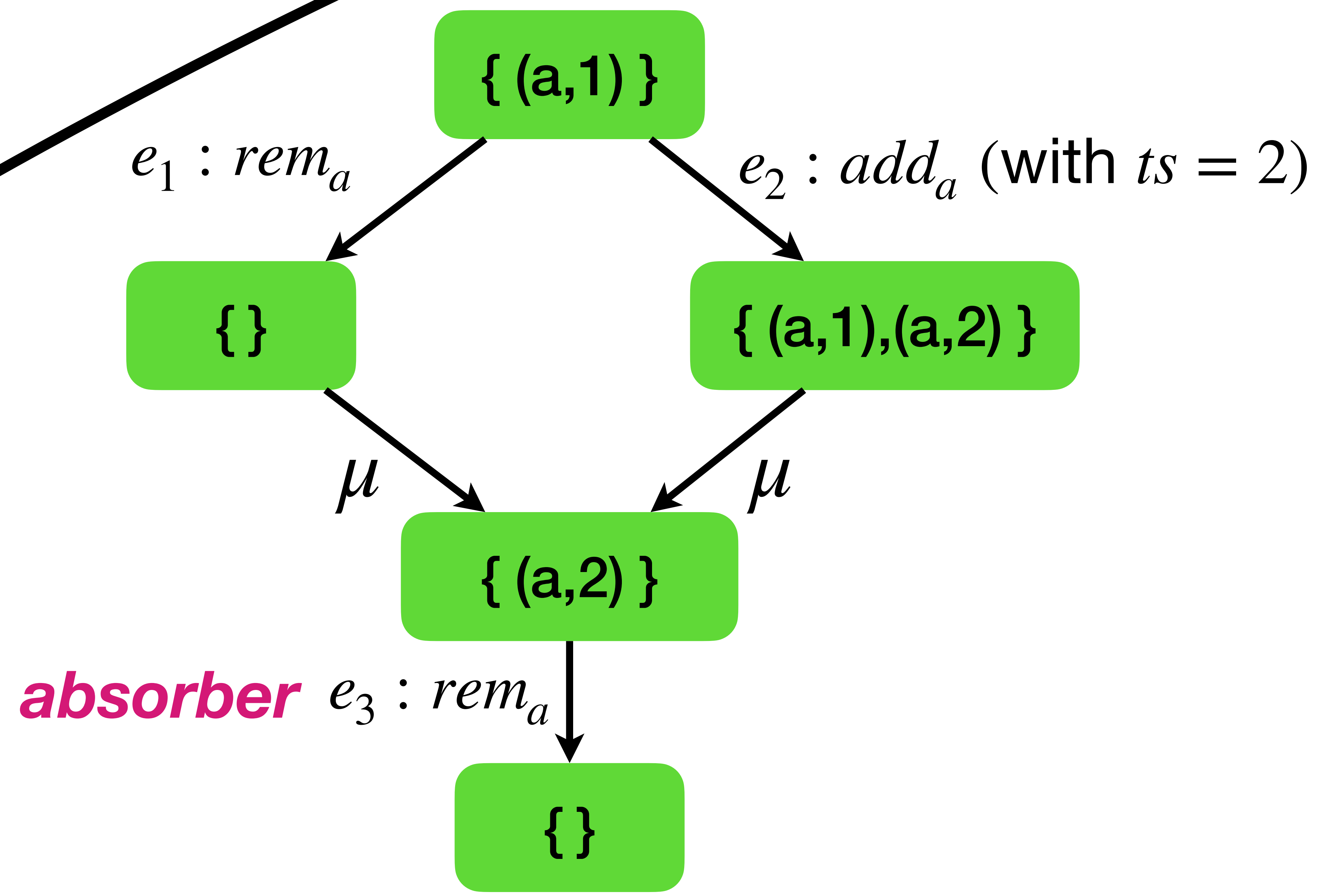
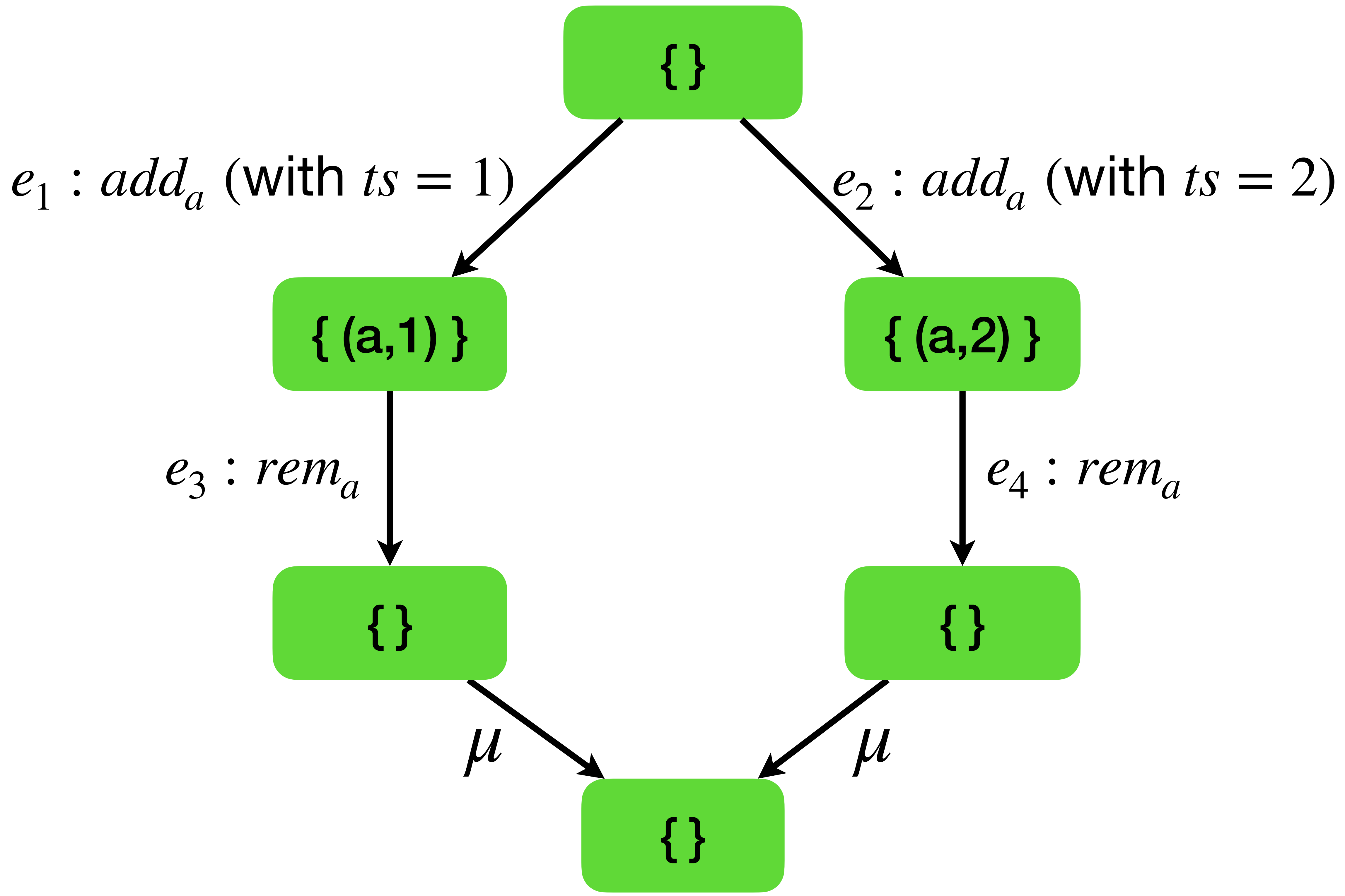
Unless there is a subsequent "absorber"



$$e_3(e_2(e_1\{(a,1)\}))) = e_3(e_2(e_1(\{(a,1)\}))) = \varnothing$$

Where absorber helps



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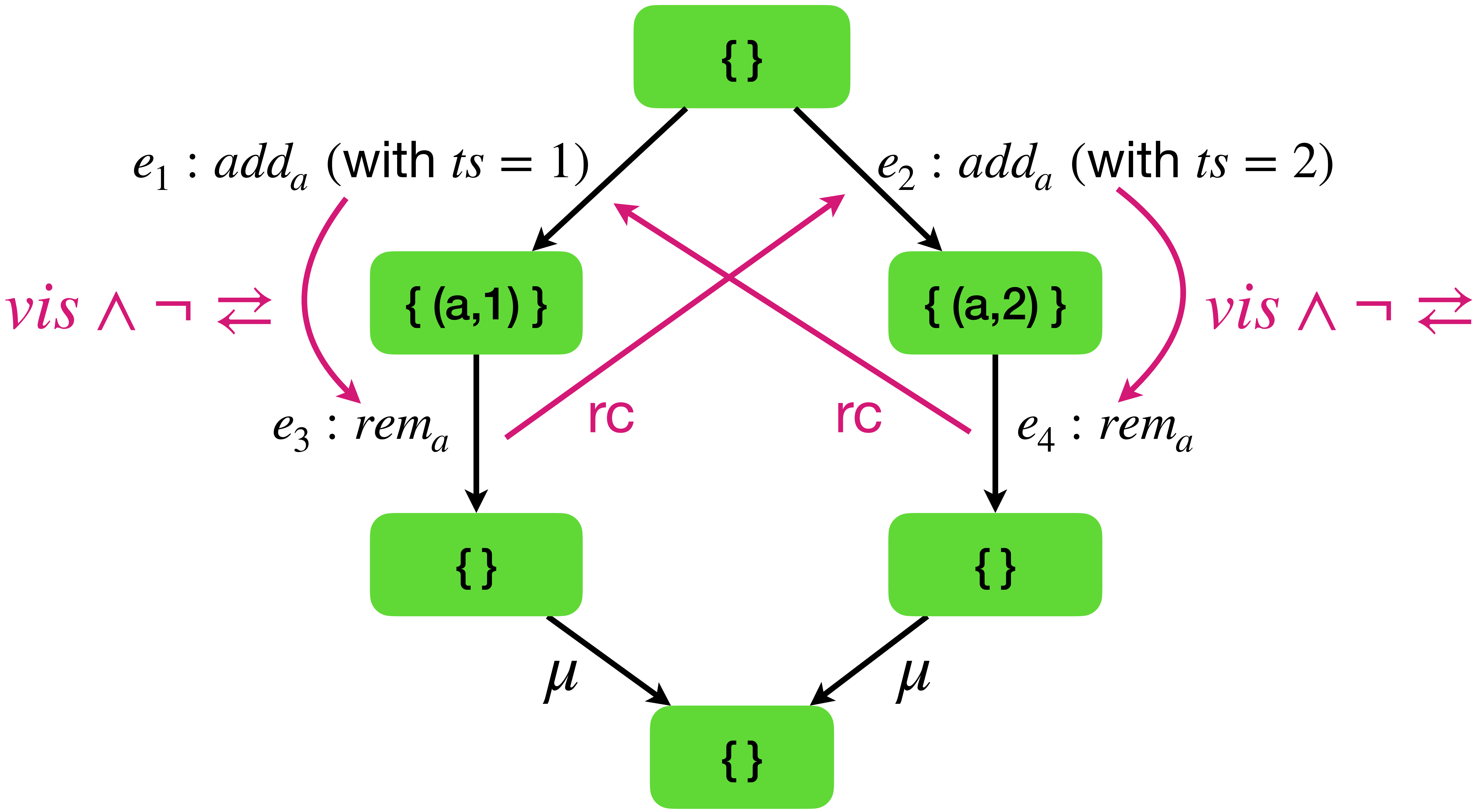

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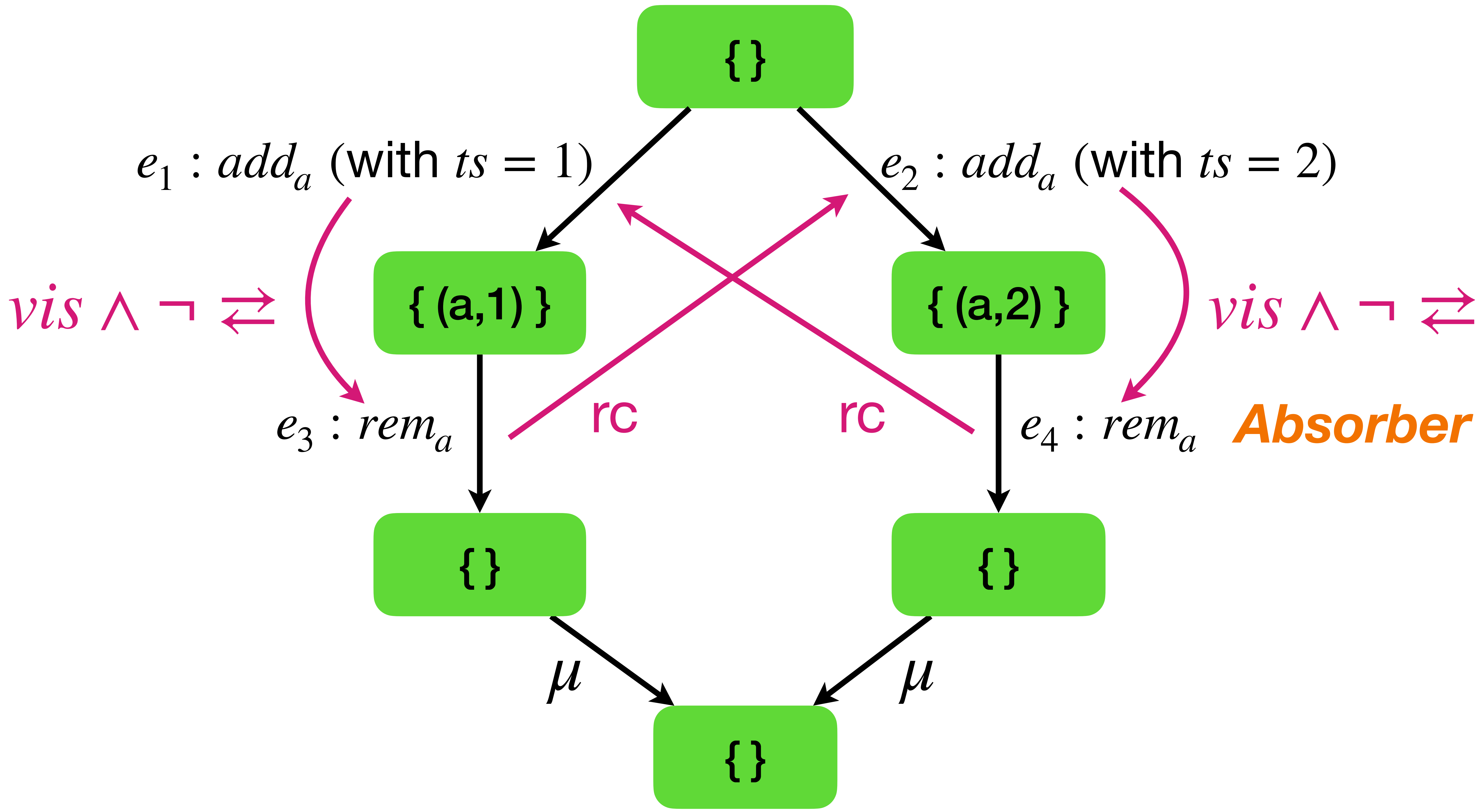

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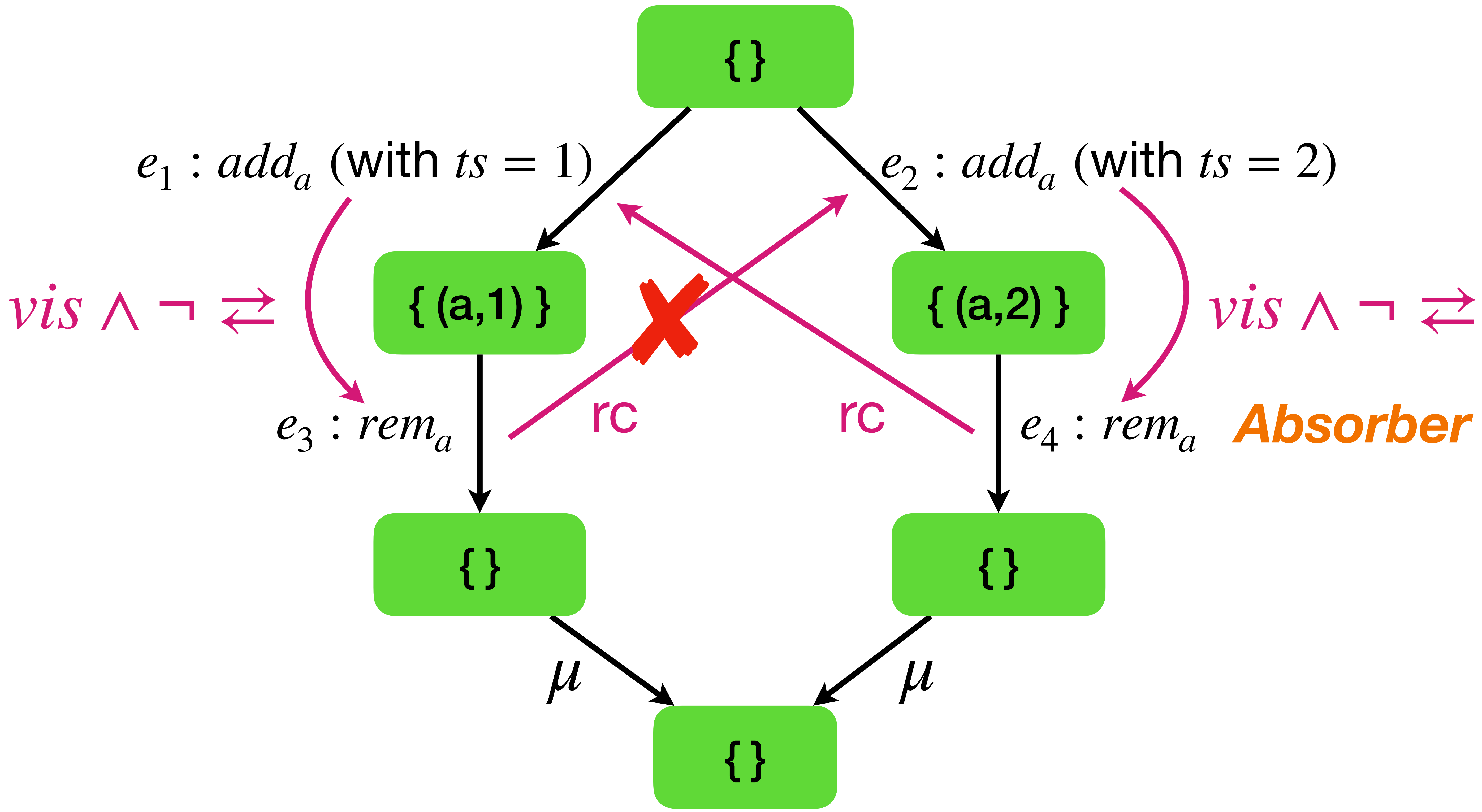

Proving RA-linearizability

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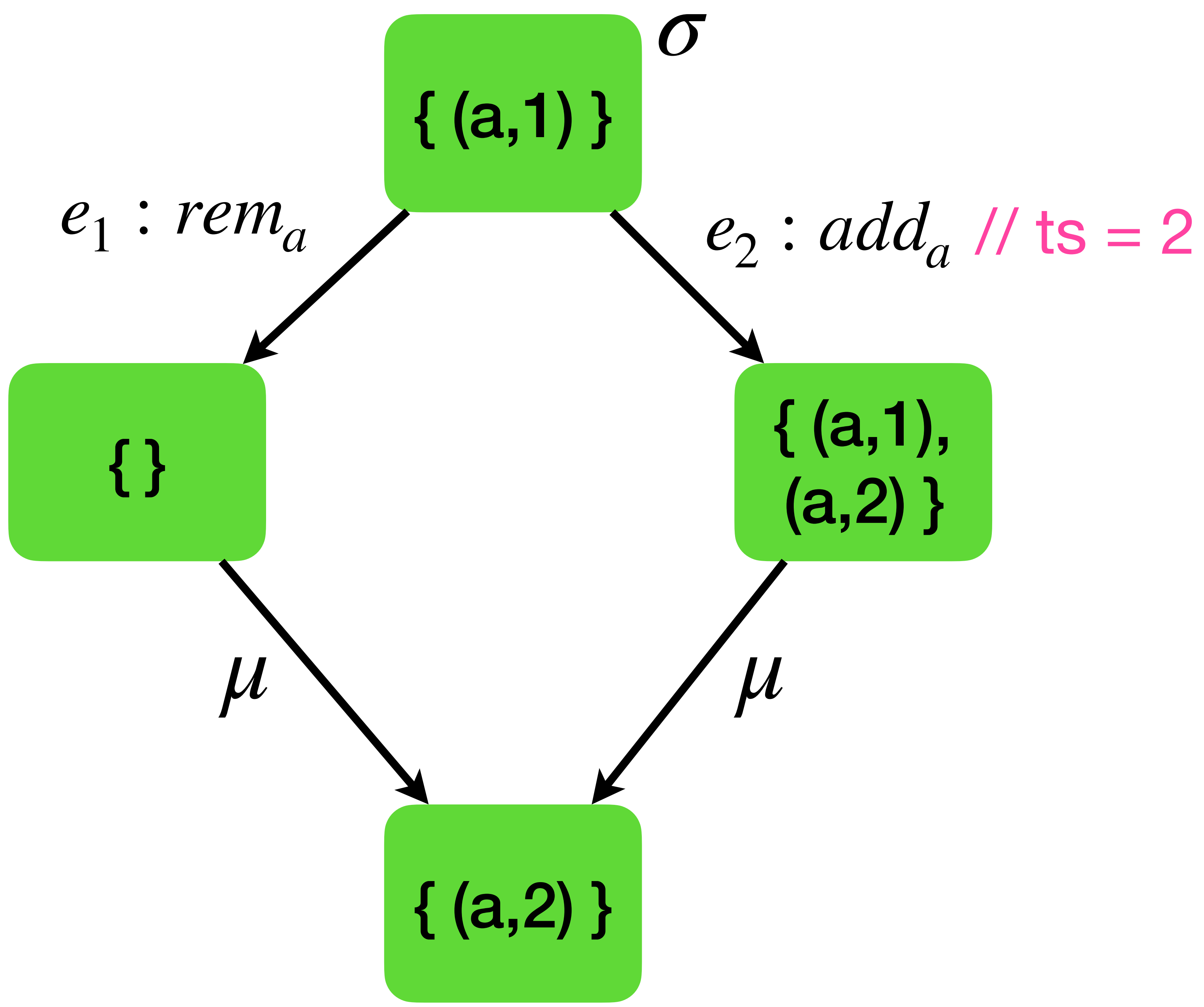
THEOREM 4.7. *If an MRDT $\mathcal{D}$ satisfies the VCs ψ^**(BOTTOMUP-2-OP), *ψ^**(BOTTOMUP-1-OP), *ψ^**(BOTTOMUP-0-OP), MERGEIDEMPOTENCE *and* MERGECOMMUTATIVITY, *then $\mathcal{D}$ is linearizable.*

  - Meta theory — 24 VCs $\Longrightarrow$ RA-Linearizability
  - Automatically checked against 24 VCs using the SMT solver

Bottom up linearisation

$$rc = \{(rem_a, add_a) \mid a \in \mathbb{E}\}$$



To show

$$\mu(\sigma, e_1(\sigma), e_2(\sigma)) = e_2(e_1(\sigma))$$

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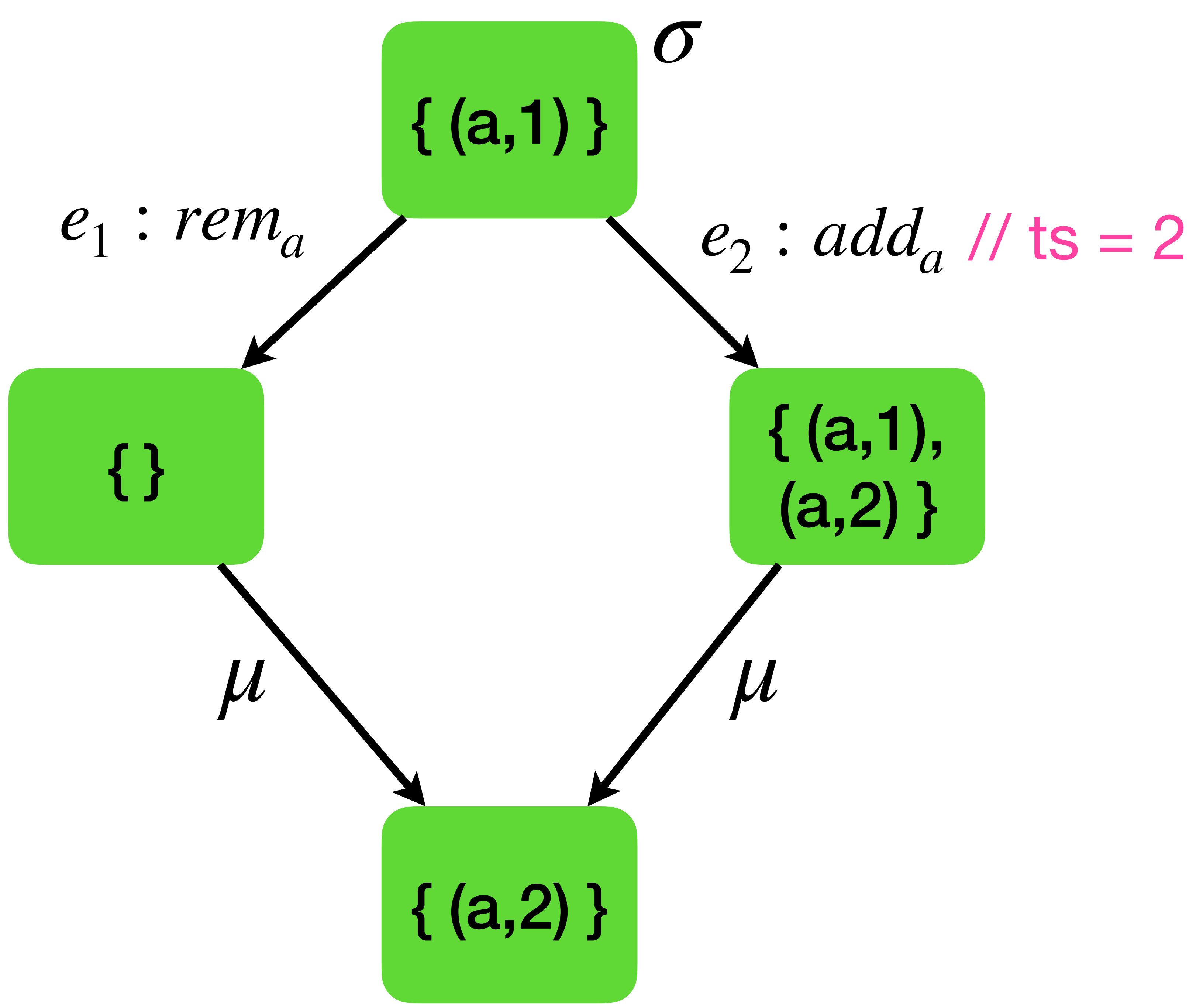

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$$\frac{e_1 \neq e_2 \qquad e_1 \xrightarrow{\mathsf{rc}} e_2 \vee e_1 \rightleftarrows e_2}{\mu(l, e_1(a), e_2(b)) = e_2(\mu(l, e_1(a), b))}$$

[BottomUp-1-OP]

$$\frac{(e_\top \neq \epsilon \wedge e_1 \neq e_\top) \vee (e_\top = \epsilon \wedge l = b)}{\mu(e_\top(l), e_1(a), e_\top(b)) = e_1(\mu(e_\top(l), a, e_\top(b)))}$$

[BottomUp-0-OP]

$$\mu(e_\top(l), e_\top(a), e_\top(b)) = e_\top(\mu(l, a, b))$$

[MergeIdempotence]

$$\mu(a, a, a) = a$$

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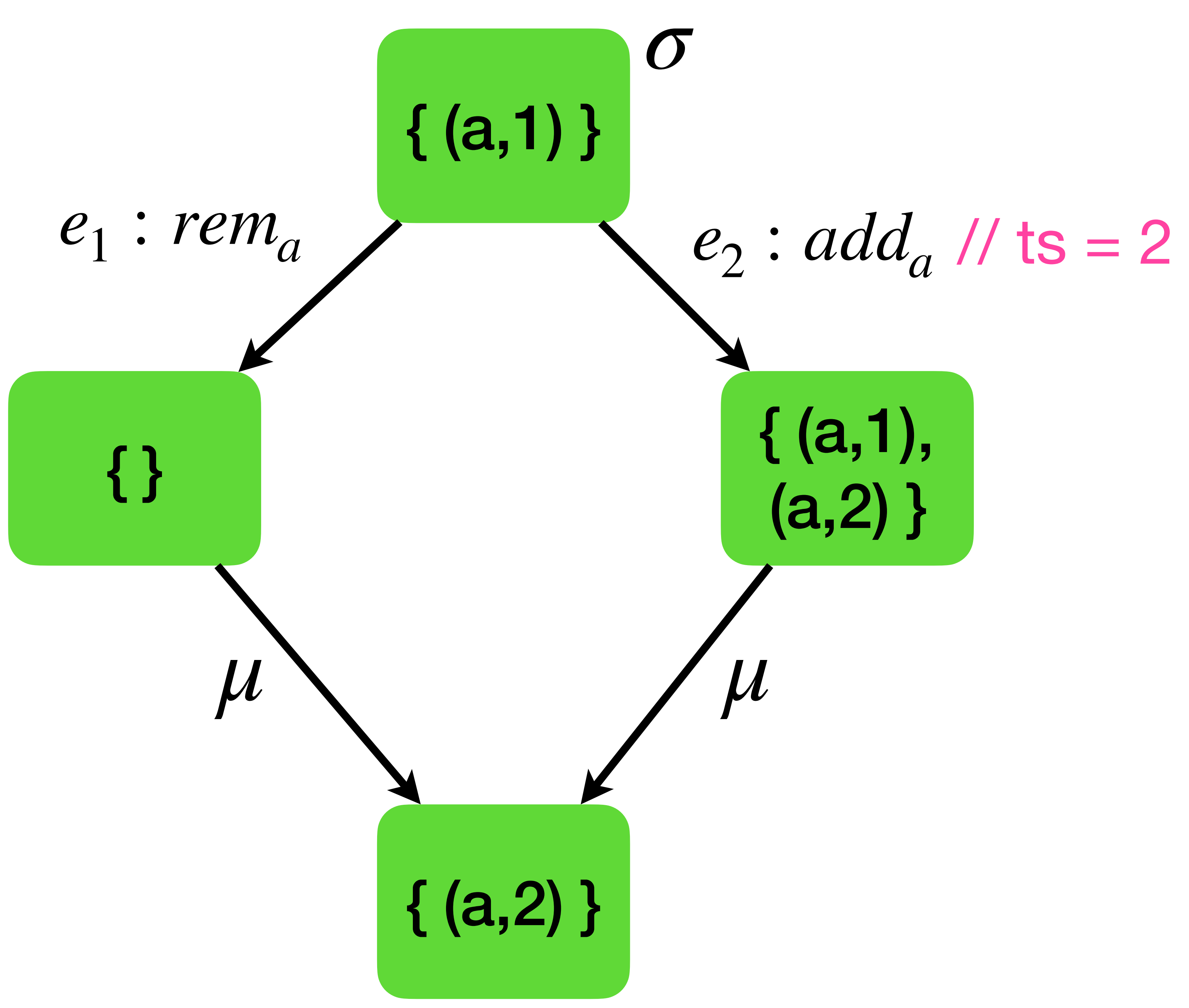

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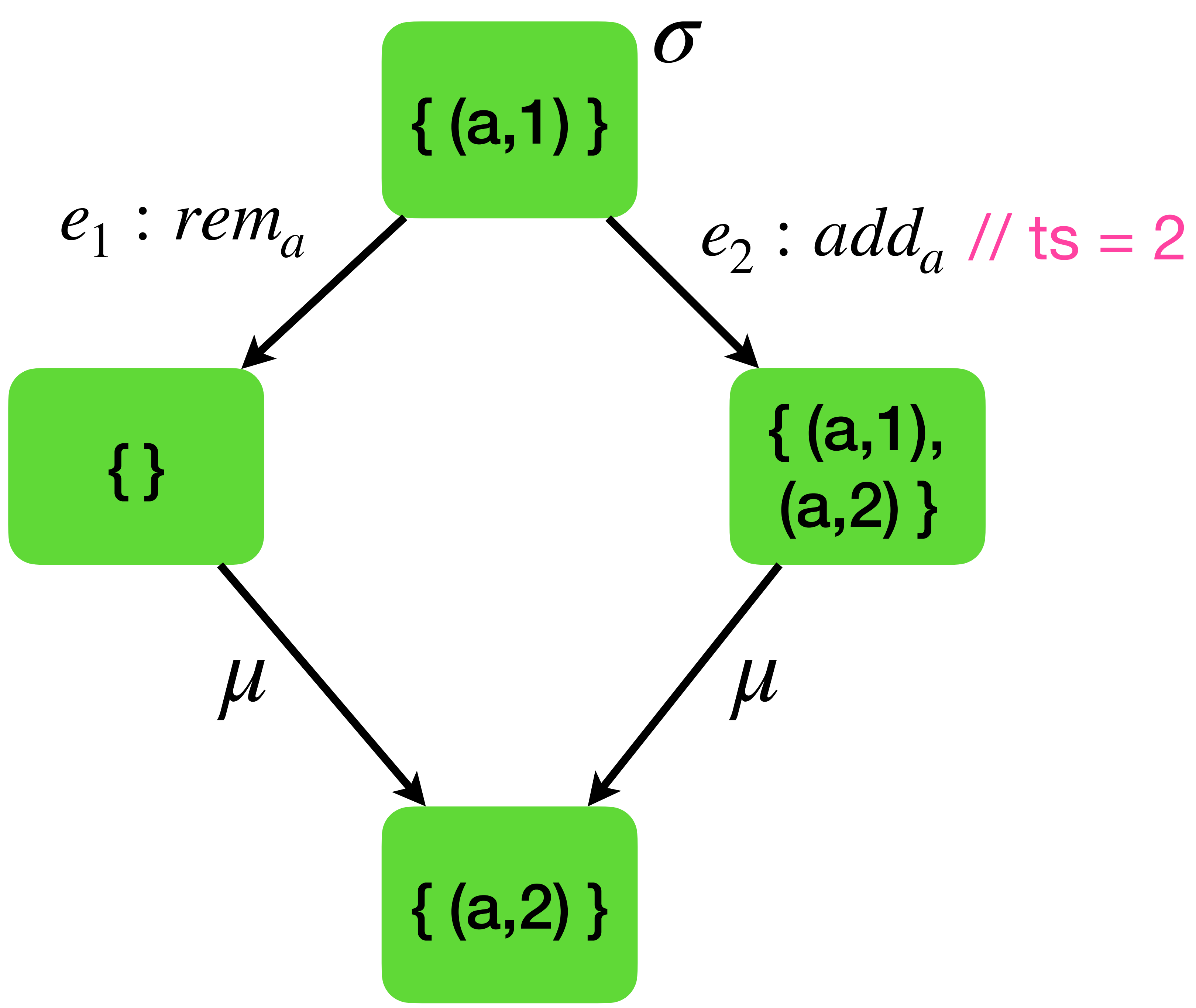

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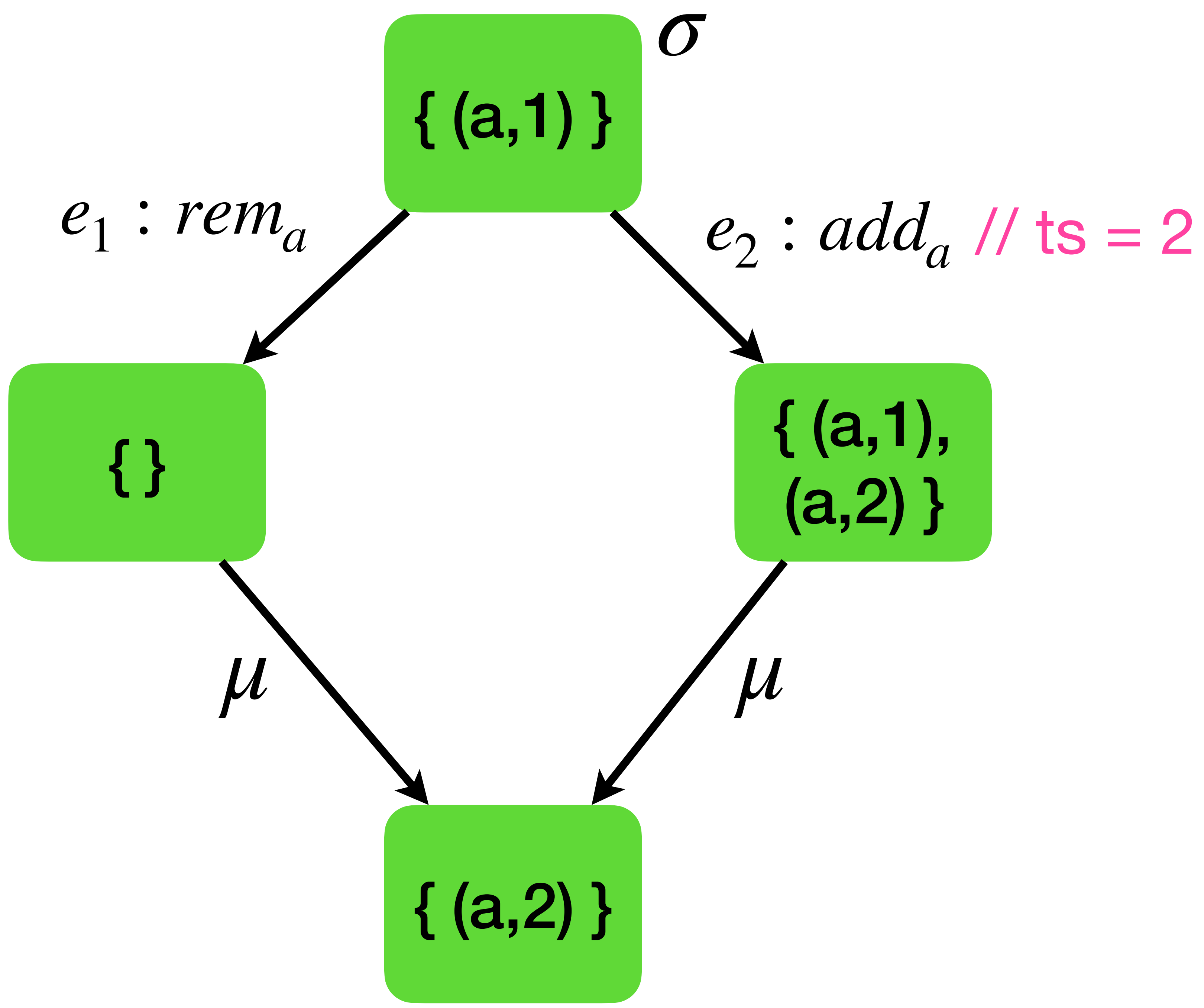

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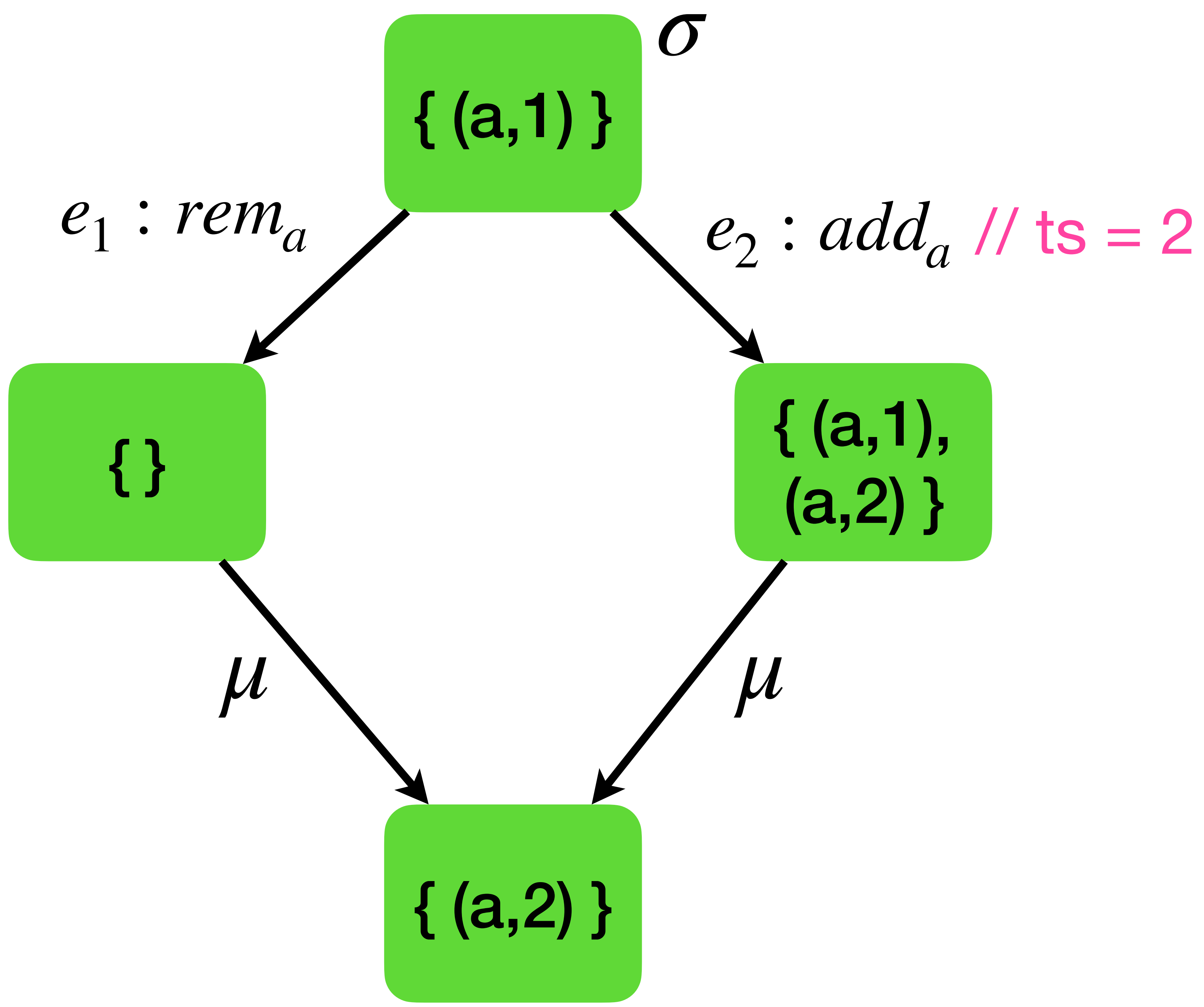

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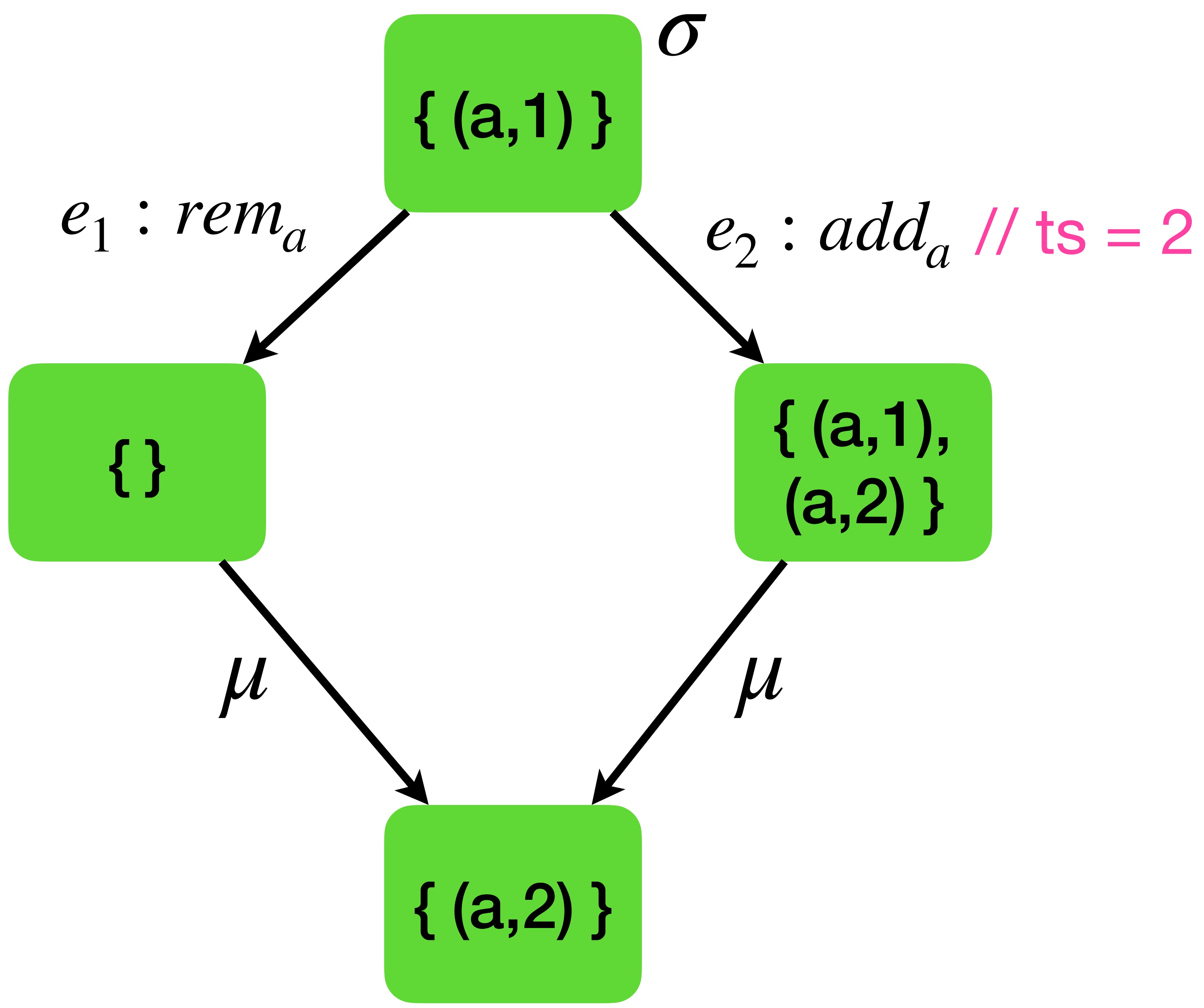

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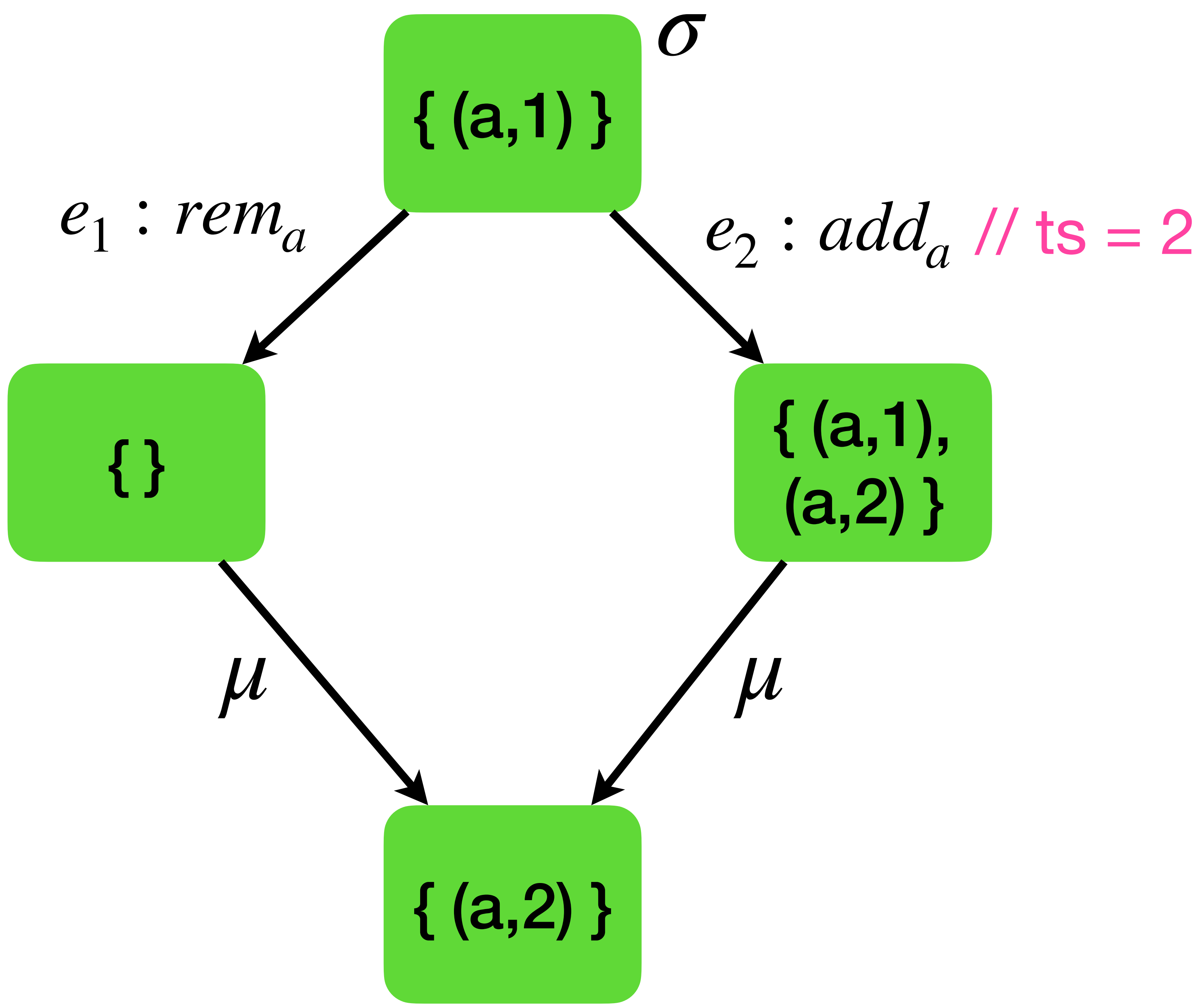

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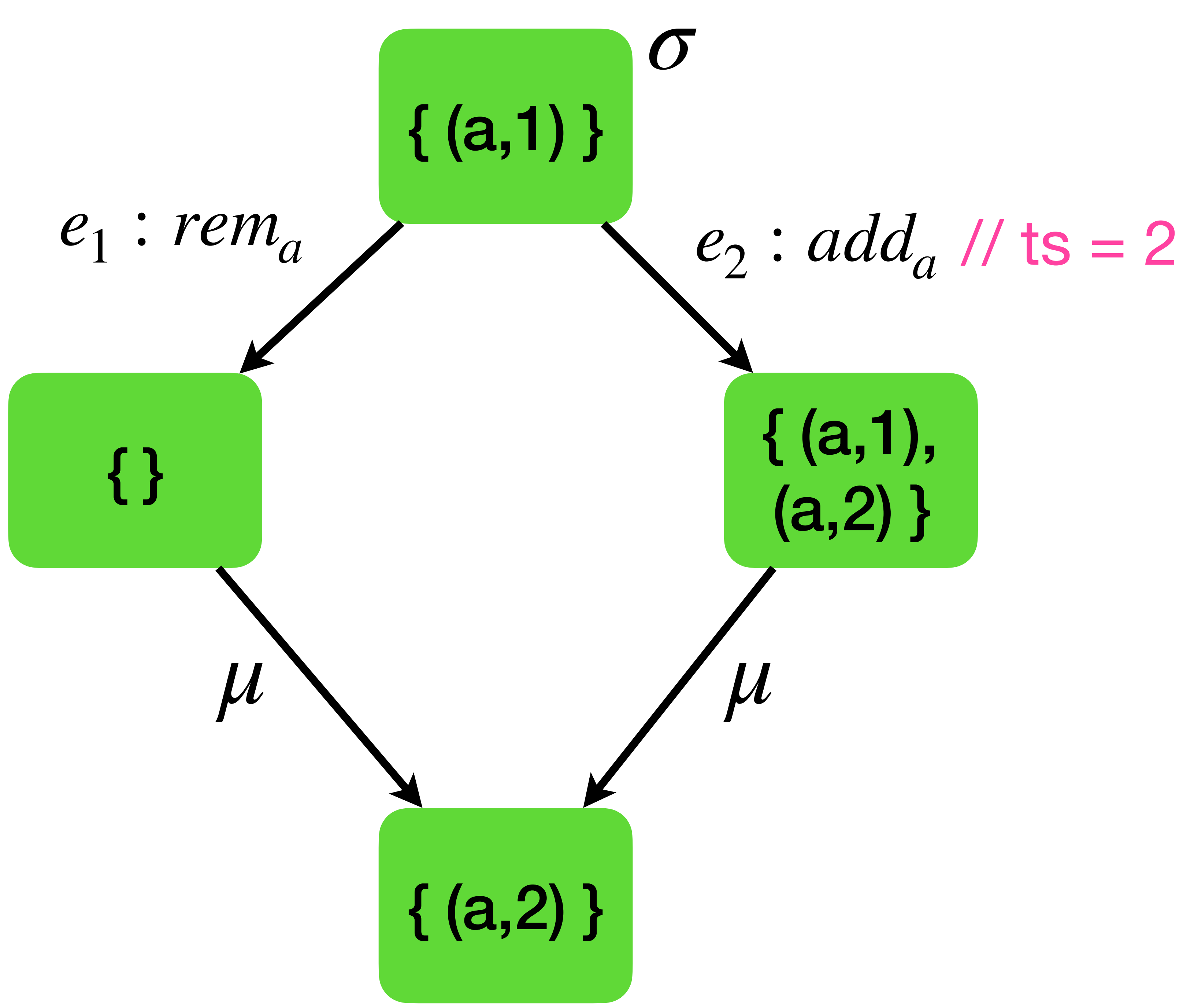

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VCs are inductive versions over arbitrary length traces

Verified MRDTs

MRDT	rc **Policy**	**#LOC**	**Verification Time (s)**
Increment-only counter [12]	none	6	0.72
PN counter [23]	none	10	1.64
Enable-wins flag*	disable $\xrightarrow{\text{rc}}$ enable	30	29.80
Disable-wins flag*	enable $\xrightarrow{\text{rc}}$ disable	30	37.91
Grows-only set [12]	none	6	0.45
Grows-only map [23]	none	11	4.65
OR-set [23]	$\text{rem}_a \xrightarrow{\text{rc}} \text{add}_a$	20	4.53
OR-set (efficient)*	$\text{rem}_a \xrightarrow{\text{rc}} \text{add}_a$	34	660.00
Remove-wins set*	$\text{add}_a \xrightarrow{\text{rc}} \text{rem}_a$	22	9.60
Set-wins map*	$\text{del}_k \xrightarrow{\text{rc}} \text{set}_k$	20	5.06
Replicated Growable Array [1]	none	13	1.51
Optional register*	unset $\xrightarrow{\text{rc}}$ set	35	200.00
Multi-valued Register*	none	7	0.65
JSON-style MRDT*	Fig. 13	26	148.84

Neem also supports verification of RA-linearizability of state-based CRDTs

https://github.com/prismlab/neem

Challenges with Neem (F*)

- Automated verification in F* returns yes / no / ¯_(ツ)_/¯
  - Not pleasant for engineering
  - No counterexamples!

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- TCB includes the SMT solver
- F* SMT-aided proofs are brittle
  - Some lemmas need specific heuristics for fuel, unfolding, etc.
  - z3 upgrade breaks proofs!

Sal: Multi-modal verification of RDTs

- Move Neem from F* to Lean

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- Advantages
  - Smaller TCB
  - Multi-modal verification
    - Property-based testing — counter-example generation and visualisation
    - SMT proofs, deductive verification

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  - Multi-modal verification
    - Property-based testing — counter-example generation and visualisation
    - SMT proofs, deductive verification
- ***Agentic proof-oriented programming***
  - Fundamentally changing how we construct software
  - Coding Agents are great at Lean

Agentic Proof Oriented Programming in Sal

- Claude (Opus 4.7, Opus 4.8 and Fable) and Aristotle

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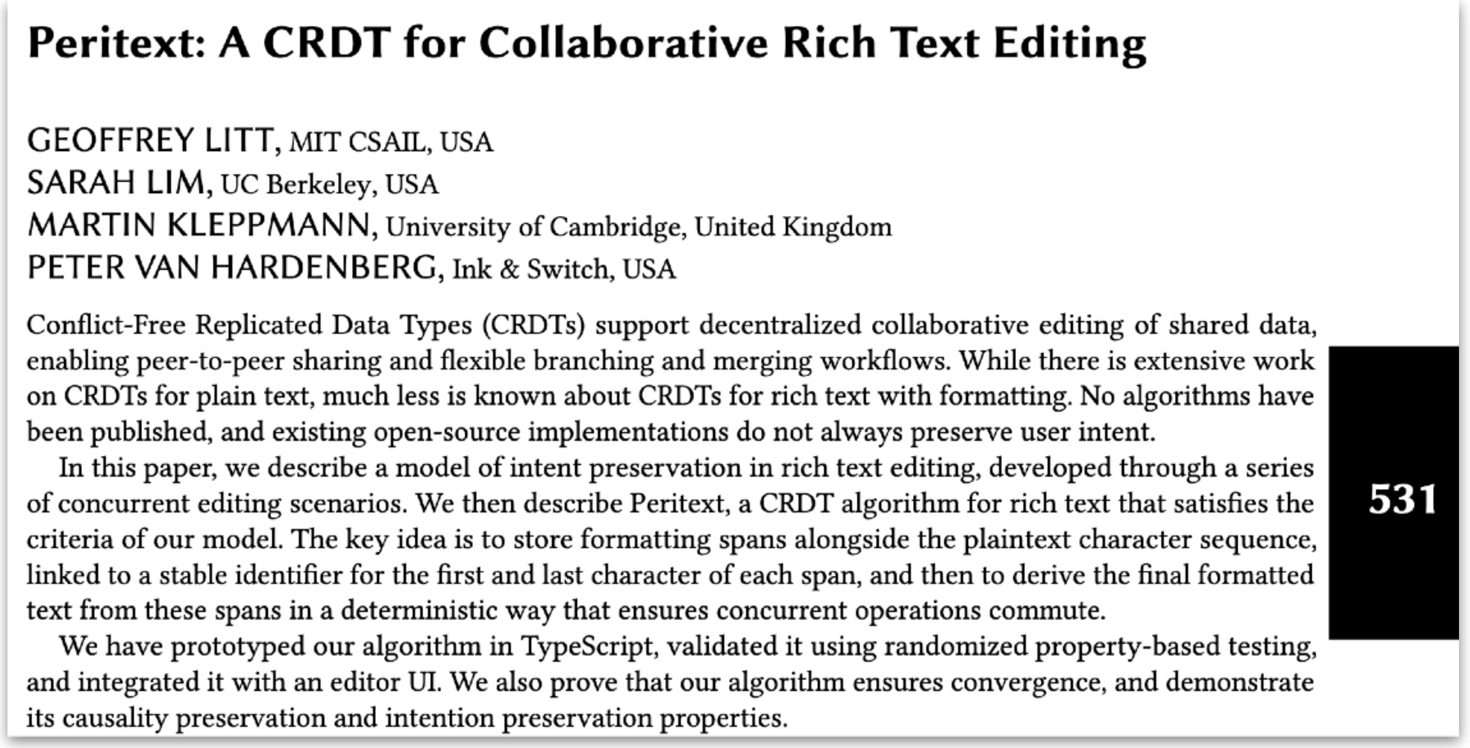
- **13 new CRDTs and 4 new MRDTs** have been verified
- Including Peritext, Priority Queue, Bounded Counters



Peritext: A CRDT for Collaborative Rich Text Editing

GEOFFREY LITT, MIT CSAIL, USA
SARAH LIM, UC Berkeley, USA
MARTIN KLEPPMANN, University of Cambridge, United Kingdom
PETER VAN HARDENBERG, Ink & Switch, USA

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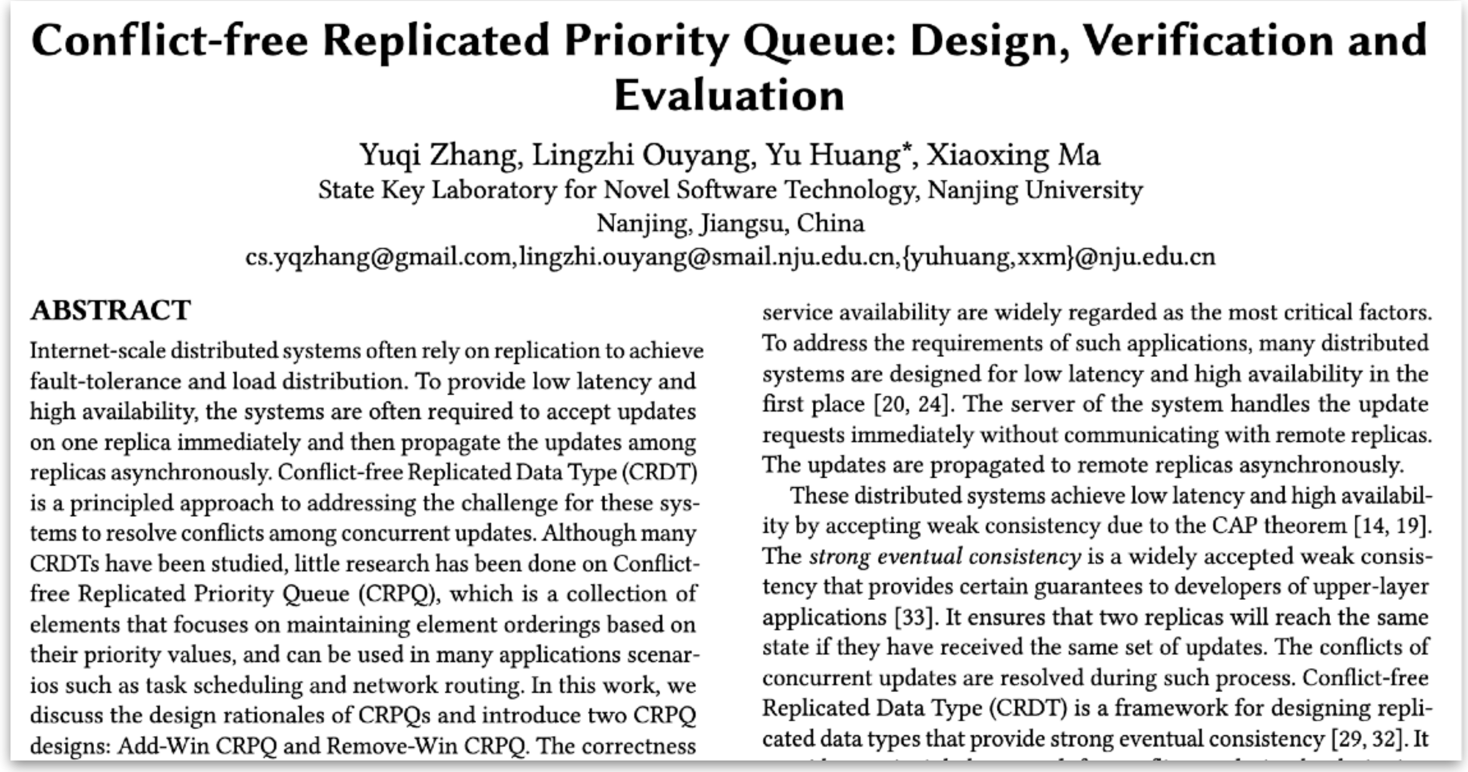

Conflict-free Replicated Priority Queue: Design, Verification and Evaluation

Yuqi Zhang, Lingzhi Ouyang, Yu Huang*, Xiaoxing Ma
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These distributed systems achieve low latency and high availability by accepting weak consistency due to the CAP theorem [14, 19]. The *strong eventual consistency* is a widely accepted weak consistency that provides certain guarantees to developers of upper-layer applications [33]. It ensures that two replicas will reach the same state if they have received the same set of updates. The conflicts of concurrent updates are resolved during such process. Conflict-free Replicated Data Type (CRDT) is a framework for designing replicated data types that provide strong eventual consistency [29, 32]. It

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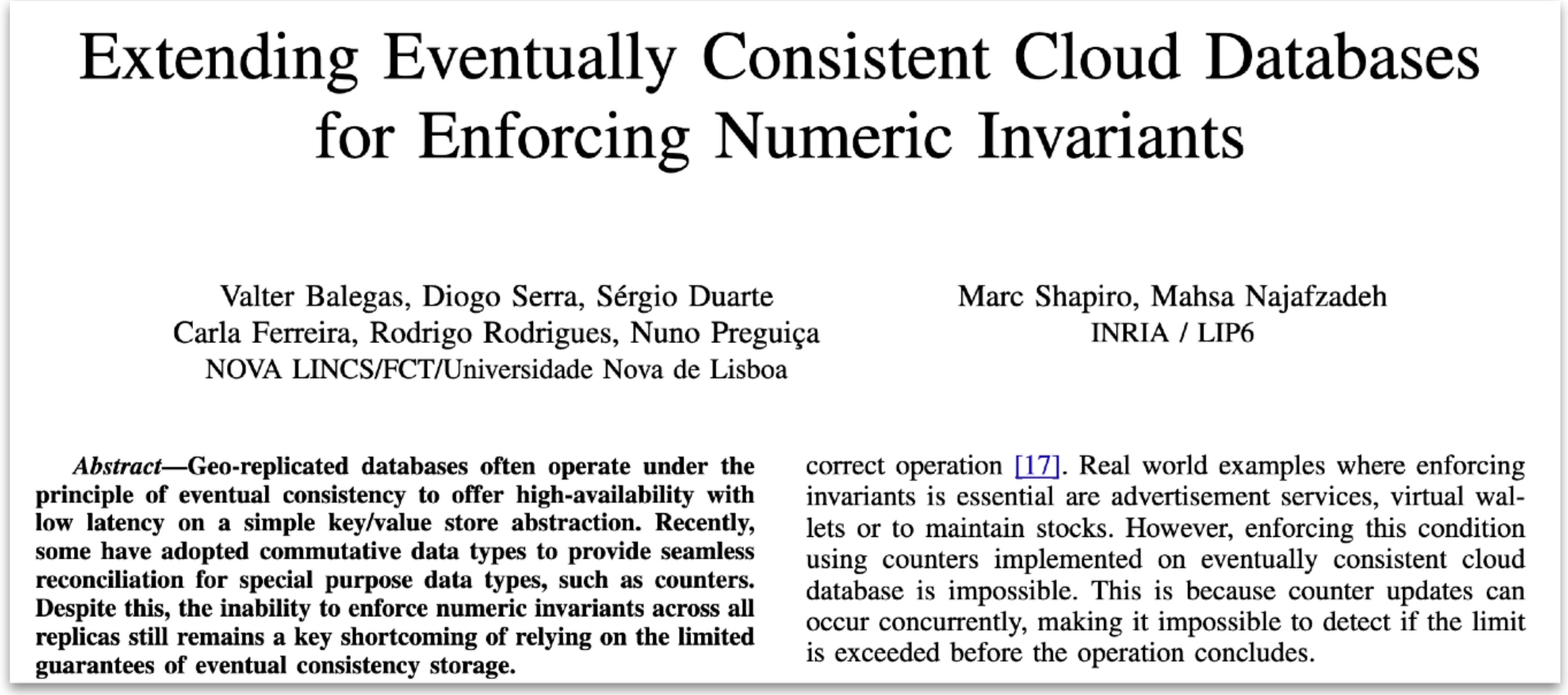

Extending Eventually Consistent Cloud Databases for Enforcing Numeric Invariants

Valter Balegas, Diogo Serra, Sérgio Duarte
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NOVA LINCS/FCT/Universidade Nova de Lisboa

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***Abstract*—Geo-replicated databases often operate under the principle of eventual consistency to offer high-availability with low latency on a simple key/value store abstraction. Recently, some have adopted commutative data types to provide seamless reconciliation for special purpose data types, such as counters. Despite this, the inability to enforce numeric invariants across all replicas still remains a key shortcoming of relying on the limited guarantees of eventual consistency storage.**

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Agentic Proof Oriented Programming in Sal

- Claude (Opus 4.7, Opus 4.8 and Fable) and Aristotle
- **13 new CRDTs and 4 new MRDTs** have been verified
- Including Peritext, Priority Queue, Bounded Counters

Peritext: A CRDT for Collaborative Rich Text Editing

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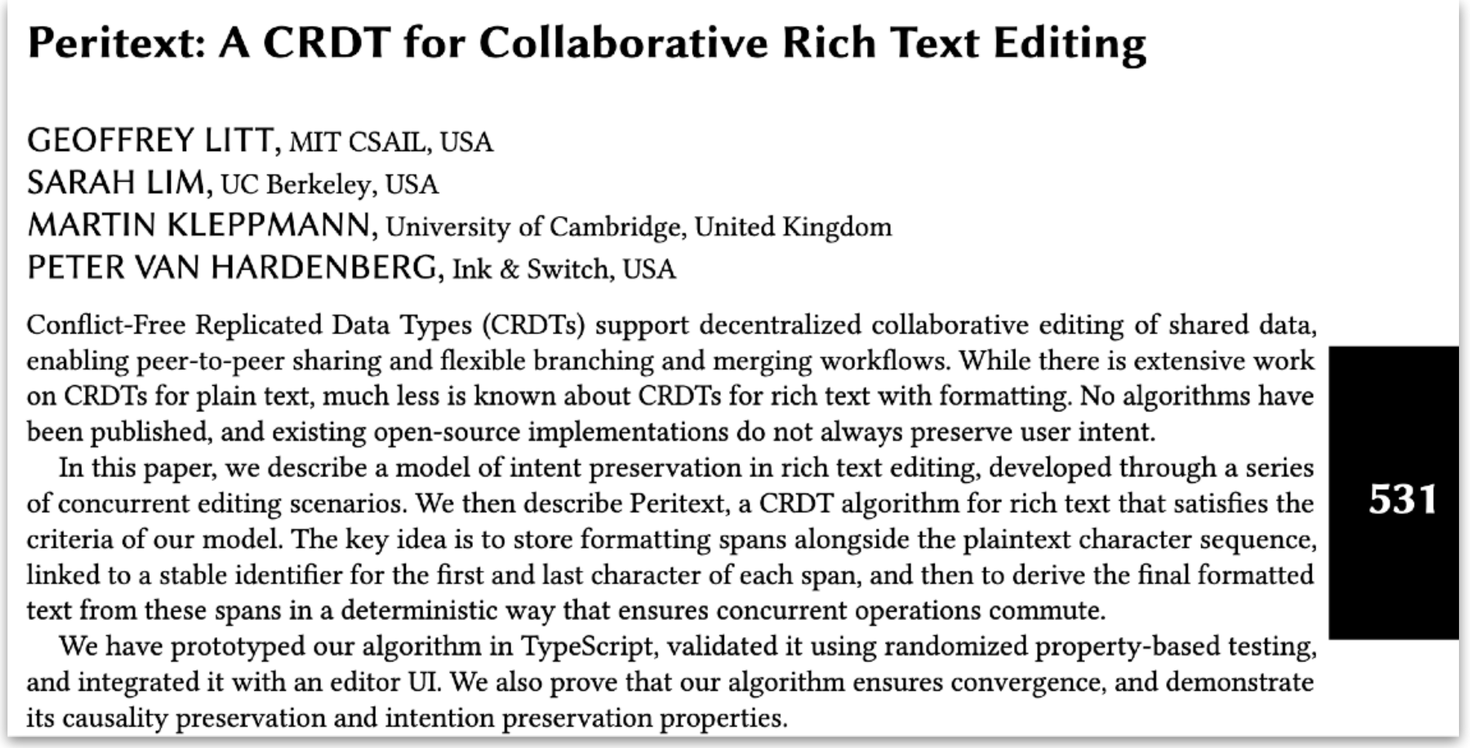

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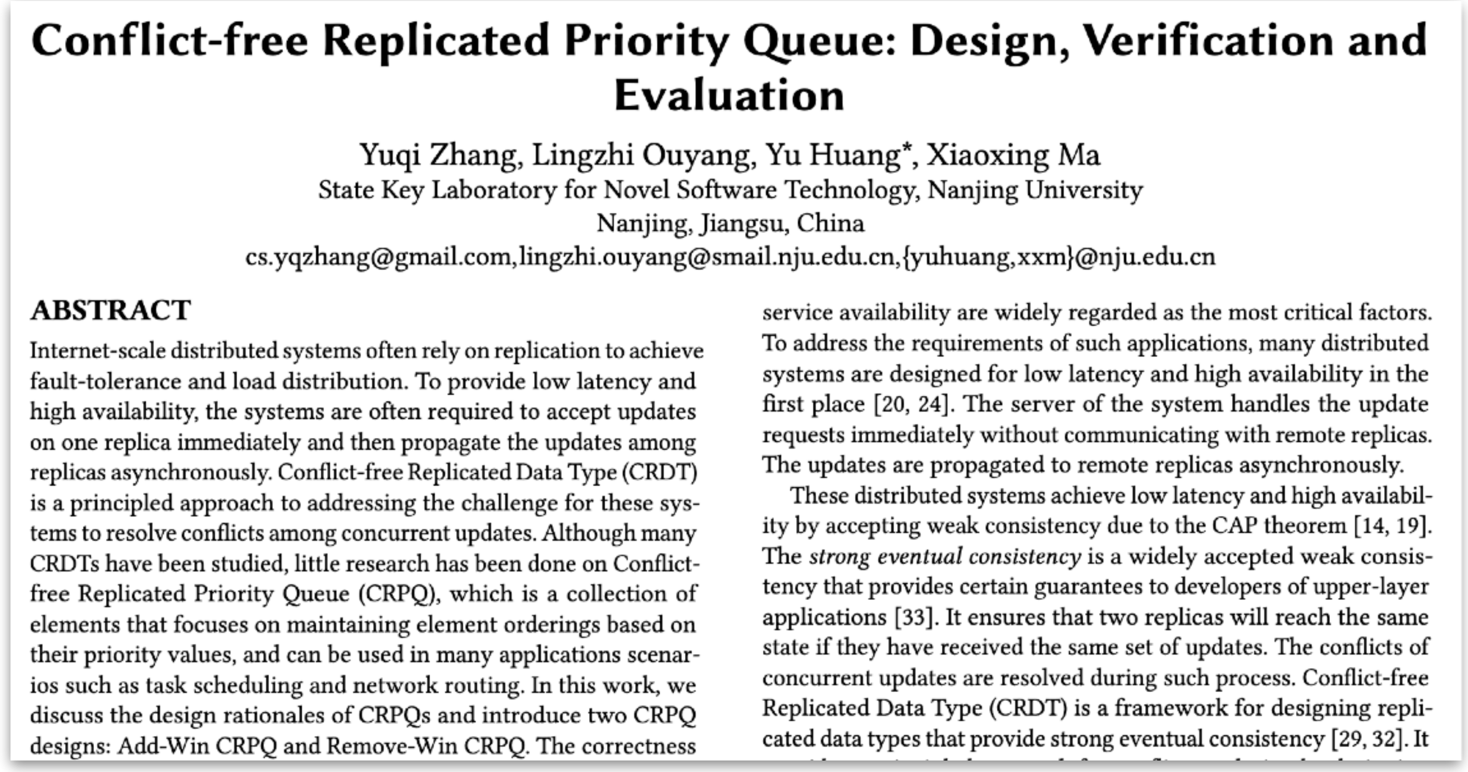

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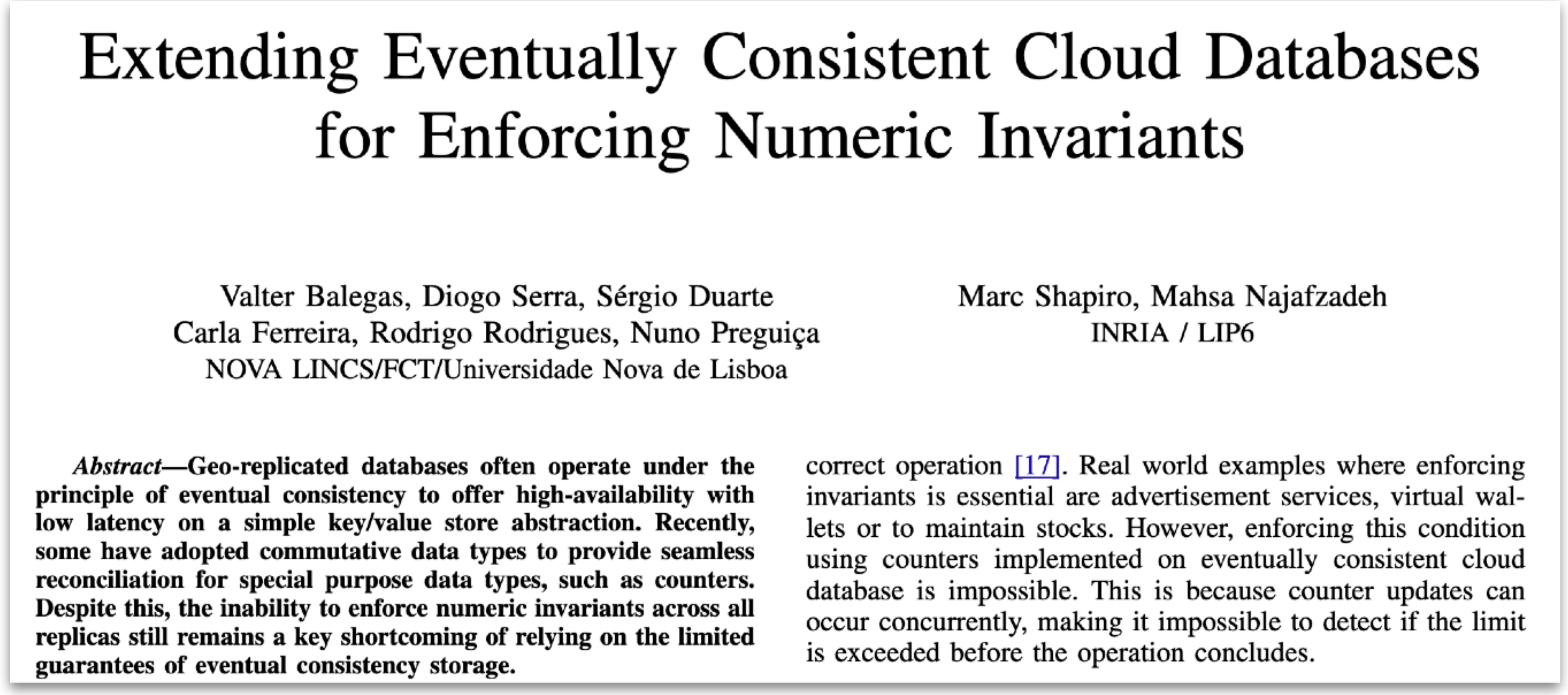

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ReadSide Proofs — Peritext

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The $\underbrace{\textbf{fox jumped}}_{\text{bold}}$.

The quick $\underbrace{\textbf{fox jumped over the dog}}_{\text{bold}}$.

Text insertion at span boundaries

ReadSide Proofs — Peritext

Paper claim	Sal artifact	Status / notes
§A.1 Convergence	24 RA-linearizability VCs in `Peritext_CRDT.lean` and `Peritext_MRDT.lean` (`merge_comm`, `merge_idem`, `base_2op`, `ind_lca_2op`, `inter_*`, `ind_*`, `lem_0op`)	✅ All closed via `by sal`, kernel-checked. Universally quantified over op interleavings.
§3.1 Causality preservation	Framework assumption (causal delivery); `afters` map structurally encodes insert causality; `causal_order_visible_lt` (RGA readside) lifts it to visible-order	✅ Same convention as paper §3.1.
§A.2 Intent preservation — convergence at the read	`readRichText_visible_convergent`	✅ Read-side analogue of merge convergence.
§A.2 Ex 1 — insertion within a span	`insert_within_span_in_span_visible` + propagation lemmas	✅
§A.2 Ex 2 — overlapping same-type Adds	`partial_overlap_all_adds_formatted_visible`	✅
§A.2 Ex 3 — different mark types coexist	`different_type_adds_coexist_visible`	✅
§A.2 Ex 4 — same-type, different values (colors)	not formalised	⚠️ Requires `markValue` field on `MarkOp` — state-shape change.
§A.2 Ex 5 — bold vs non-bold conflict	`add_wins_over_concurrent_remove_visible` + `no_add_cover_implies_unformatted_visible`	✅ LWW by opId (paper §4.4).
§A.2 Ex 6 — overlapping comments via distinct markType	follows from `different_type_adds_coexist_visible`	✅
§A.2 Ex 7 — bold-boundary insertion expands	`ex7_bold_older_sibling_in_span` + `bold_expand_in_span_visible` (via `bold_expand_reach`)	✅
§A.2 Ex 8 — link-boundary insertion does not expand	`ex8_link_descendant_visible_lt_endId` + `_not_in_span_visible_of_wf`	✅ Uses `wf_afters` acyclicity.
(Beyond paper) Anchors survive tombstones	`anchors_survive_tombstones_visible`	✅ Implicit in §4.4 paper discussion.

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https://github.com/fplaunchpad/sal