# Automatically Verifying Replicated Data Types

**KC Sivaramakrishnan**

Joint work with Vimala Soundarapandian, Aseem Rastogi and Kartik Nagar

**National University of Singapore**

**4th August 2025**

# Collaborative Applications

Airtable Google Docs Figma Notion Overleaf

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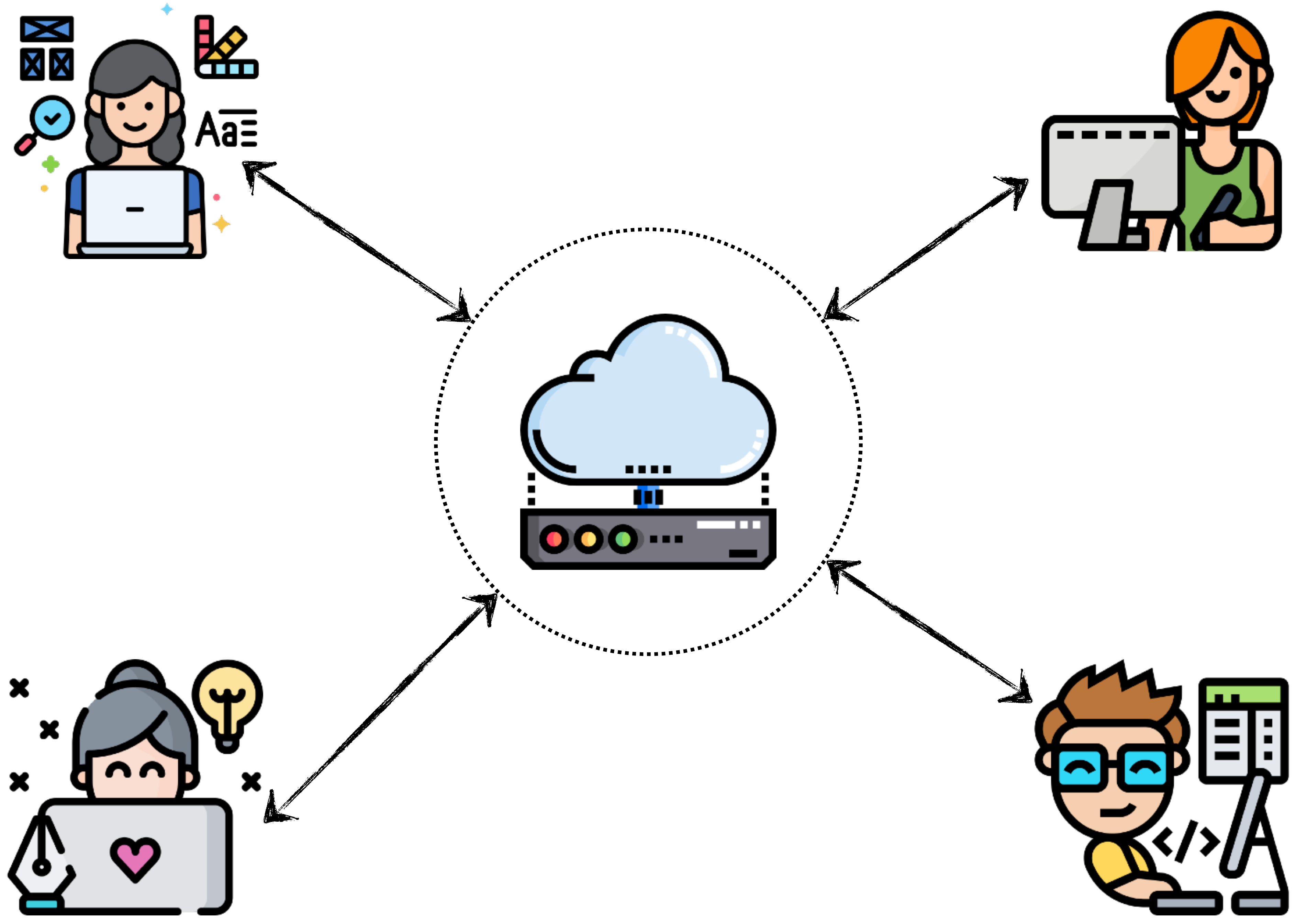

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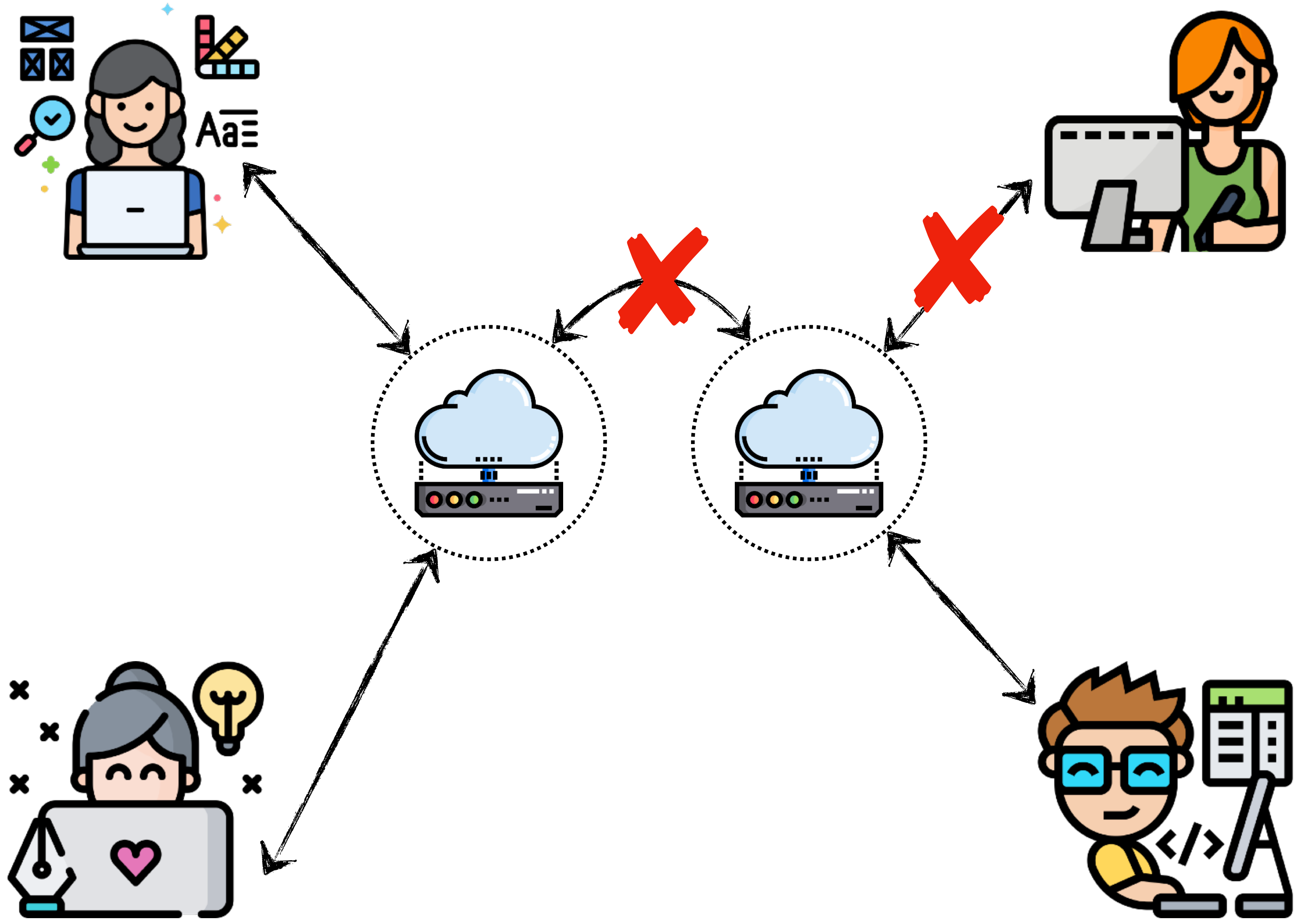
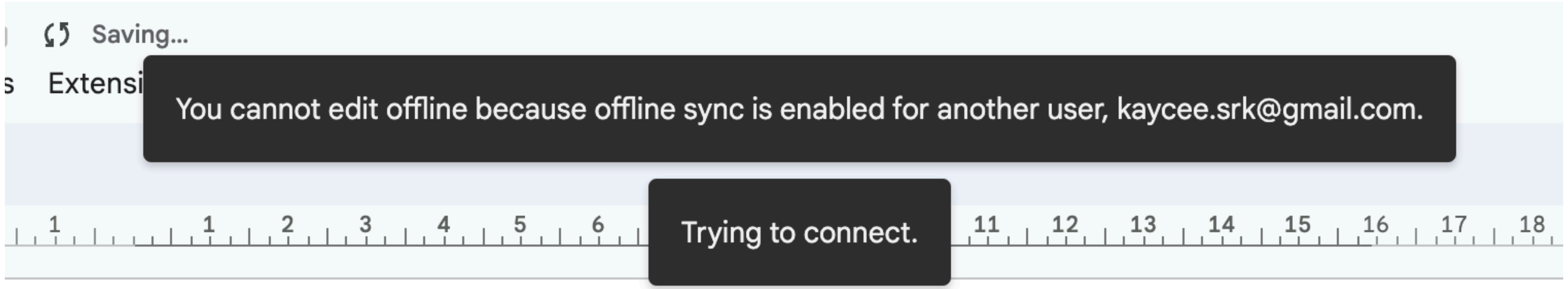


# Network Partitions

- Centralised Apps provide limited support for offline editing




*Enabling offline sync for one account prevents other accounts from working offline*

# Local-first software

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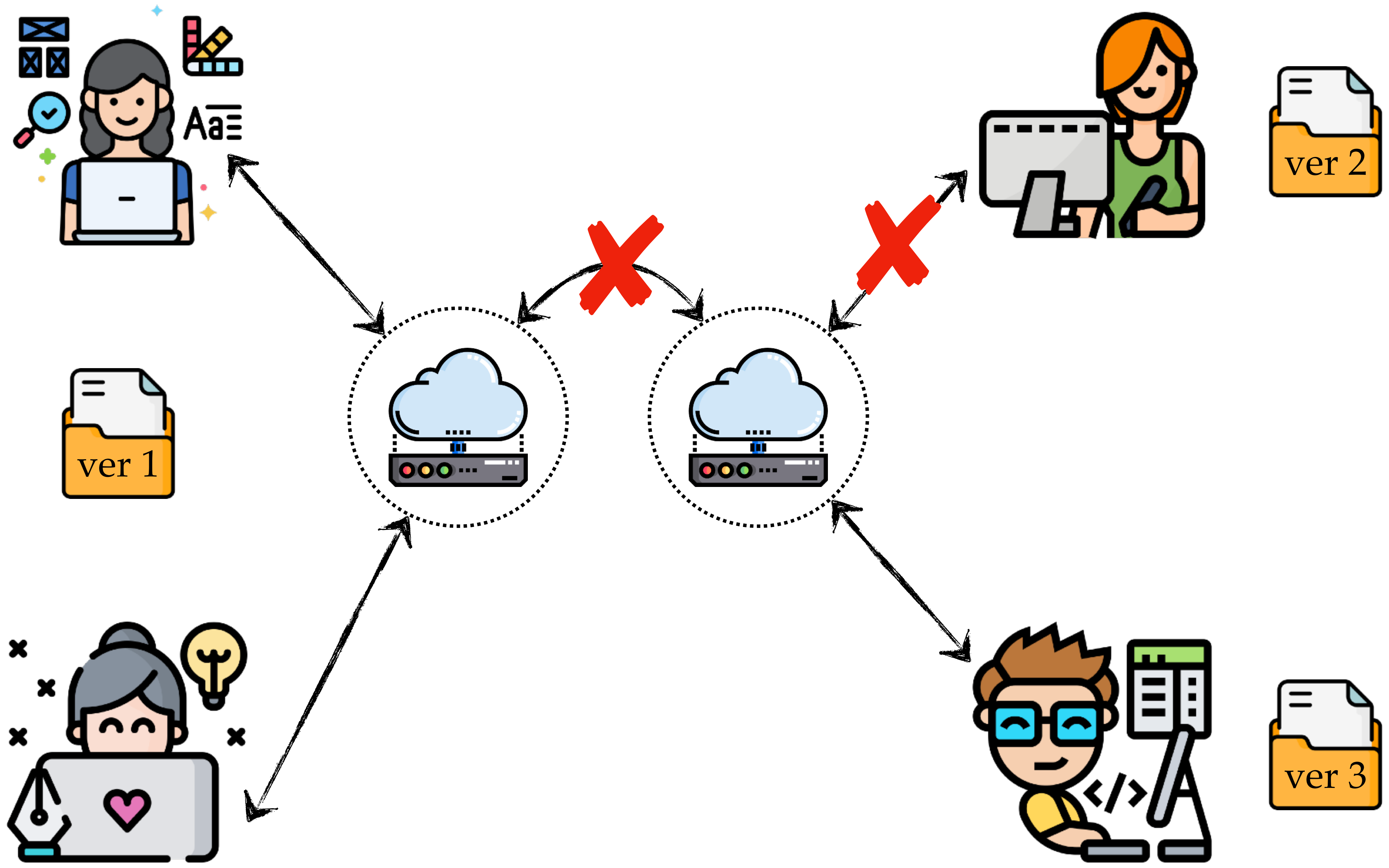

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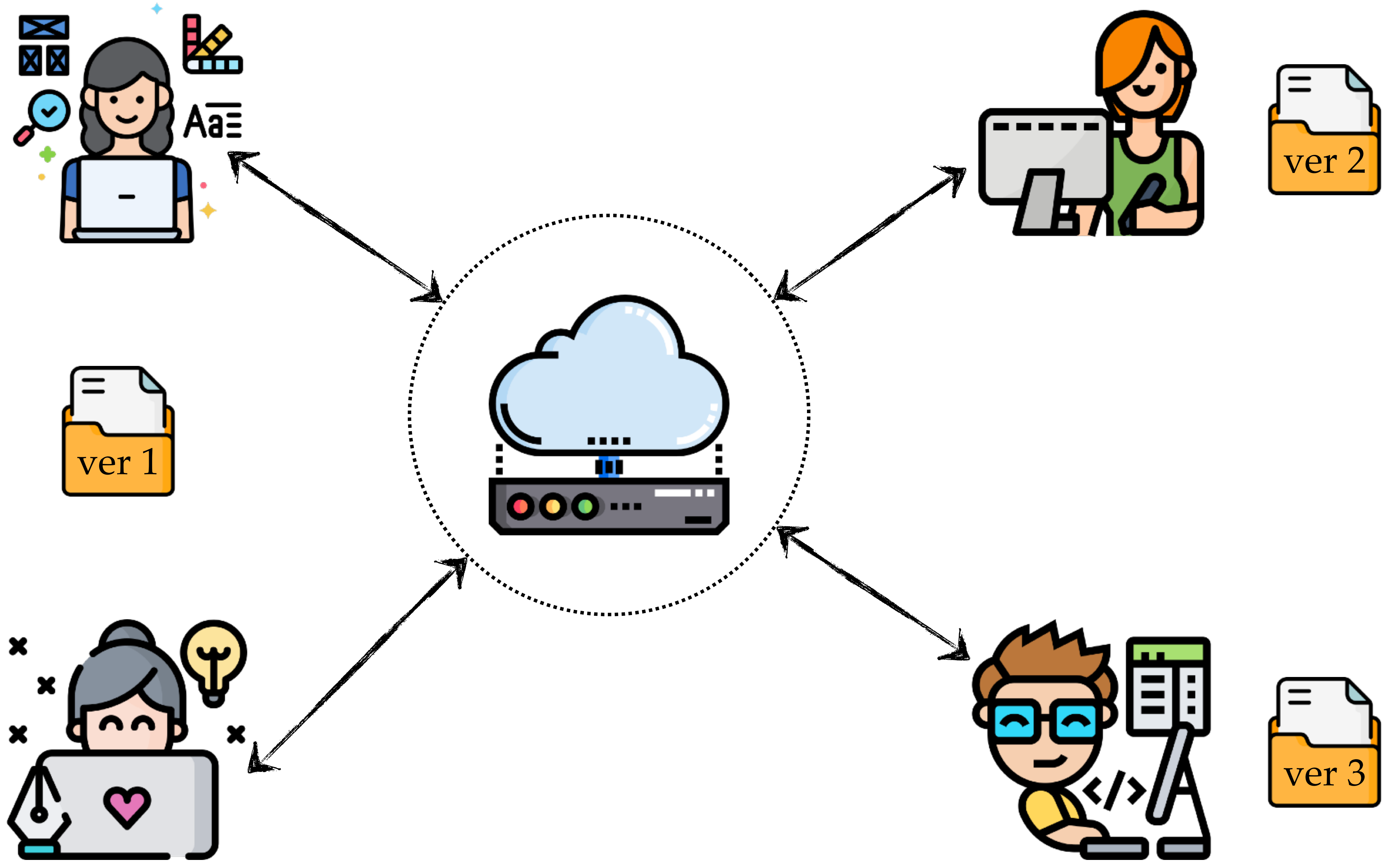

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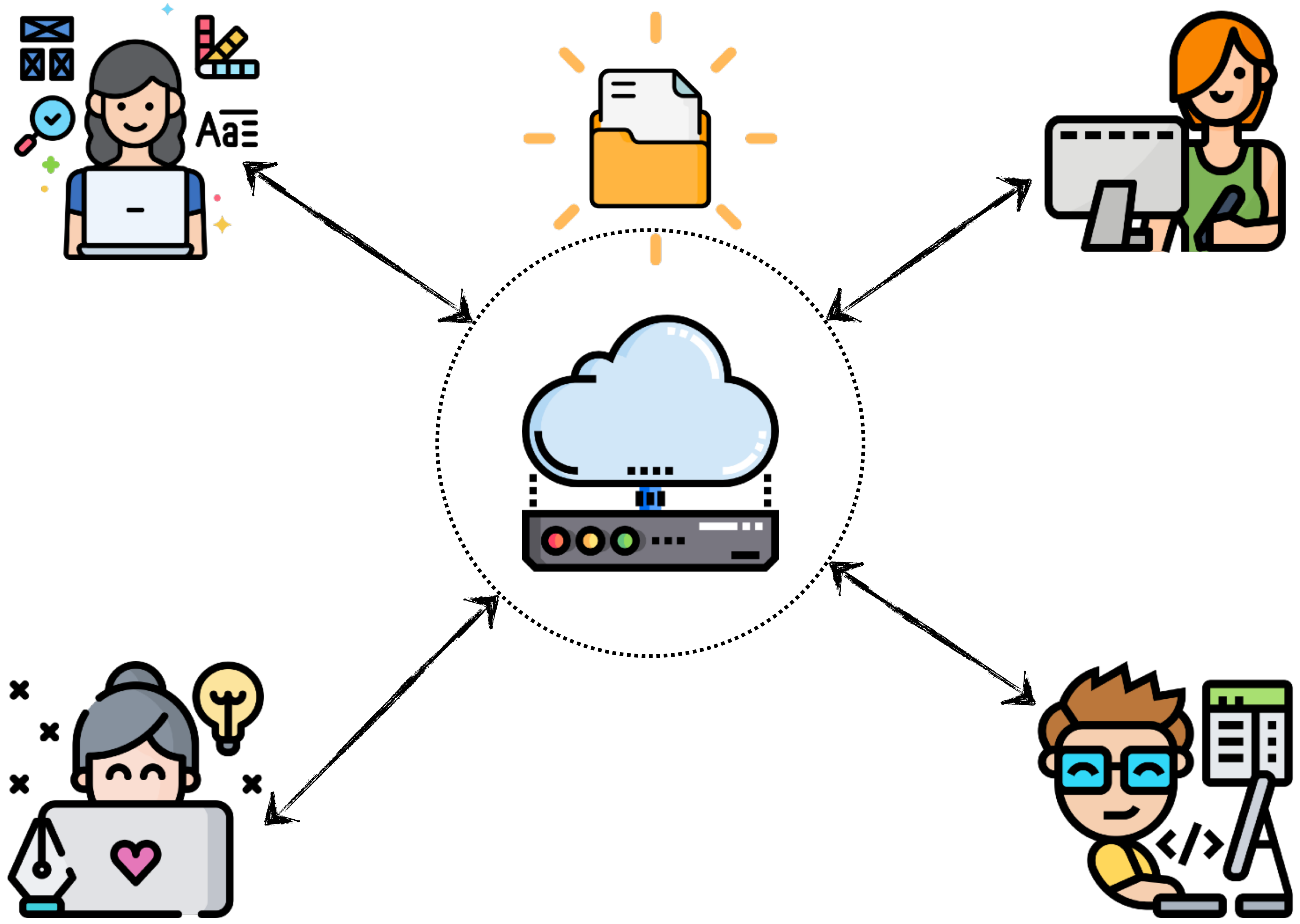

*How do we build such applications?*

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Embed the notion of replication into the data types

# Mergeable Replicated Data Types (MRDTs)

- MRDTs = Sequential data types + 3-way merge function à la Git

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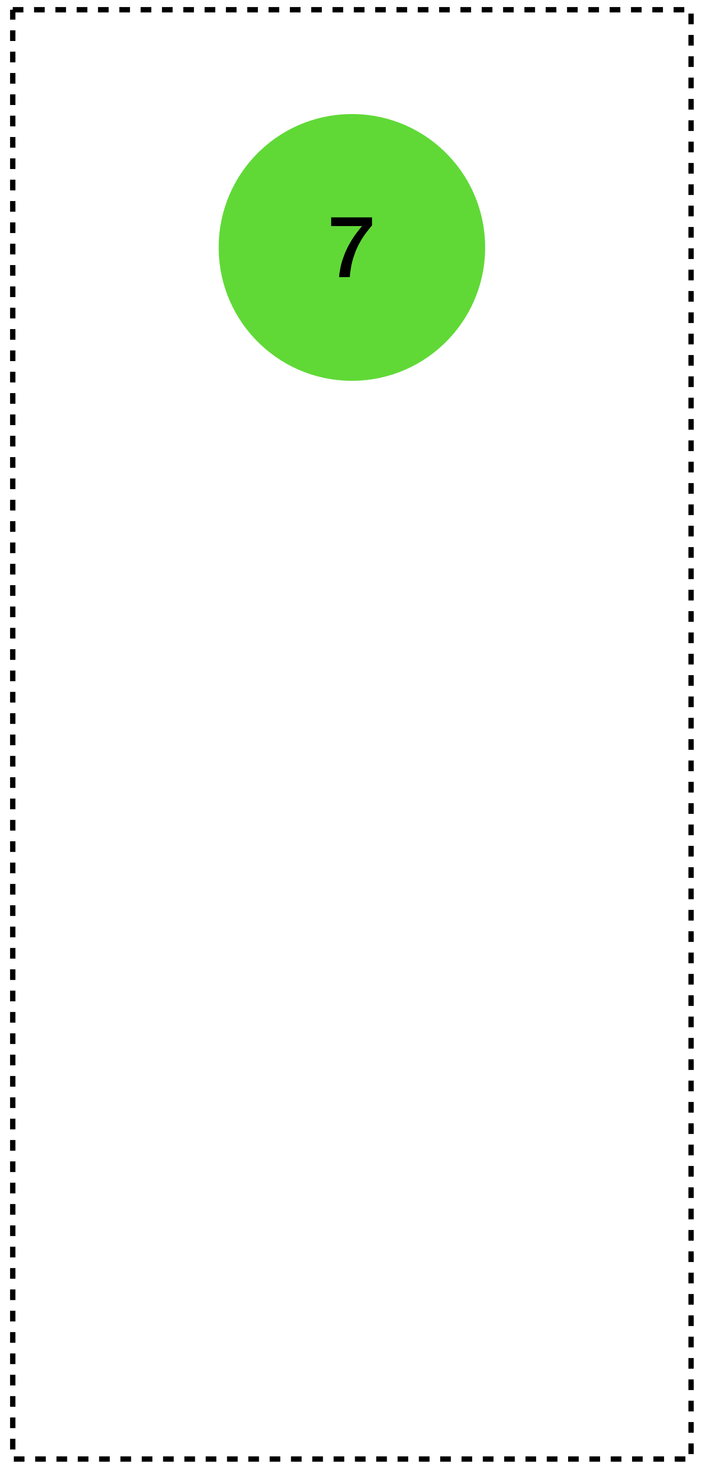
**Increment-only counter**




**Replica 1**

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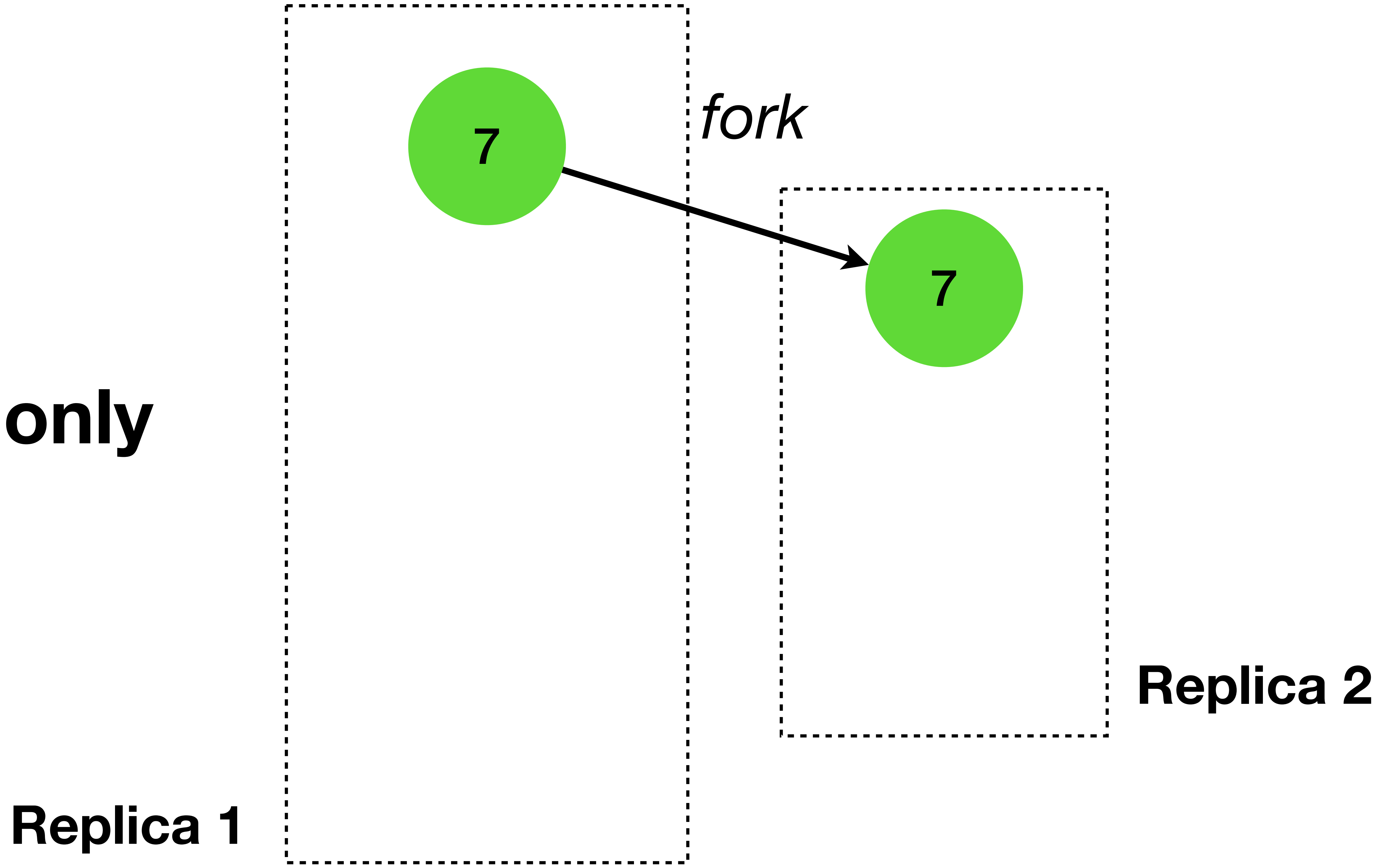

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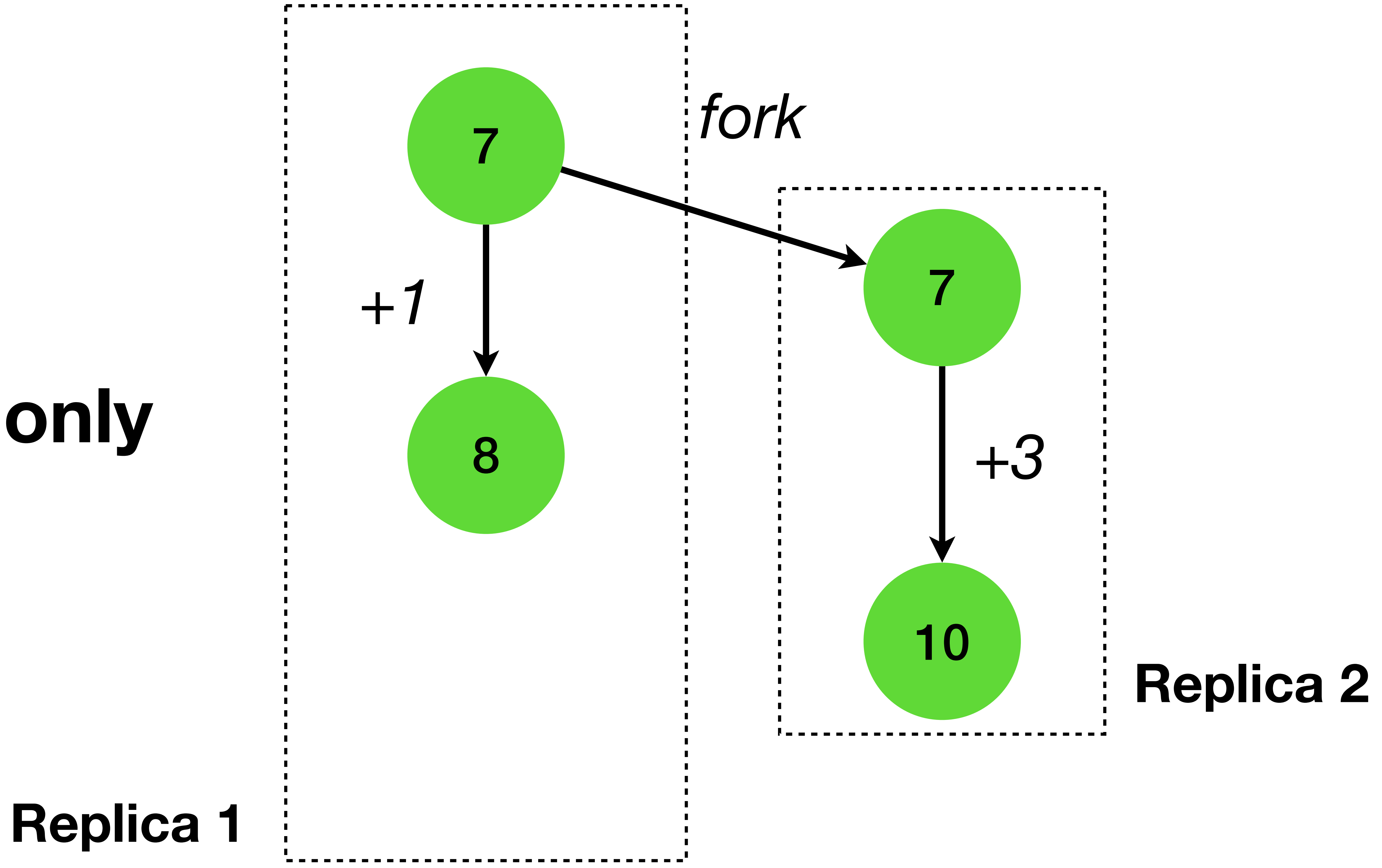

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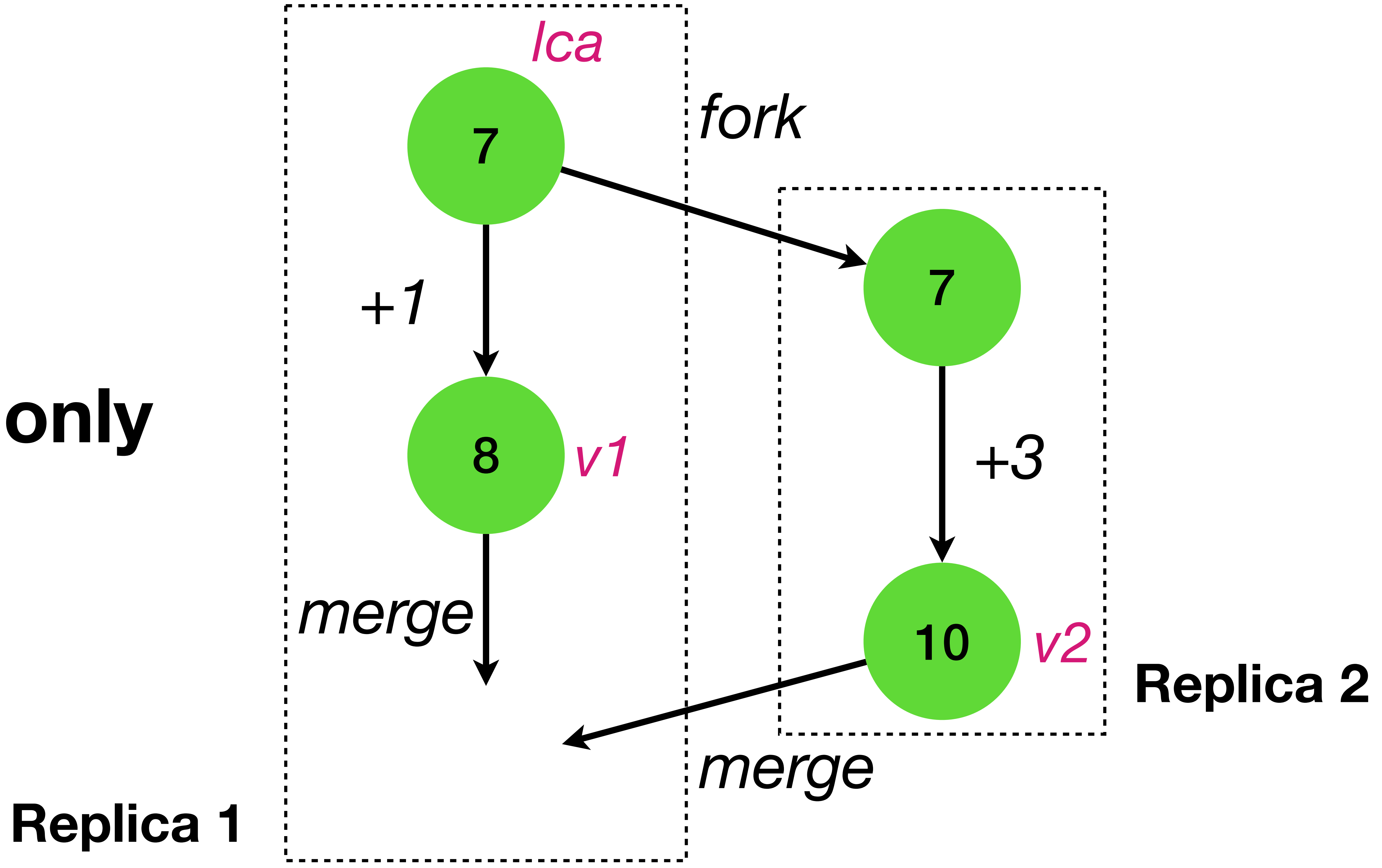


```
let merge lca v1 v2 =
  lca + (v1 - lca) + (v2 - lca)
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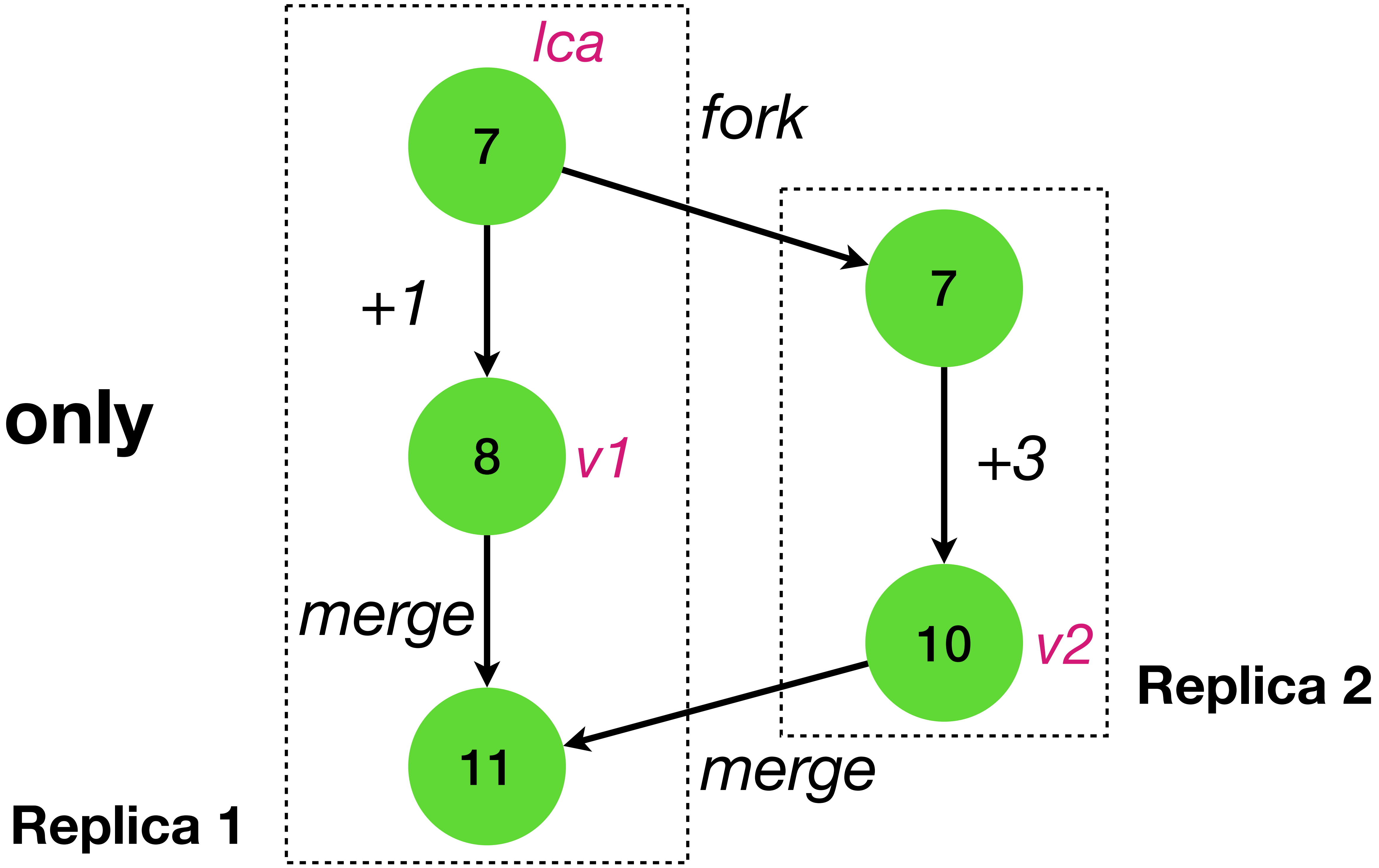


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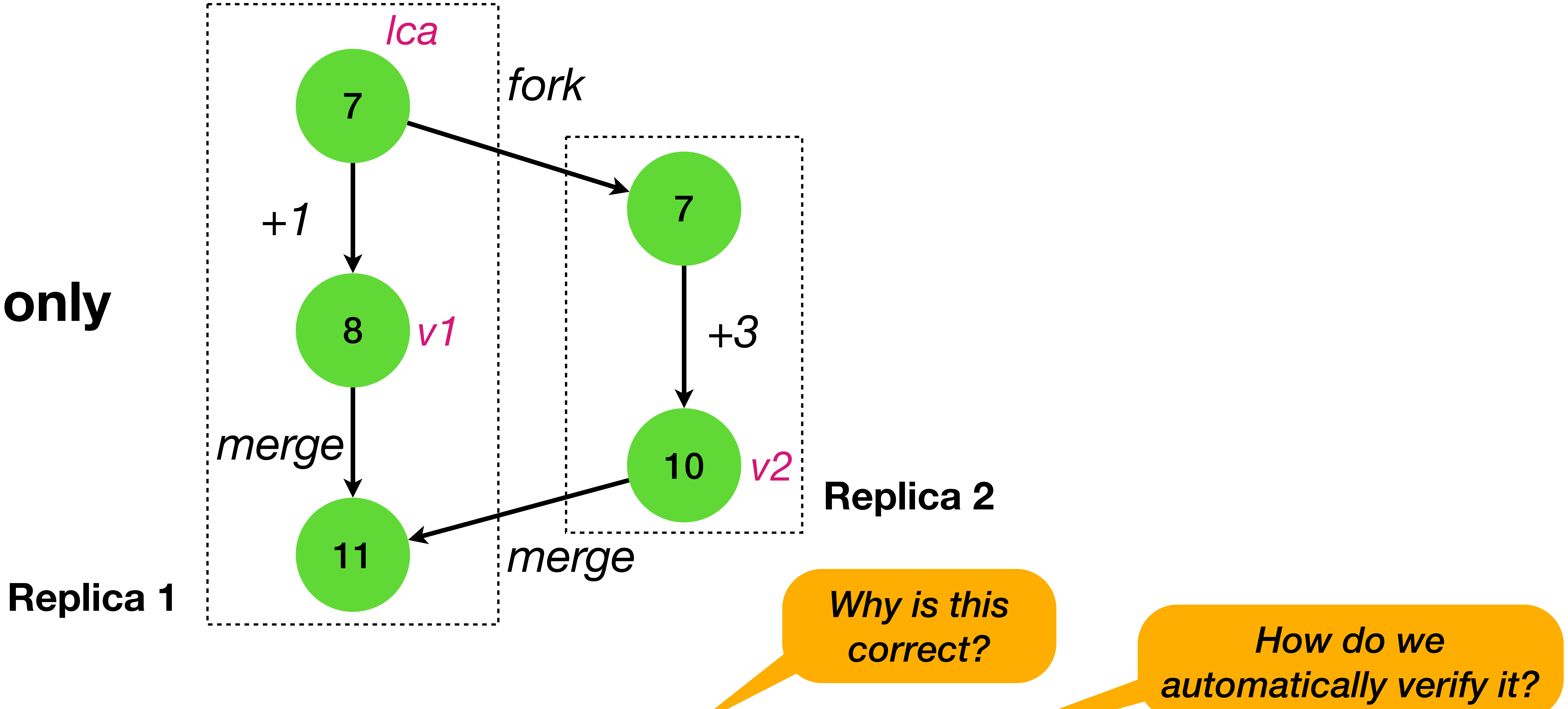


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# Verification using Algebraic Properties

- State-based Convergent Replicated Data Types (CRDTs)
  - Merge is 2-way $-\mu(v_1, v_2)$
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| $\mu(a, a) = a$ | Idempotence |
| $\mu(\mu(a, b), c) = \mu(a, \mu(b, c))$ | Associativity |

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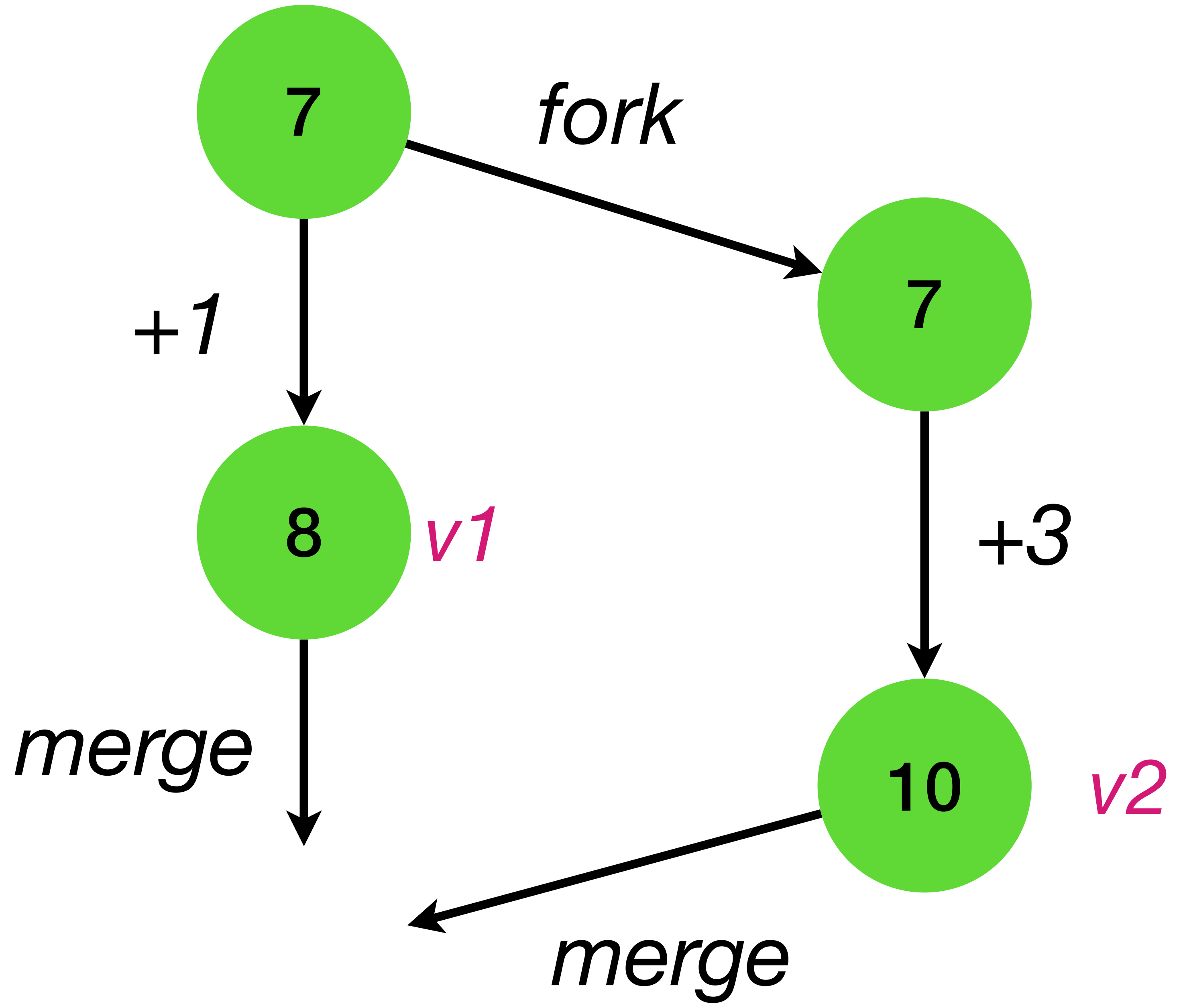

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***Satisfies algebraic properties***

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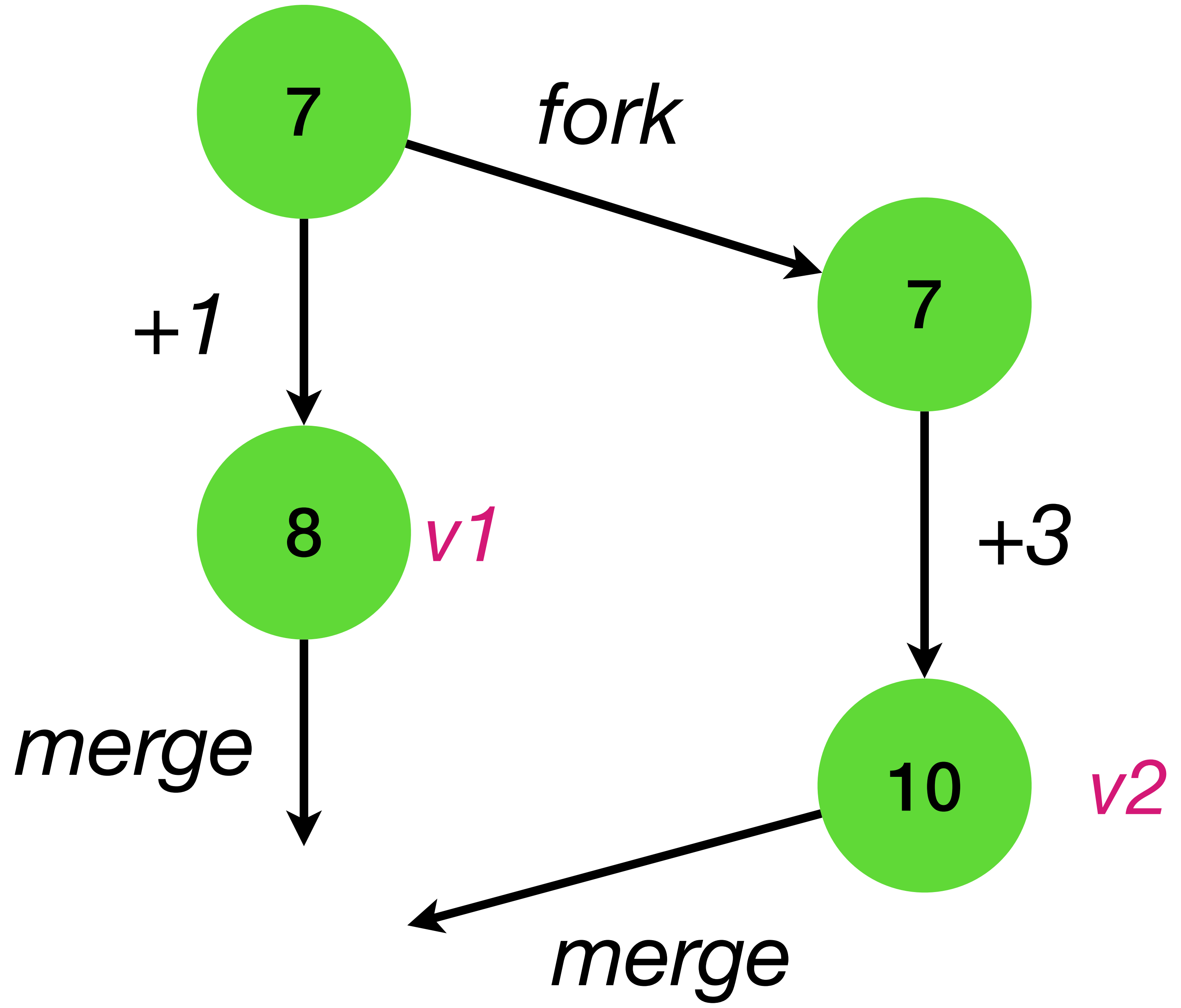

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let merge v1 v2 = max v1 v2
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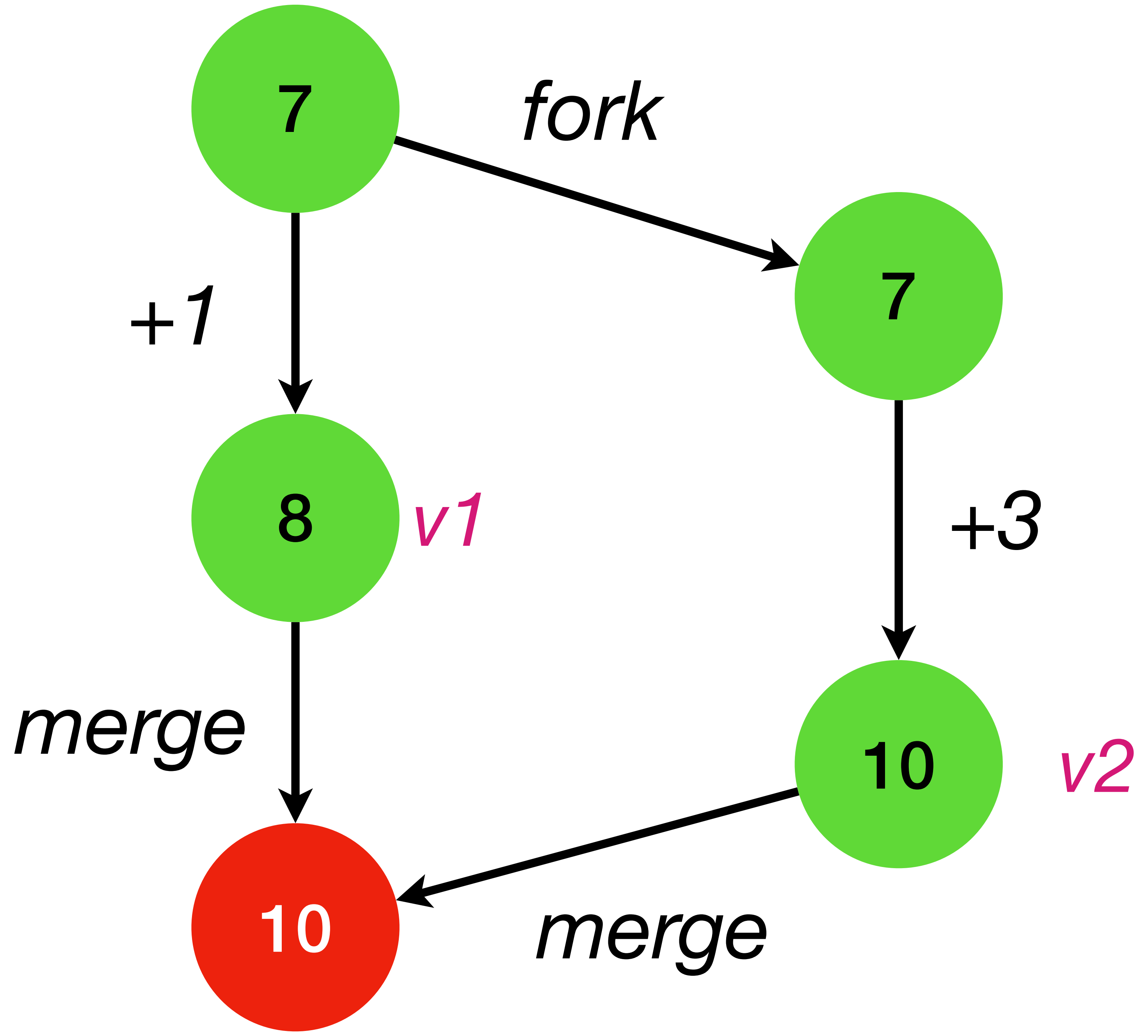
***Intent is not captured***



# Prior work: Capturing Intent through Axiomatic Spec

RESEARCH-ARTICLE | OPEN ACCESS

## Certified mergeable replicated data types

**Authors**: Vimala Soundarapandian, Adharsh Kamath, Kartik Nagar, KC Sivaramakrishnan | Authors Info & Claims

PLDI 2022: Proceedings of the 43rd ACM SIGPLAN International Conference on Programming Language Design and Implementation
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- **Execution graph** = events + visibility (partial order)
- **Operations** = folds over execution graphs

***Specification***

***Simulation relation***

Sequential data type + operations + 3-way merge function

***Implementation***

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***Specification***

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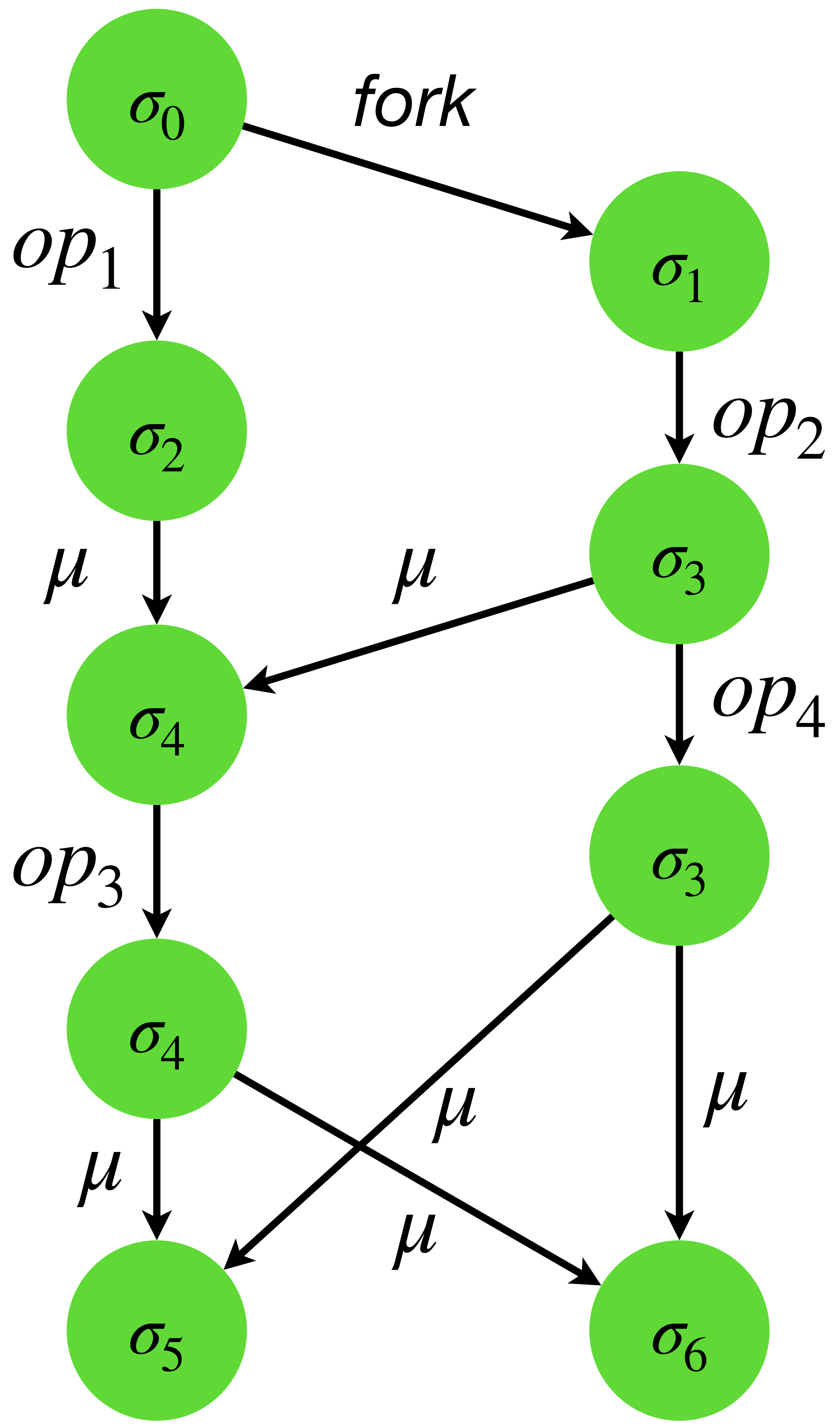
Sequential data type + operations + 3-way merge function

***Implementation***

*Complex*

*Manual*

# Is there a more natural spec?




$$\sigma_5 = \sigma_6 = \text{linearization}(\{op_1, op_2, op_3, op_4\})\ \sigma_0$$

# Replication-aware Linearizability

RESEARCH-ARTICLE

**Replication-aware linearizability**

**Authors**: Chao Wang, Constantin Enea, Suha Orhun Mutluergil, Gustavo Petri | Authors Info & Clai

PLDI 2019: Proceedings of the 40th ACM SIGPLAN Conference on Programming Language Design and Implementation
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- Replica states should be a ***linearisation*** of observed ***update*** operations
    - Use commutativity and asynchrony → Replication-aware (RA) linearizability
- All replicas should ***converge*** to the same state — Strong Eventual Consistency

# Add-wins set CRDT

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$$\Sigma \xrightarrow{\text{add}(a)} \Sigma \ \cup \ \{(a, id)\} \quad \text{where id is fresh}$$

$$\Sigma \xrightarrow{\text{rem}(a)} \{ (e, id) \in \Sigma \mid e \neq a \}$$

$$\Sigma \xrightarrow{\text{read}()} A \quad \text{where } A = \{ e \mid \exists\, id\,.\,(e, id) \in \Sigma \}$$

**Add-wins set *sequential* specification**

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**Add-wins set *sequential* specification**

- **Operation-based CRDT**
  - Operations are asynchronously transmitted to other replicas
  - Applied in *causal order*

# Replication-aware Linearizability

A history $h = (E, vis)$, $E \subseteq \mathrm{Queries} \uplus \mathrm{Updates}$, is RA-linearizable w.r.t. a sequential specification $\mathrm{Spec}$ if there exists a total order $seq$ on $E$ (same events) such that:

$(i)$ $vis \cup seq$ is acyclic;

$(ii)$ $seq{\downarrow}_{Updates} \in \mathrm{Spec};$

$(iii)$ $\forall \ell_{qr} \in E, \left(seq{\downarrow}_{\mathrm{vis}^{-1}(\ell_{qr}) \cap Updates}\right) \cdot \ell_{qr} \in \mathrm{Spec}\,.$

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- A CRDT is said to be RA-linearizable if every history *h* is RA-linearizable
- Add-wins set is RA-linearizable
- *RA-linearizability makes program reasoning easier!*

# Using RA-linearizability for verification

```
add(a);          ||  add(a);

rem(a);          ||

X = read();      ||  Y = read();
```

$$a \in X \implies a \in Y$$

# Using RA-linearizability for verification

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| | | | | |
|---|---|---|---|---|
| `add(a);` | `rem(a);` | `add(a);` | `X = read();` | `Y = read()` |
| `{(a,i)}` | `{ }` | `{(a,j)}` | `X = {A}` | `Y = {A}` |

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  - Make $a \in X$ true and $a \in Y$ false

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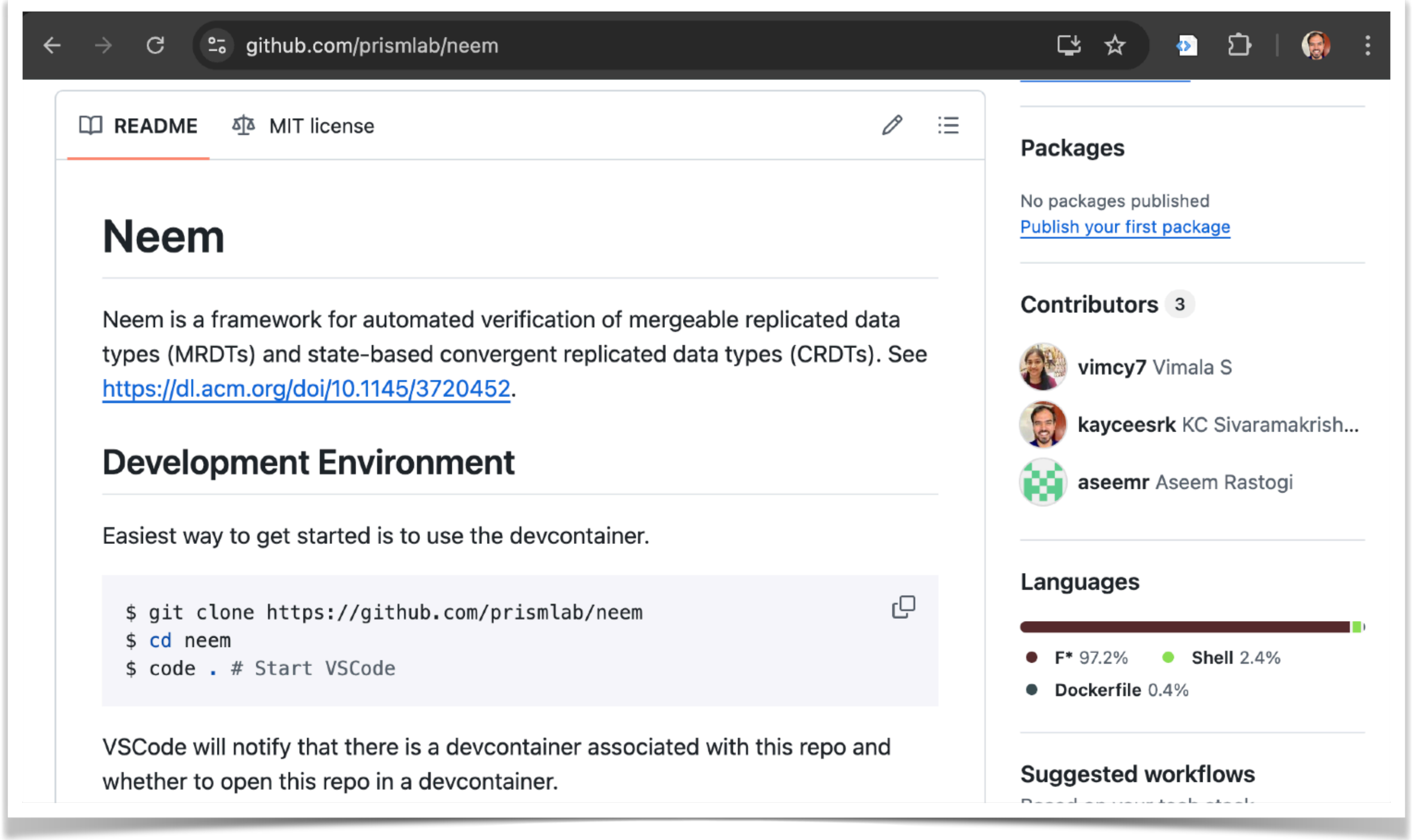
- Presented a proof methodology to show that a CRDT is linearisable
- Not automated or mechanised

# Neem — Automatic verification of RDTs

- **What's in the box?**
  - Definition of RA-linearizability for MRDTs
  - A novel induction scheme for MRDTs and state-based CRDTs to ***automatically*** verify RA-linearizability
  - Implemented in F*

# Resolving conflicts

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- Not all operations commute
  - **Add-wins set** — `add(a)` and `rem(a)` do not commute
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- Neem developers provide
  - MRDT = Sequential Data Type + 3-way merge
  - Conflict Resolution $rc$ relation

# Increment-only Counter

| | | |
|---|---|---|
| **State** | 1: | $\Sigma = \mathbb{N}$ |
| **Updates** | 2: | $O = \{\mathrm{inc}\}$ |
| **Queries** | 3: | $Q = \{\mathrm{rd}\}$ |
| **Init State** | 4: | $\sigma_0 = 0$ |
| **Update behaviour** | 5: | $\mathrm{do}(\sigma, \_, \_, \mathrm{inc}) = \sigma + 1$ |
| **Merge** | 6: | $\mathrm{merge}(\sigma_\top, \sigma_1, \sigma_2) = \sigma_\top + (\sigma_1 - \sigma_\top) + (\sigma_2 - \sigma_\top)$ |
| **Query behaviour** | 7: | $\mathrm{query}(\sigma, rd) = \sigma$ |
| **Resolve conflict** | 8: | $\mathrm{rc} = \emptyset$ |

Unique timestamp

Replica ID

# Add-wins Set

**State** 1: $\Sigma = \mathcal{P}(\mathbb{E} \times \mathcal{T})$

**Updates** 2: $O = \{\mathrm{add}_a, \mathrm{rem}_a \mid a \in \mathbb{E}\}$

**Queries** 3: $Q = \{\mathrm{rd}\}$

**Init State** 4: $\sigma_0 = \{\}$

**Update behaviour**

5: $\mathrm{do}(\sigma, t, \_, \mathrm{add}_a) = \sigma \cup \{(a, t)\}$

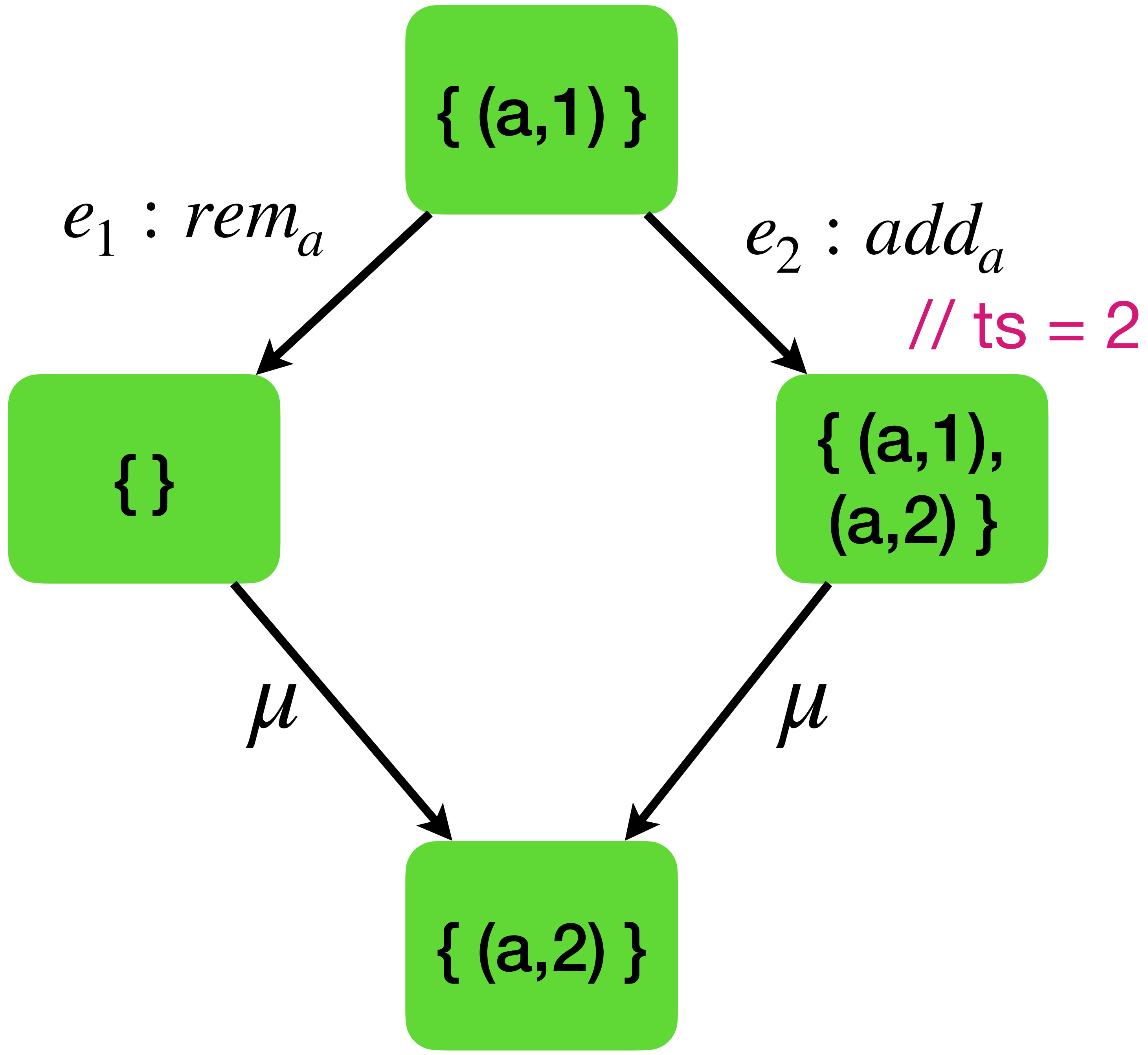
6: $\mathrm{do}(\sigma, \_, \_, \mathrm{rem}_a) = \sigma \backslash \{(a, i) \mid (a, i) \in \sigma\}$

**Merge**

7: $\mathrm{merge}(\sigma_\top, \sigma_1, \sigma_2) = (\sigma_\top \cap \sigma_1 \cap \sigma_2) \cup (\sigma_1 \backslash \sigma_\top) \cup (\sigma_2 \backslash \sigma_\top)$

**Query behaviour** 8: $\mathrm{query}(\sigma, rd) = \{a \mid (a, \_) \in \sigma\}$

**Resolve conflict** 9: $\mathrm{rc} = \{(\mathrm{rem}_a, \mathrm{add}_a) \mid a \in \mathbb{E}\}$



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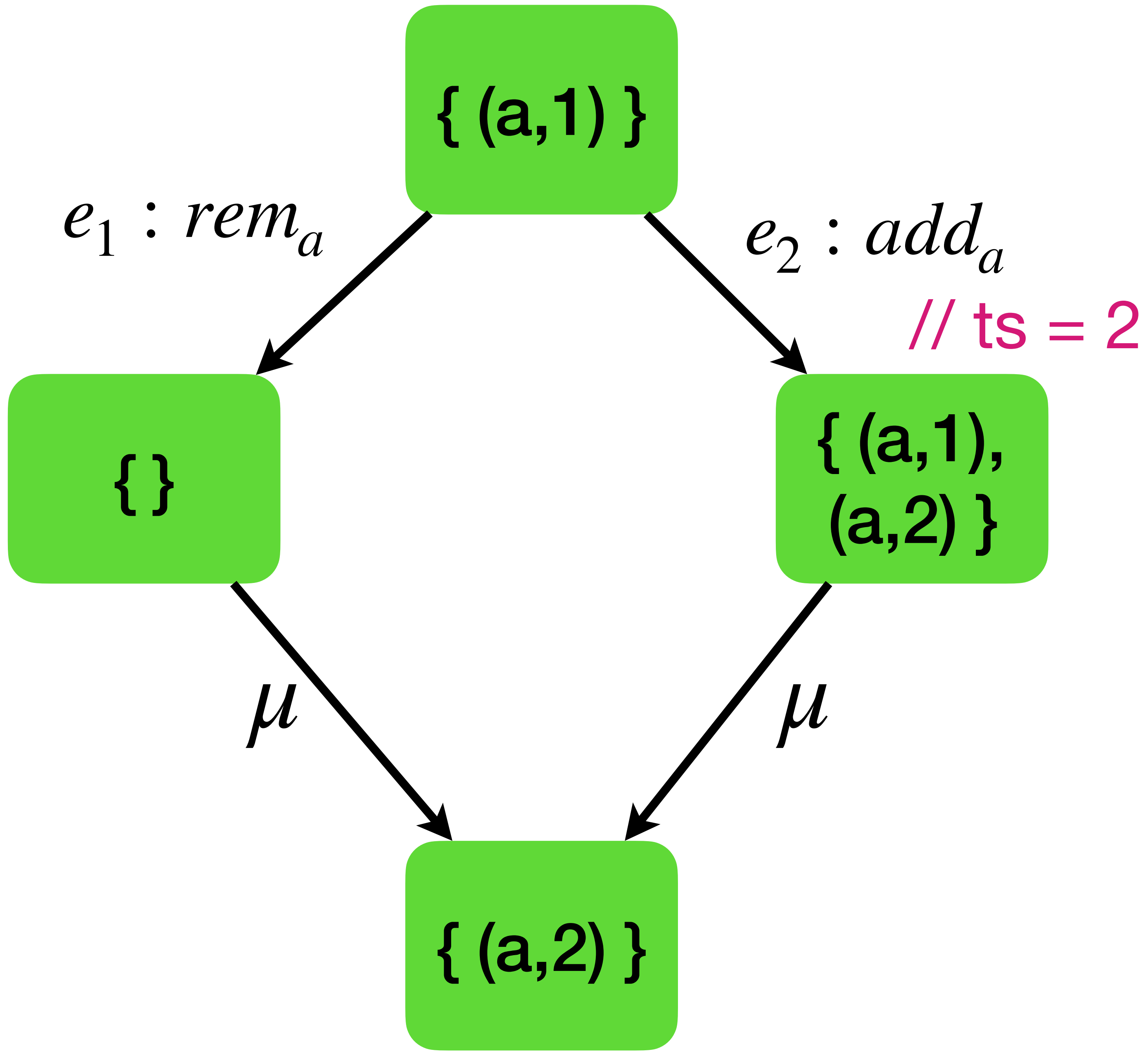


$\{(a,2)\} = add_a(rem_a\{(a,1)\})$

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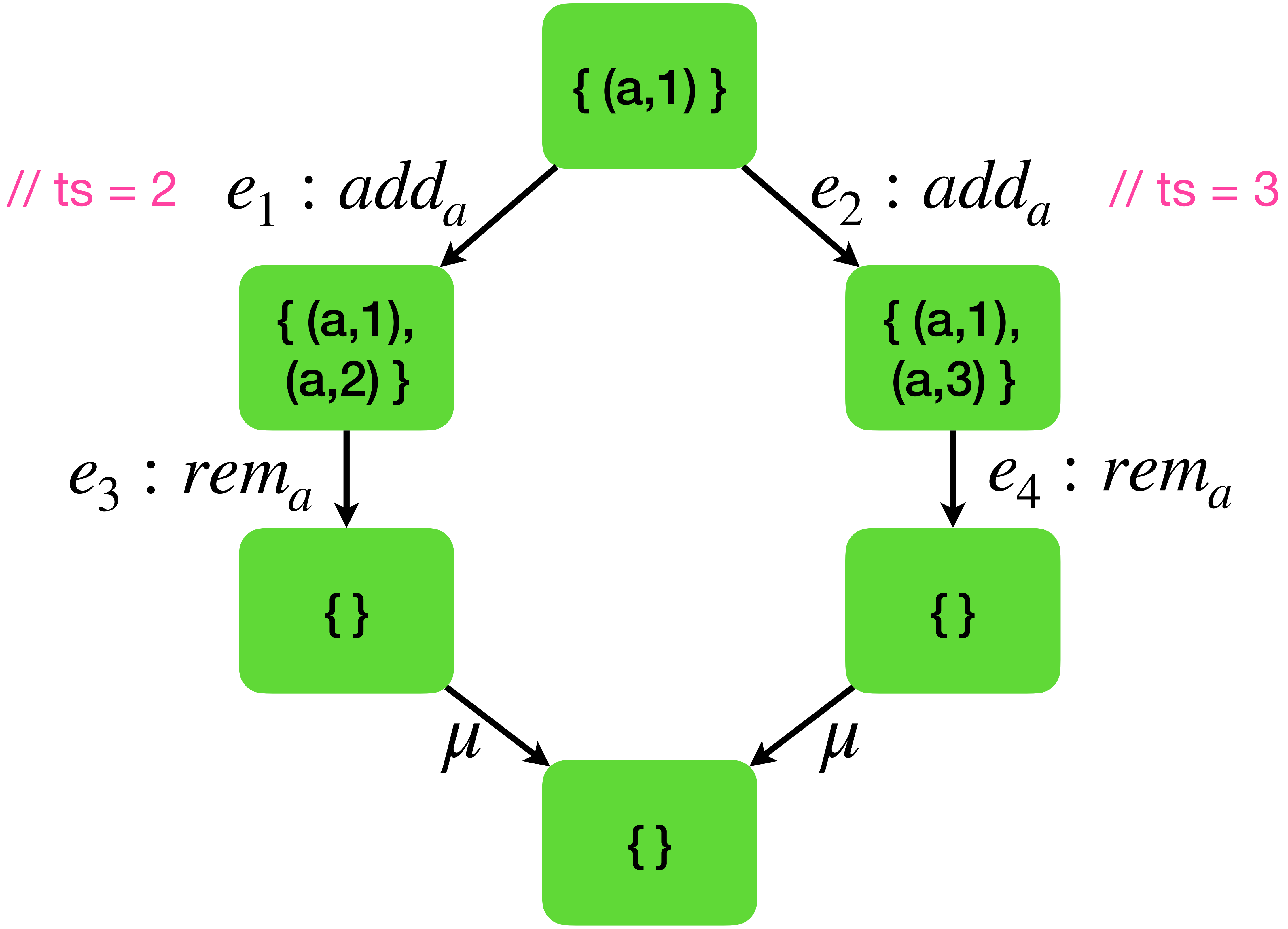

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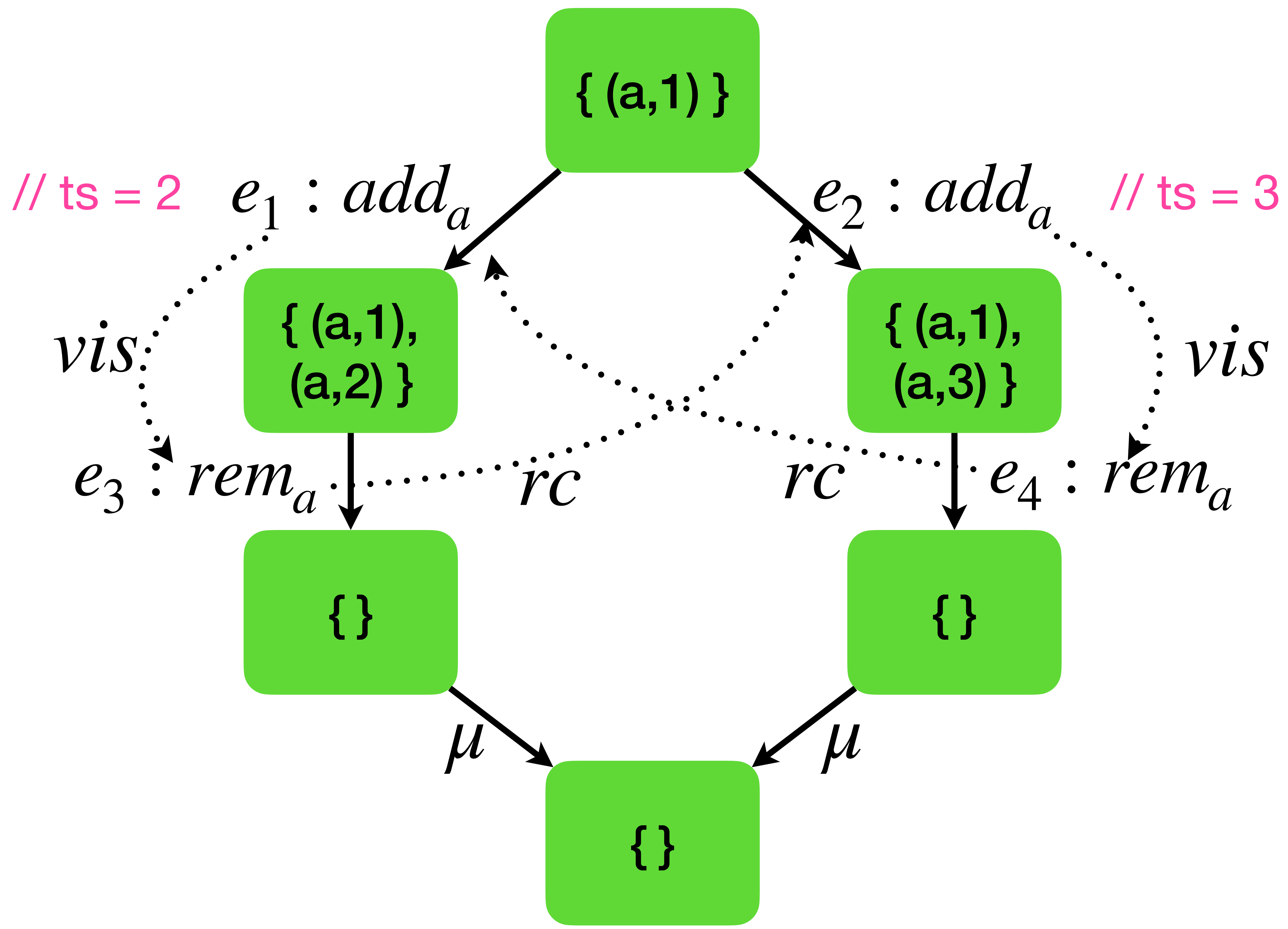


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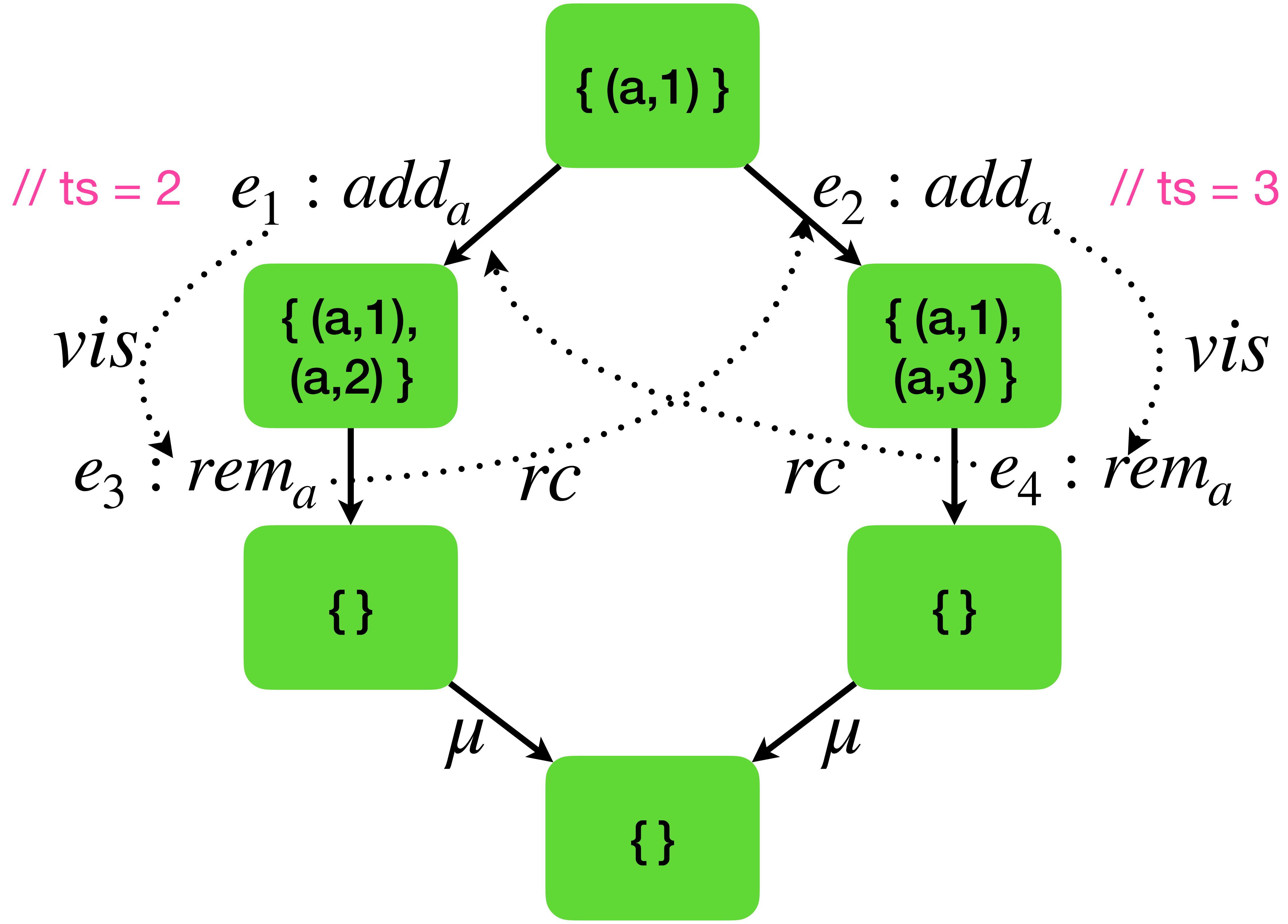


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**No totally ordered $lo$ since $(rc \cap \|) \cup vis$ forms a cycle**

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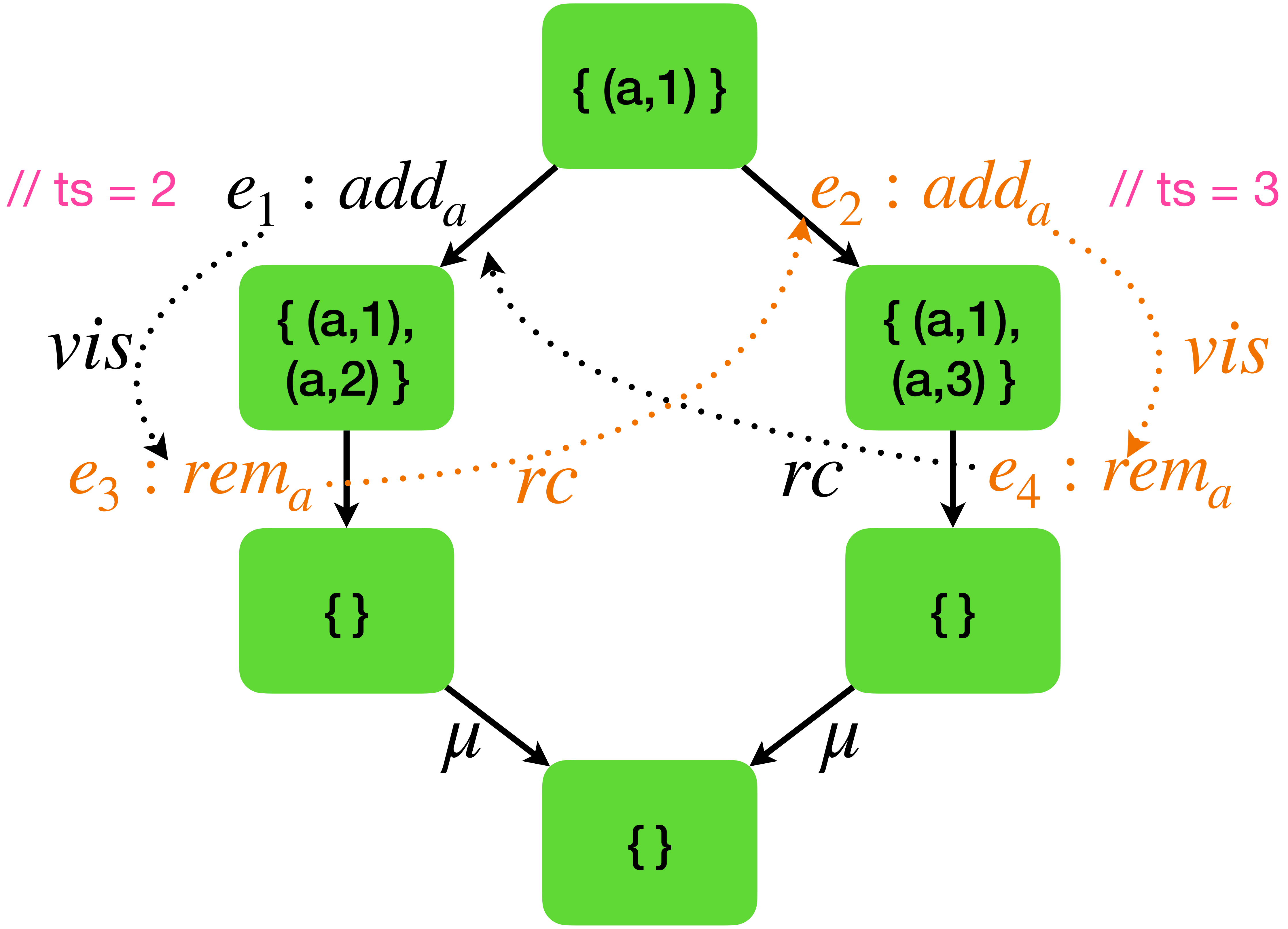


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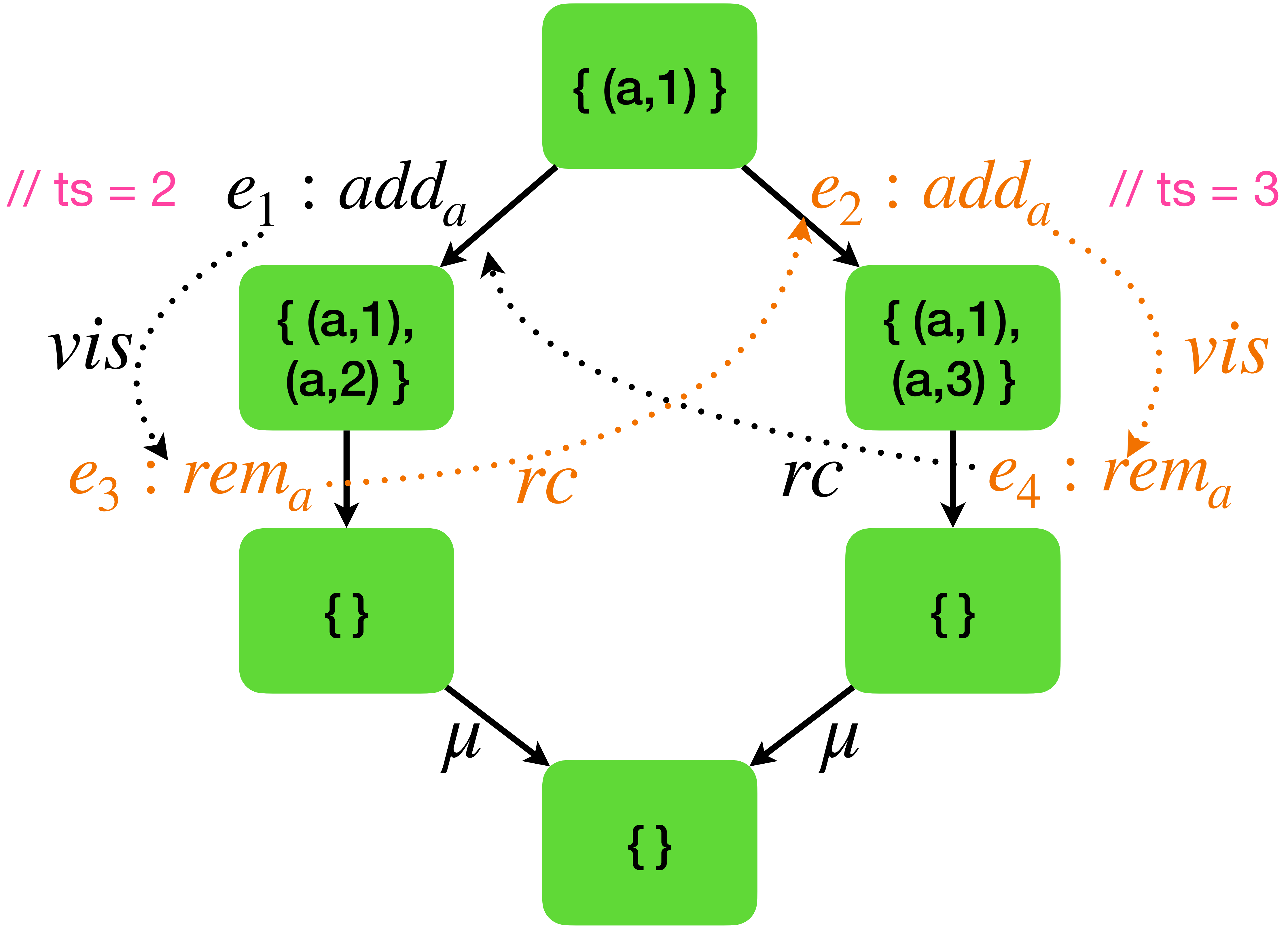
# RA-Linearizability Challenge

- Having the linearisation ***total*** order consistent with both visibility and resolve conflict orders is ***problematic***

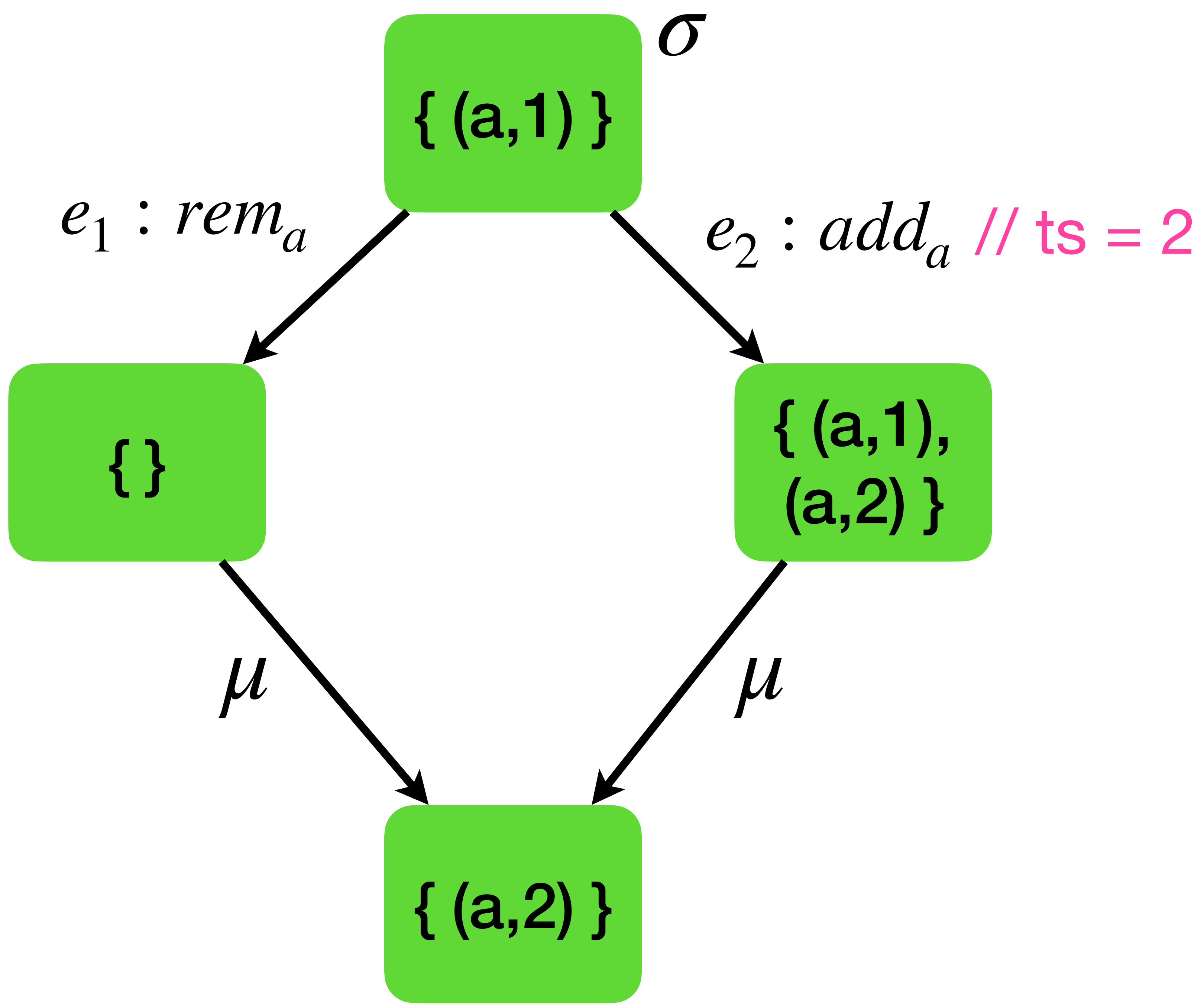



$$rc = \{(rem_a, add_a) \mid a \in \mathbb{E}\}$$

$e_3$ ***conditionally commutes*** wrt $e_2$ due to $e_4$

# Bottom up linearisation

$$rc = \{(rem_a, add_a) \mid a \in \mathbb{E}\}$$




To show

$$\mu(\sigma, e_1(\sigma), e_2(\sigma)) = e_2(e_1(\sigma))$$

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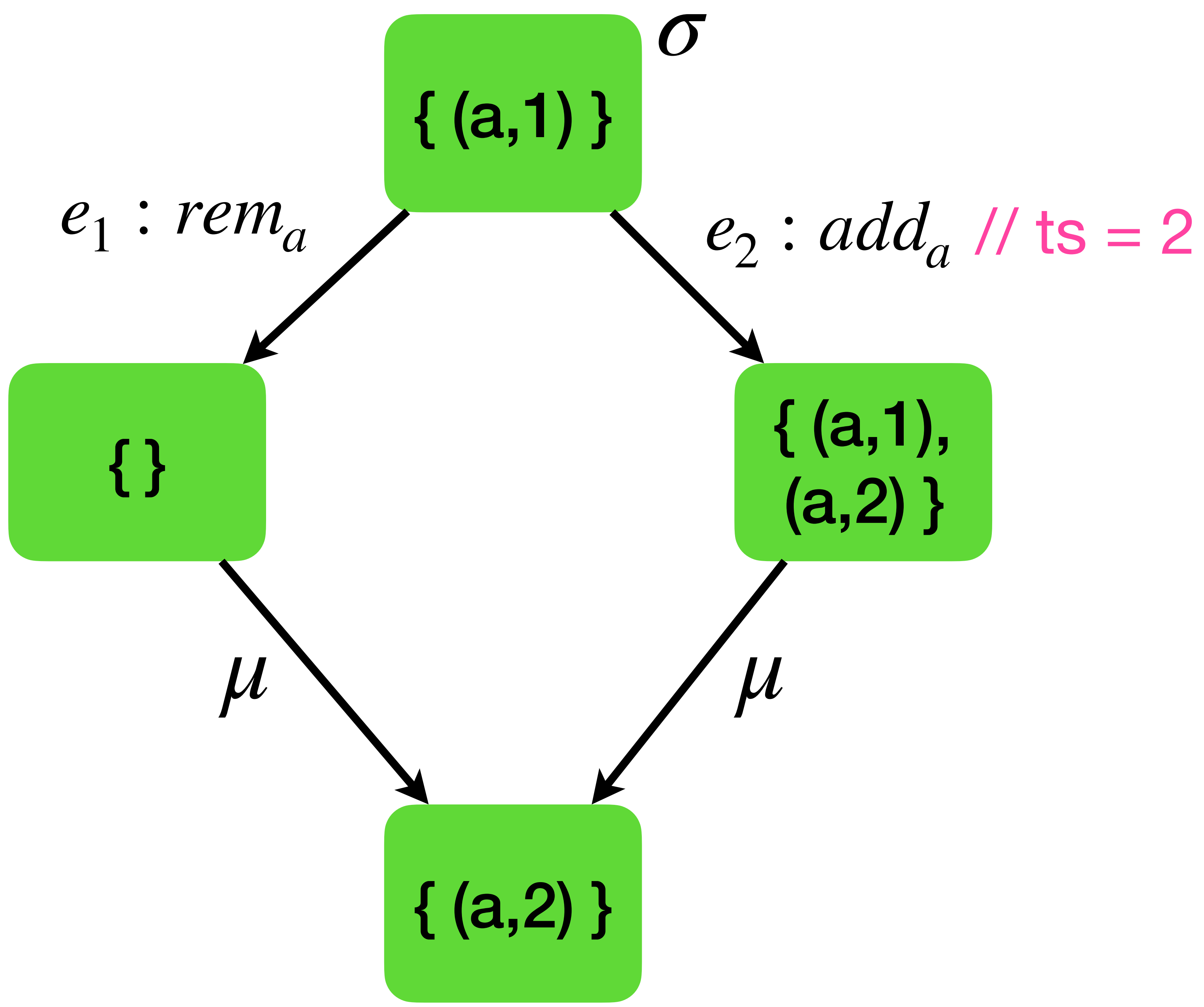


To show

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[BottomUp-2-OP]

$$\frac{e_1 \neq e_2 \quad e_1 \xrightarrow{\mathsf{rc}} e_2 \vee e_1 \rightleftarrows e_2}{\mu(l, e_1(a), e_2(b)) = e_2(\mu(l, e_1(a), b))}$$

[BottomUp-1-OP]

$$\frac{(e_\top \neq \epsilon \wedge e_1 \neq e_\top) \vee (e_\top = \epsilon \wedge l = b)}{\mu(e_\top(l), e_1(a), e_\top(b)) = e_1(\mu(e_\top(l), a, e_\top(b)))}$$

[BottomUp-0-OP]

$$\mu(e_\top(l), e_\top(a), e_\top(b)) = e_\top(\mu(l, a, b))$$

[MergeIdempotence]

$$\mu(a, a, a) = a$$

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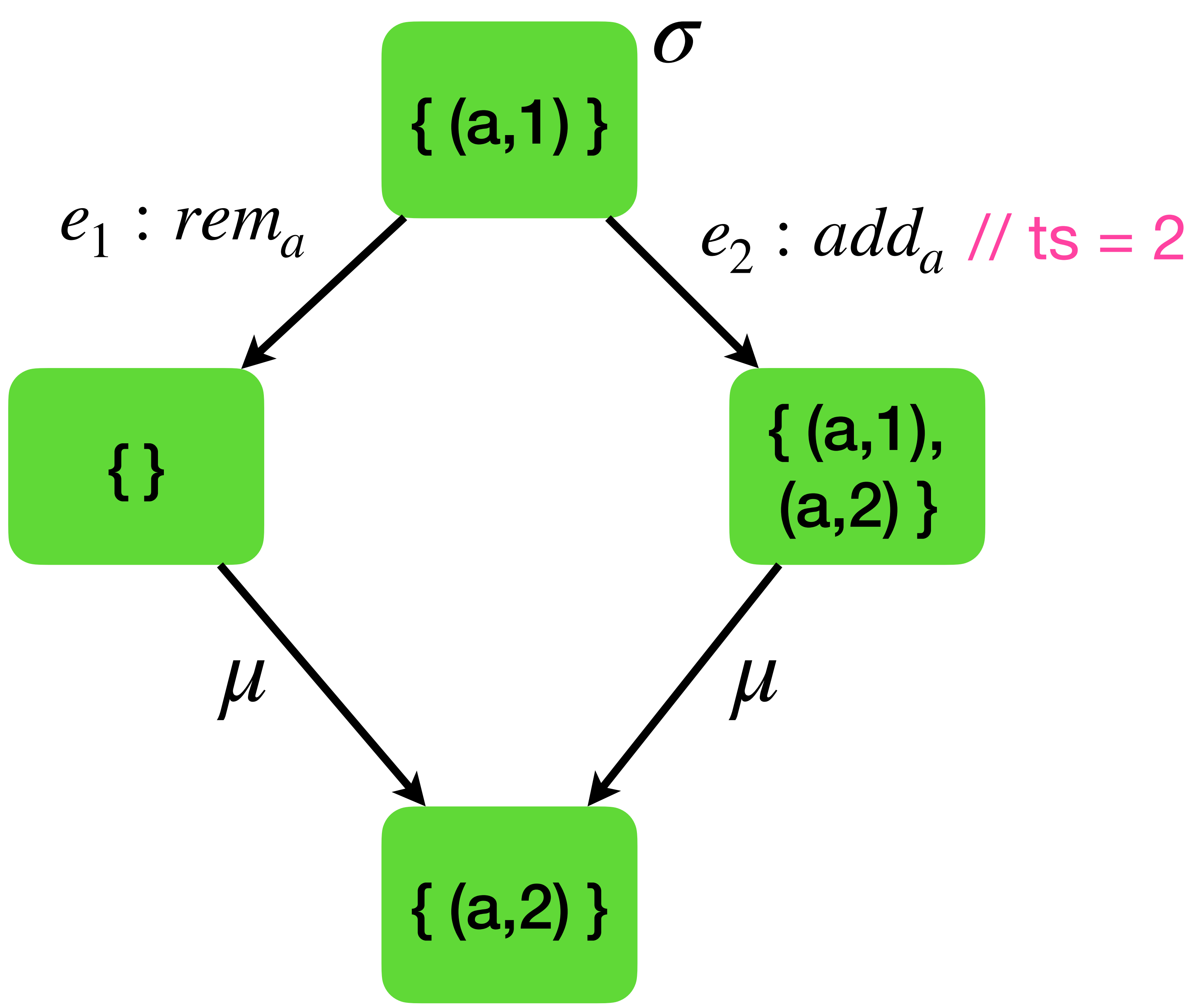


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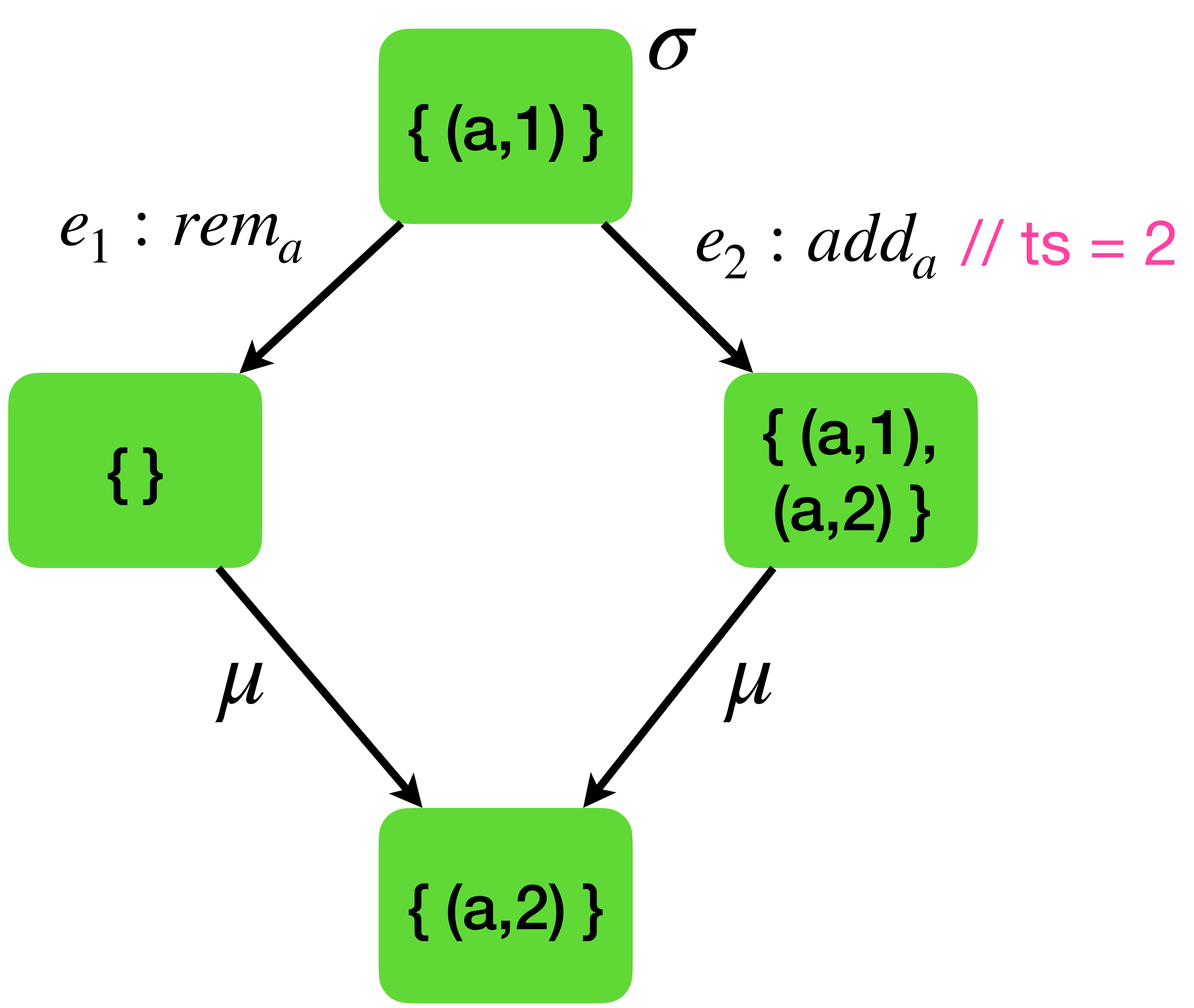


To show

$$e_2(\mu(\sigma, e_1(\sigma), \sigma)) = e_2(e_1(\sigma))$$

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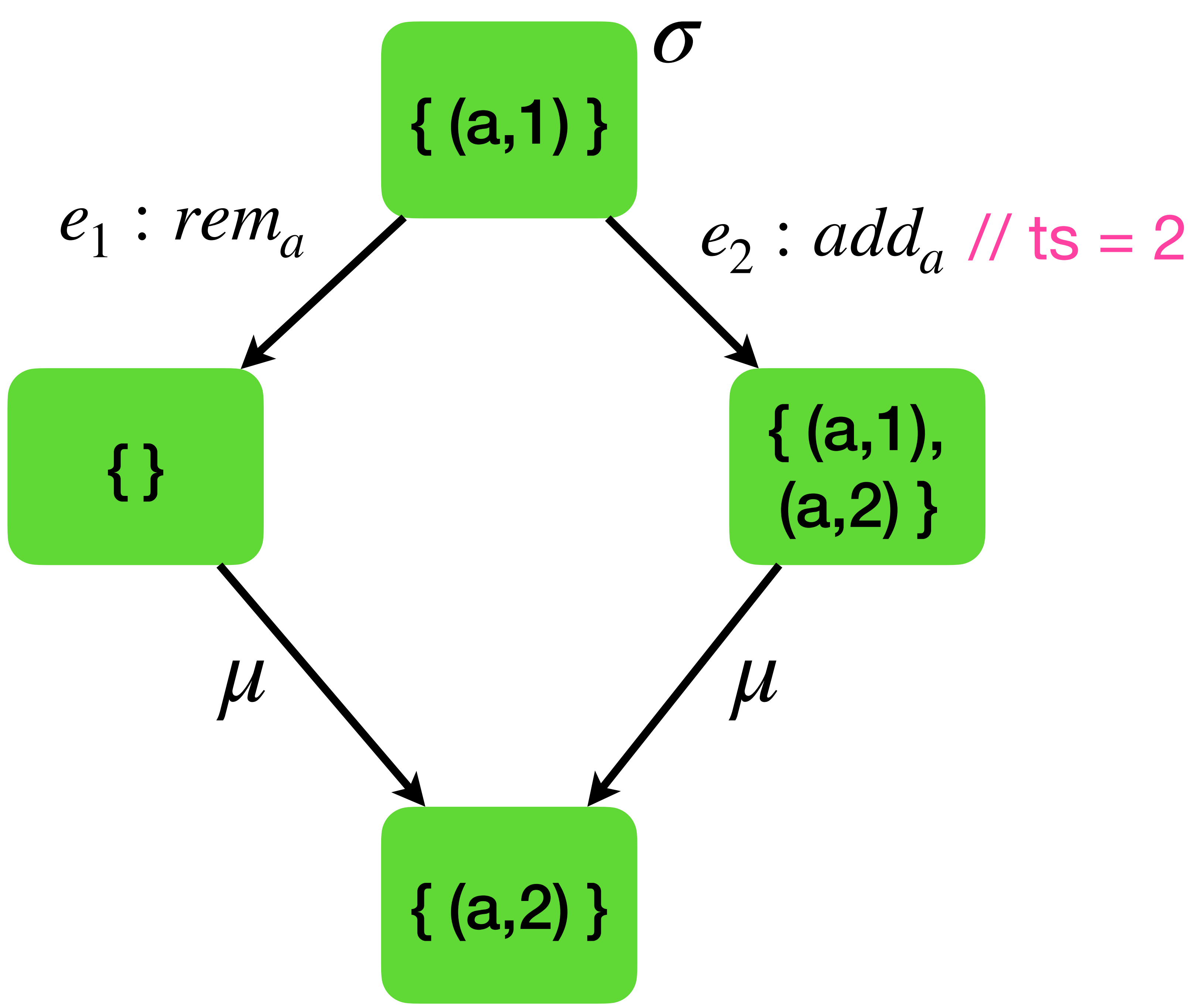


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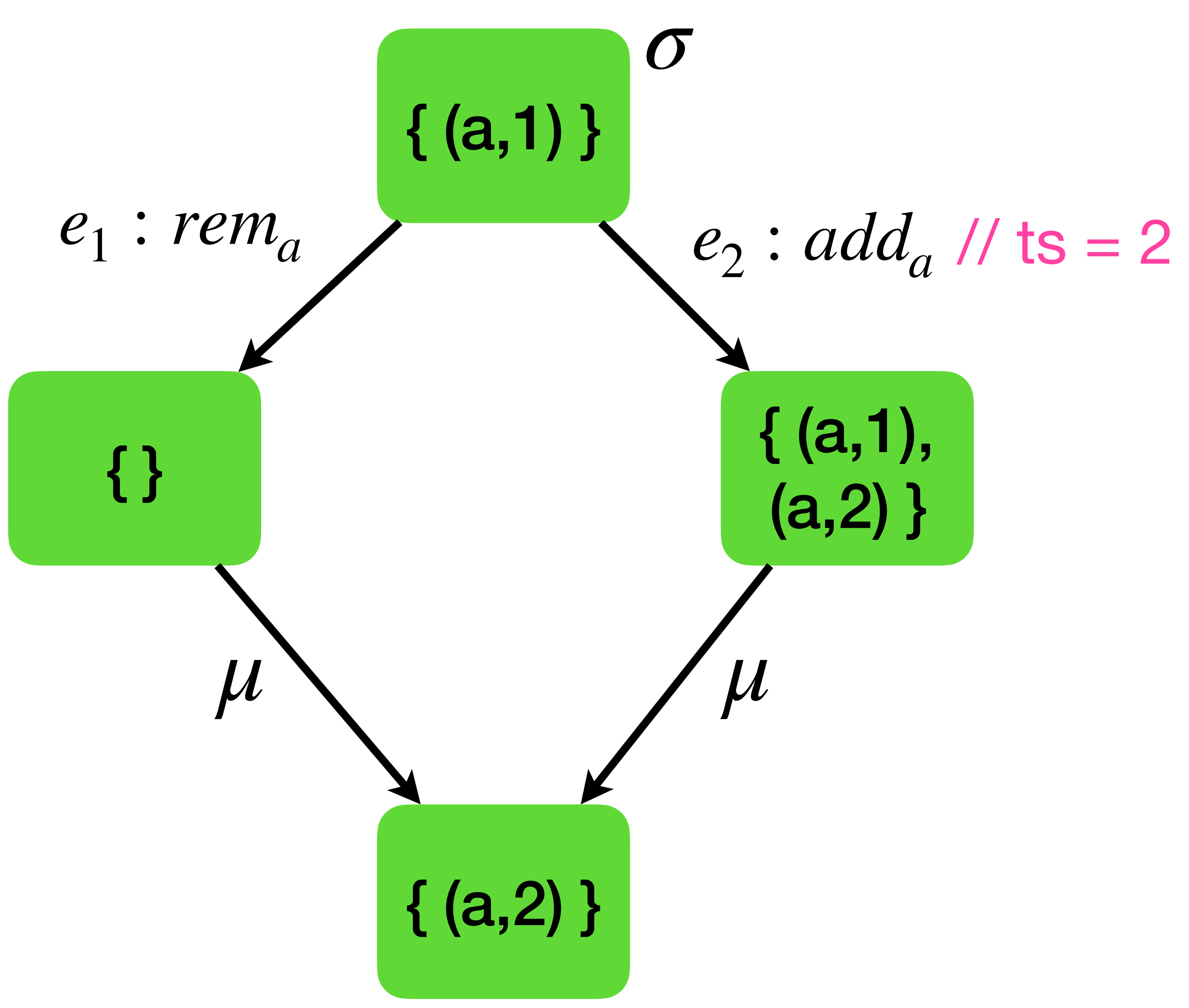


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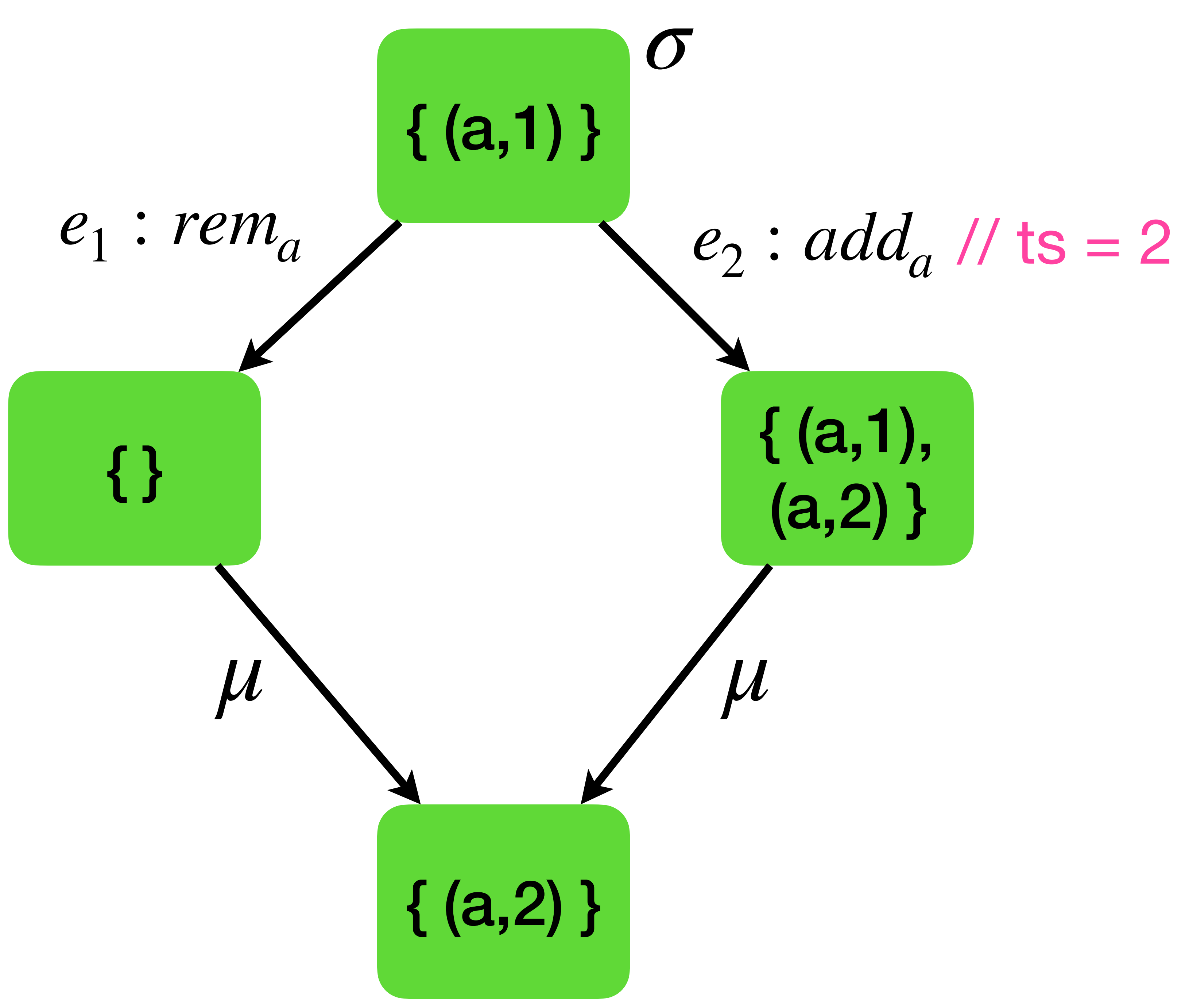


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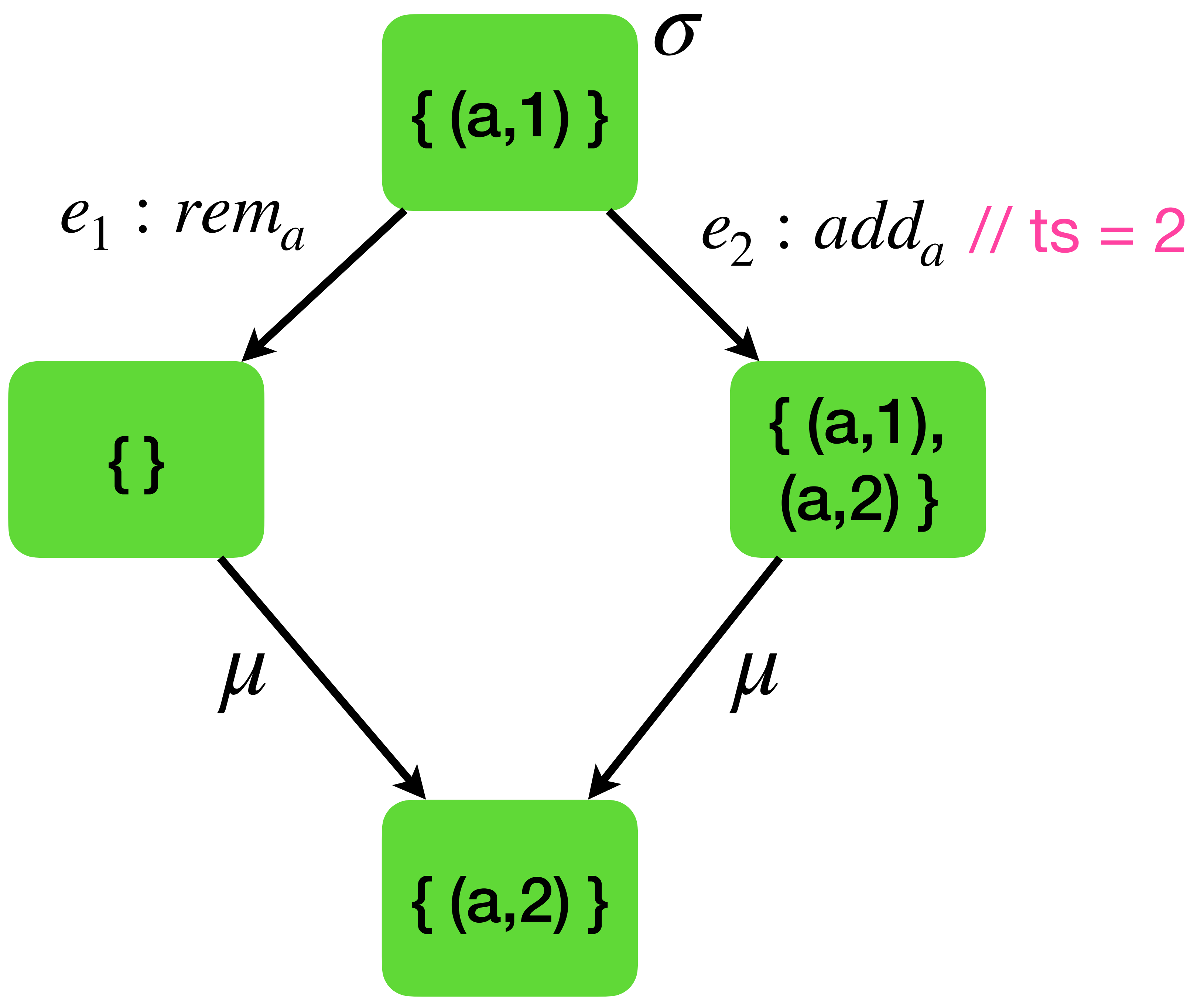


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# Making a good VC

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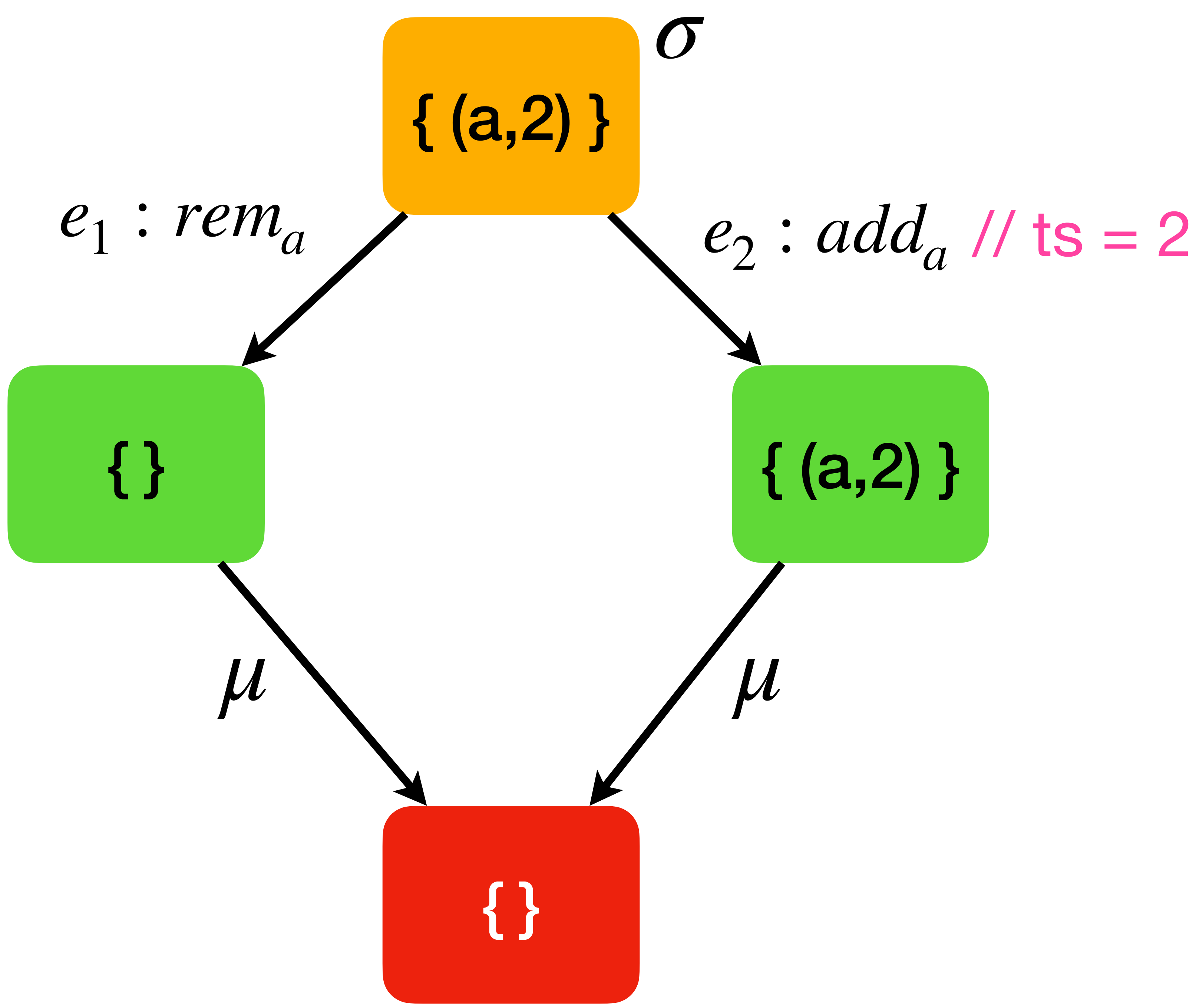


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Cannot prove for an arbitrary $l$

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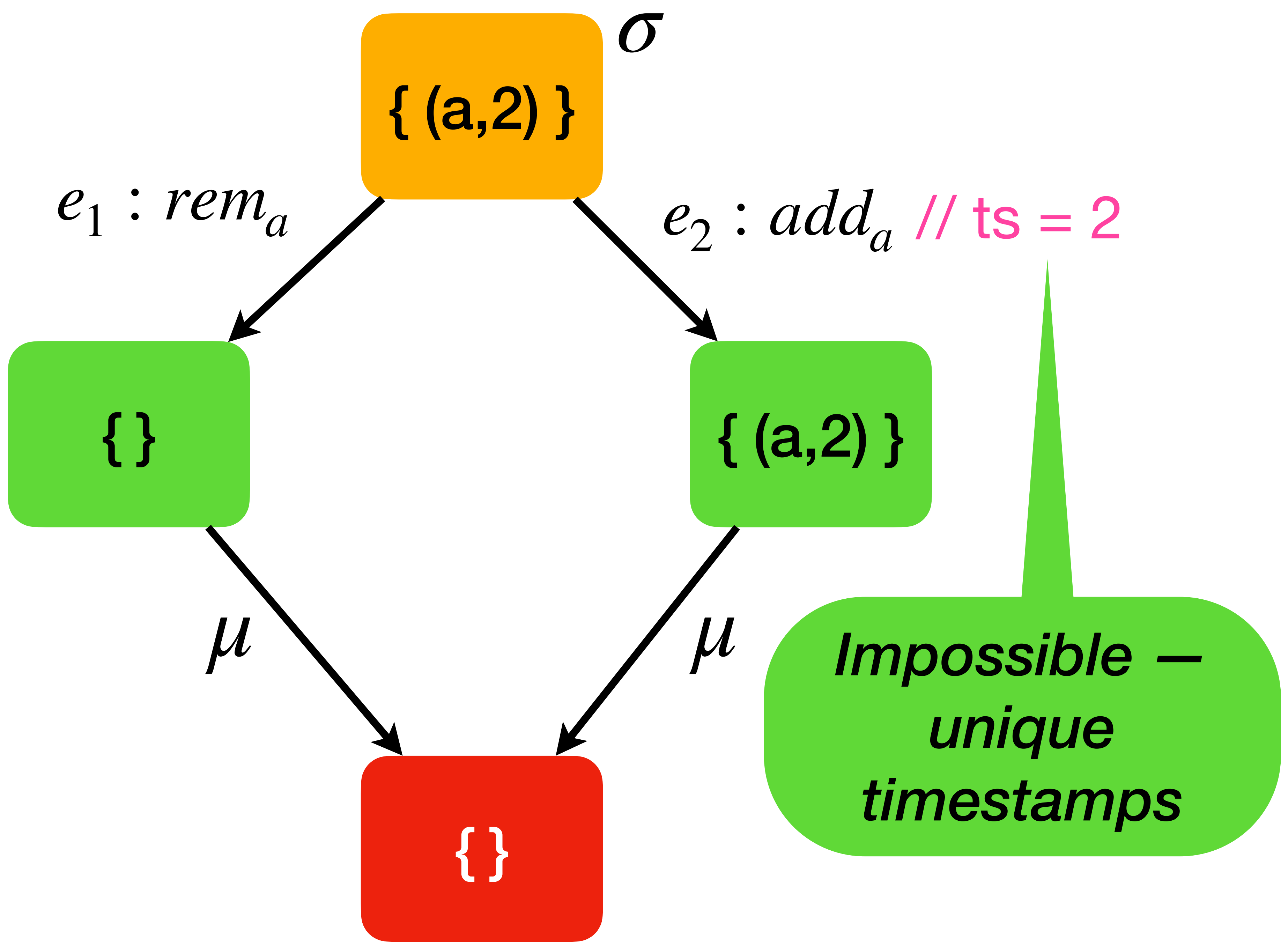


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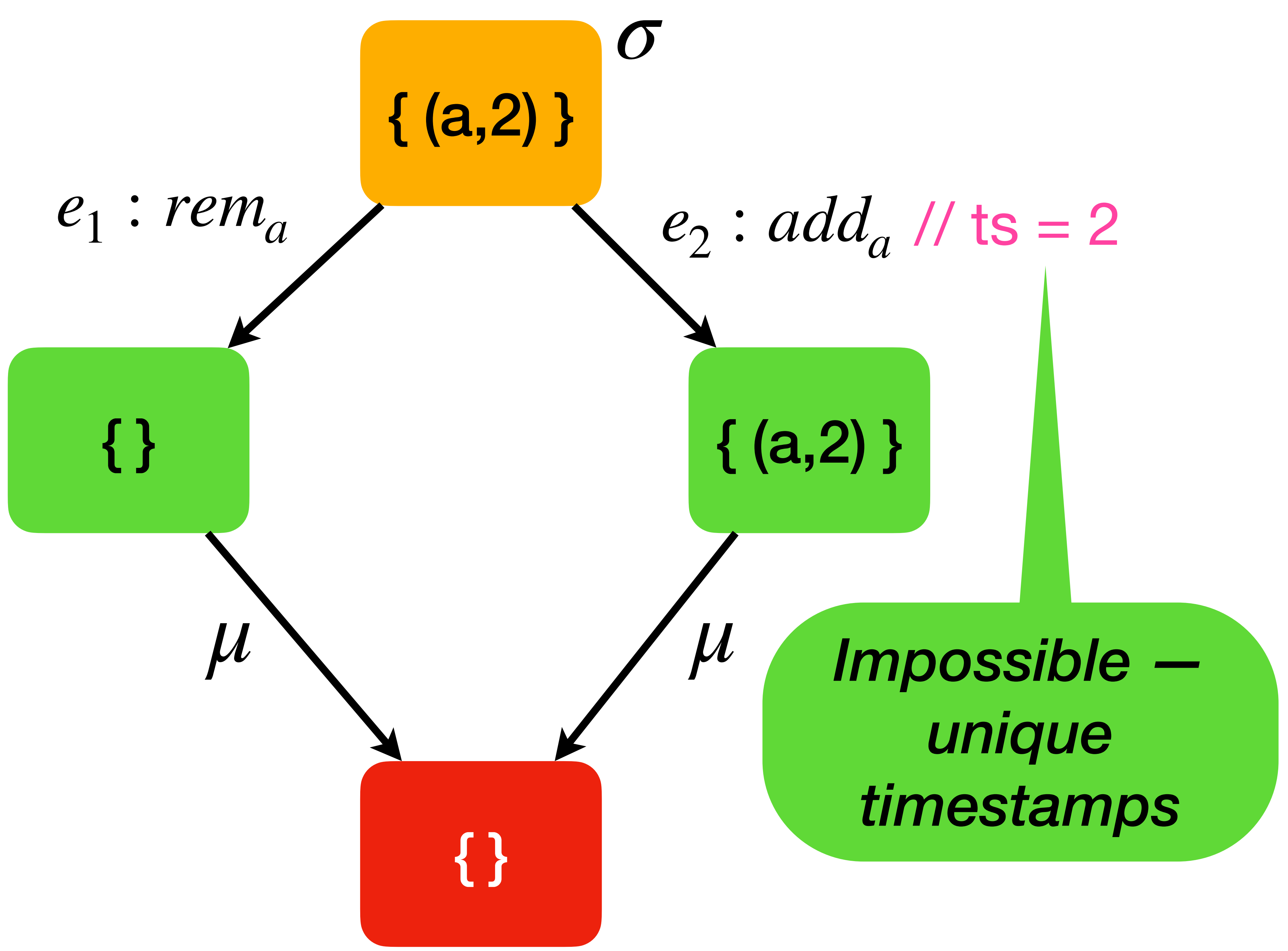


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Cannot prove for an arbitrary $l$

$l$ must be a ***feasible state***, obtained by application of updates on the initial state

# Induction over event sequences

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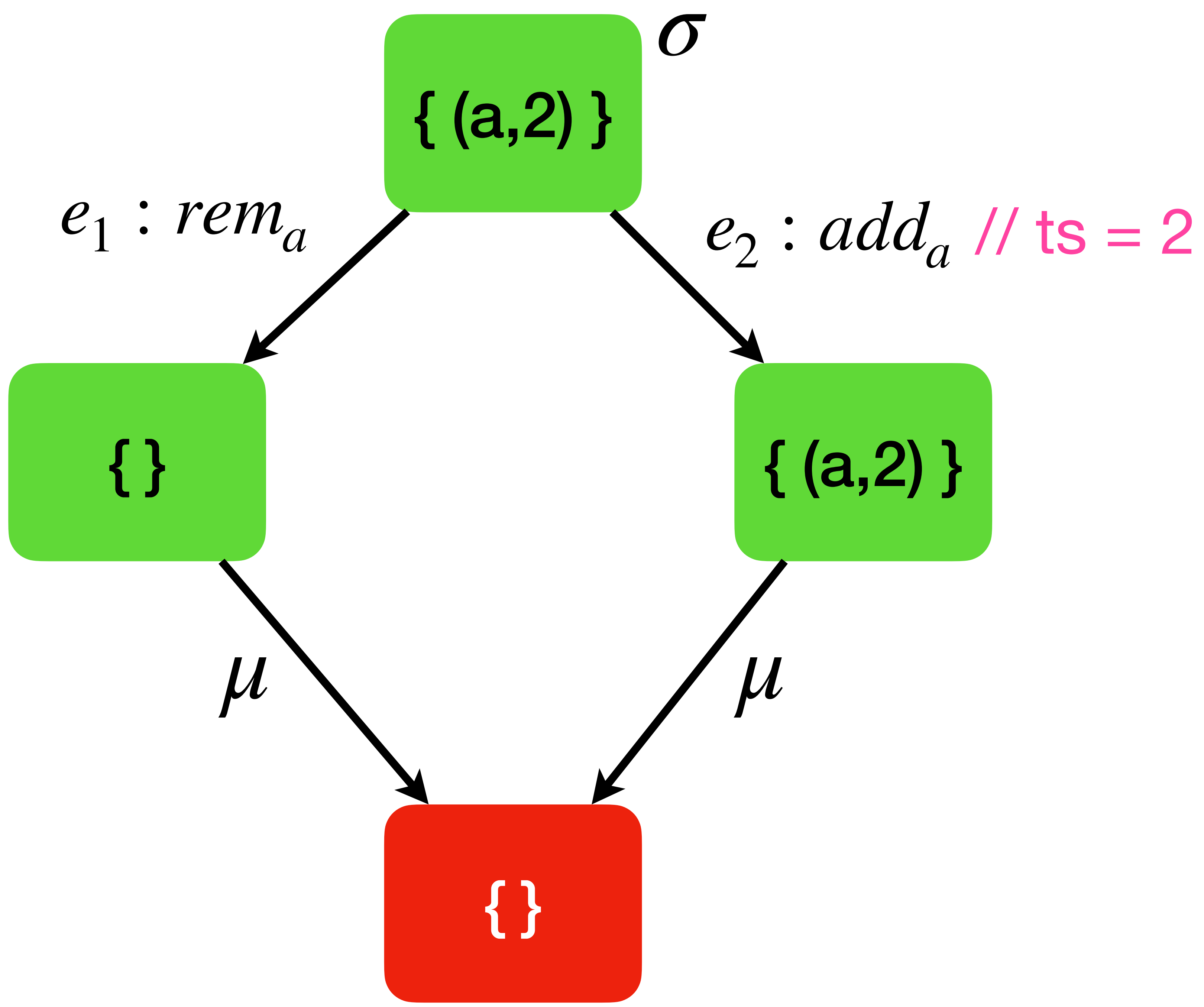


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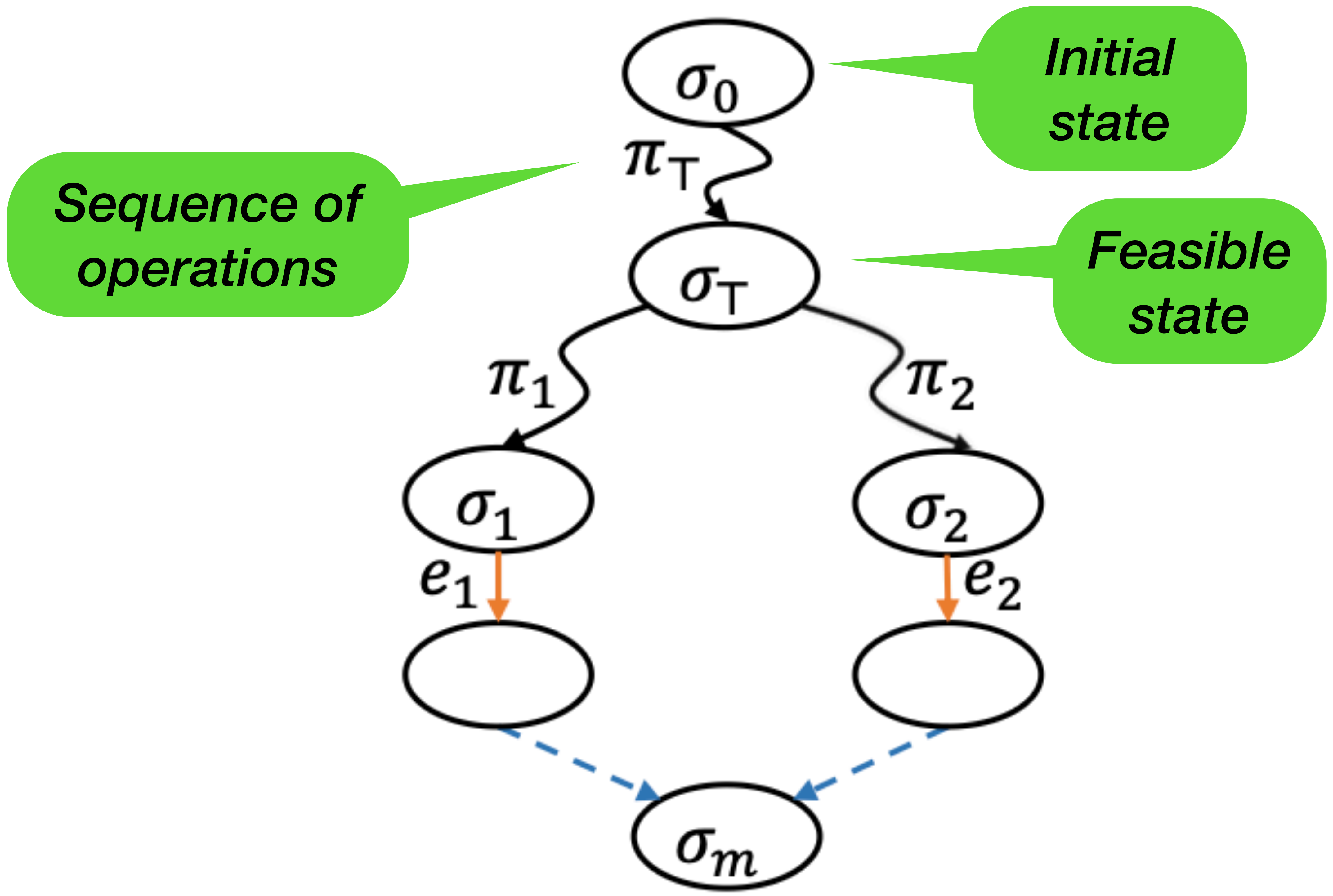

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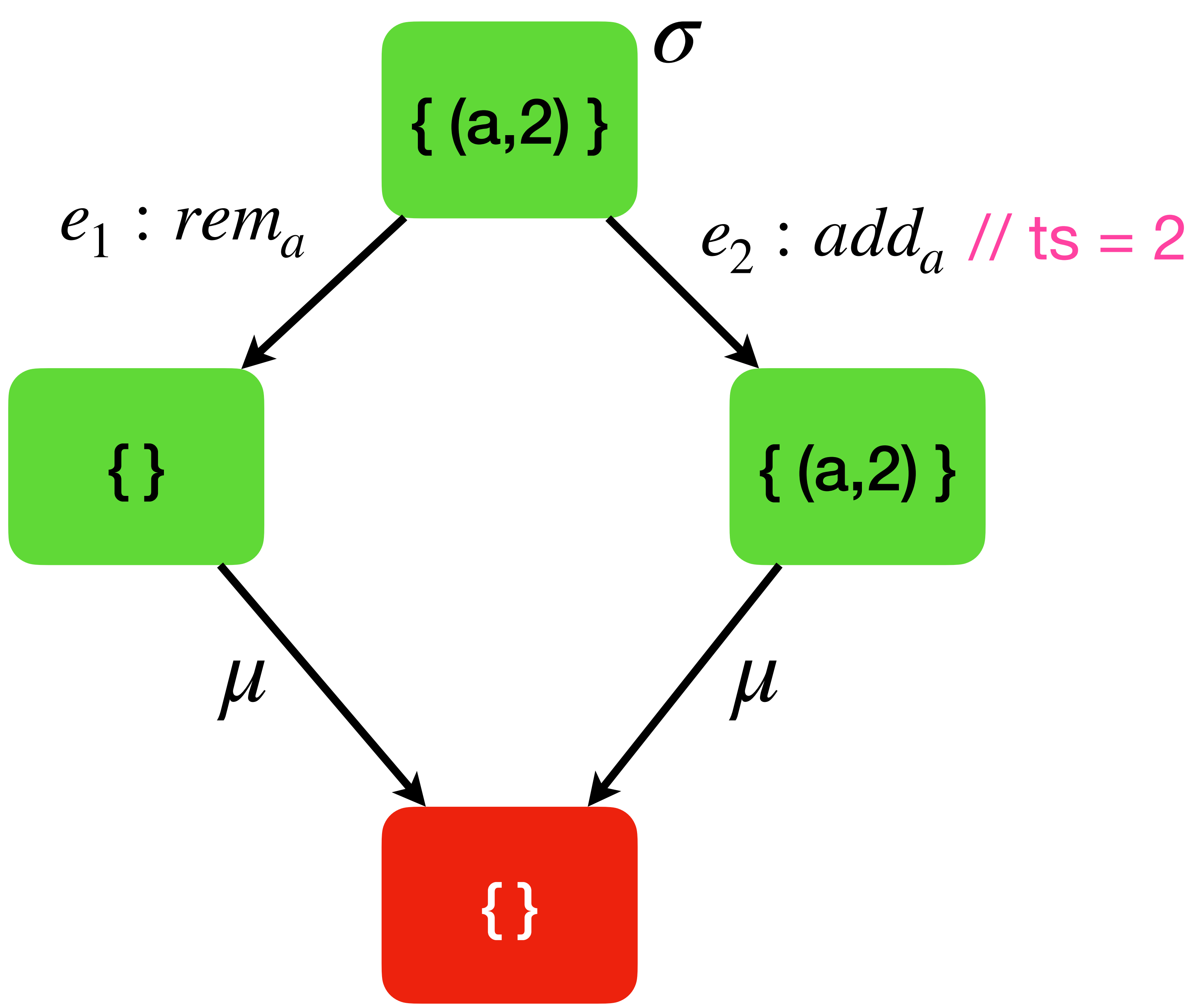


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**Induction on $\pi_\top$**

$$\frac{< pre >}{\mu(\sigma_0, e_1(\sigma_0), e_2(\sigma_0)) = e_2(\mu(\sigma_0, e_1(\sigma_0), \sigma_0))}$$

$(a,2) \notin \sigma_0$

**Base case**

# Induction over event sequences

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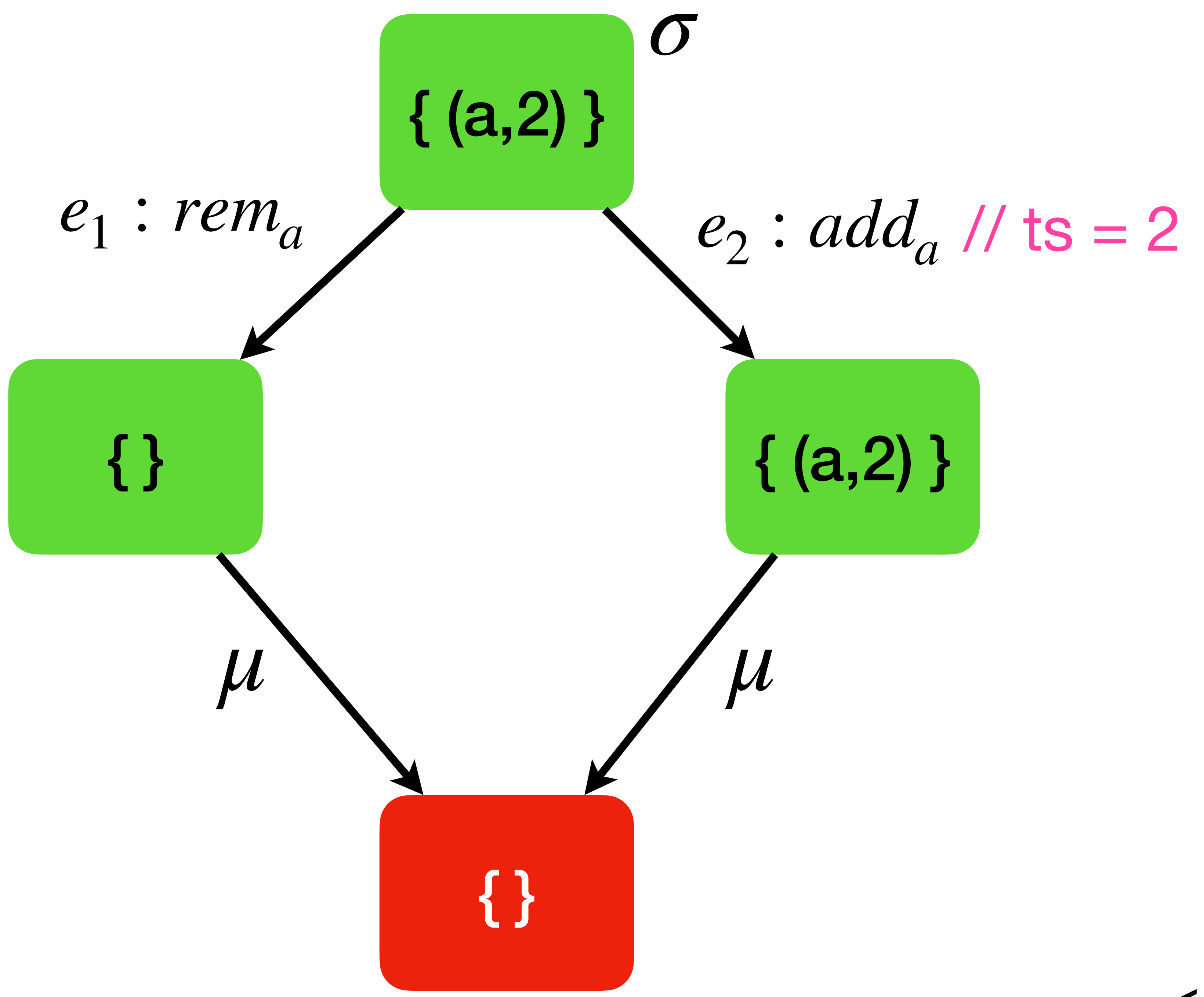


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**Inductive case**

$$\frac{< pre > \quad \mu(\sigma_t, e_1(\sigma_t), e_2(\sigma_t)) = e_2(\mu(\sigma_t, e_1(\sigma_t), \sigma_t)) \quad \sigma_t' = e(\sigma_t)}{\mu(\sigma_t', e_1(\sigma_t'), e_2(\sigma_t')) = e_2(\mu(\sigma_t', e_1(\sigma_t'), \sigma_t'))}$$

*Timestamps are unique*

# Linearizable MRDTs

THEOREM 4.7. *If an MRDT $\mathcal{D}$ satisfies the VCs $\psi^*$(BOTTOMUP-2-OP), $\psi^*$(BOTTOMUP-1-OP), $\psi^*$(BOTTOMUP-0-OP), MERGEIDEMPOTENCE and MERGECOMMUTATIVITY, then $\mathcal{D}$ is linearizable.*

*An MRDT that satisfies the algebraic properties is RA-linearizable*

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*An MRDT that satisfies the algebraic properties is RA-linearizable*

LEMMA 3.10. *If MRDT $\mathcal{D}$ is RA-linearizable, then for all executions $\tau \in [\![\mathcal{S}_{\mathcal{D}}]\!]$, for all transitions $C \xrightarrow{query(r,q,a)} C'$ in $\tau$ where $C = \langle N, H, L, G, vis \rangle$, there exists a sequence $\pi$ consisting of all events in $L(H(r))$ such that $\mathsf{lo}(C)_{|L(H(r))} \subseteq \pi$ and $a = \mathsf{query}(\pi(\sigma_0), q)$.*

*RA-linearizable MRDT query results match those obtained on the linearised updates applied to the initial state*

# Verified MRDTs

| MRDT | rc **Policy** | **#LOC** | **Verification Time (s)** |
|---|---|---|---|
| Increment-only counter [12] | none | 6 | 0.72 |
| PN counter [23] | none | 10 | 1.64 |
| Enable-wins flag* | $\text{disable} \xrightarrow{\text{rc}} \text{enable}$ | 30 | 29.80 |
| Disable-wins flag* | $\text{enable} \xrightarrow{\text{rc}} \text{disable}$ | 30 | 37.91 |
| Grows-only set [12] | none | 6 | 0.45 |
| Grows-only map [23] | none | 11 | 4.65 |
| OR-set [23] | $\text{rem}_a \xrightarrow{\text{rc}} \text{add}_a$ | 20 | 4.53 |
| OR-set (efficient)* | $\text{rem}_a \xrightarrow{\text{rc}} \text{add}_a$ | 34 | 660.00 |
| Remove-wins set* | $\text{add}_a \xrightarrow{\text{rc}} \text{rem}_a$ | 22 | 9.60 |
| Set-wins map* | $\text{del}_k \xrightarrow{\text{rc}} \text{set}_k$ | 20 | 5.06 |
| Replicated Growable Array [1] | none | 13 | 1.51 |
| Optional register* | $\text{unset} \xrightarrow{\text{rc}} \text{set}$ | 35 | 200.00 |
| Multi-valued Register* | none | 7 | 0.65 |
| JSON-style MRDT* | Fig. 13 | 26 | 148.84 |

*Neem also supports verification of RA-linearizability of state-based CRDTs*

https://github.com/prismlab/neem

# Limitations

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  - Not pleasant for engineering
  - No counterexamples!

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  - Not pleasant for engineering
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- **Current work:** model checking MRDTs against RA-linearizability
  - Fixed inputs & unrestricted concurrency
  - ***QuickCheck?***

# Neem — Automatic verification of RDTs

- **What's in the box?**
  - Definition of RA-linearizability for MRDTs
  - A novel induction scheme for MRDTs and state-based CRDTs to ***automatically*** verify RA-linearizability
  - Implemented in F*