Automatically Verifying Replicated Data Types

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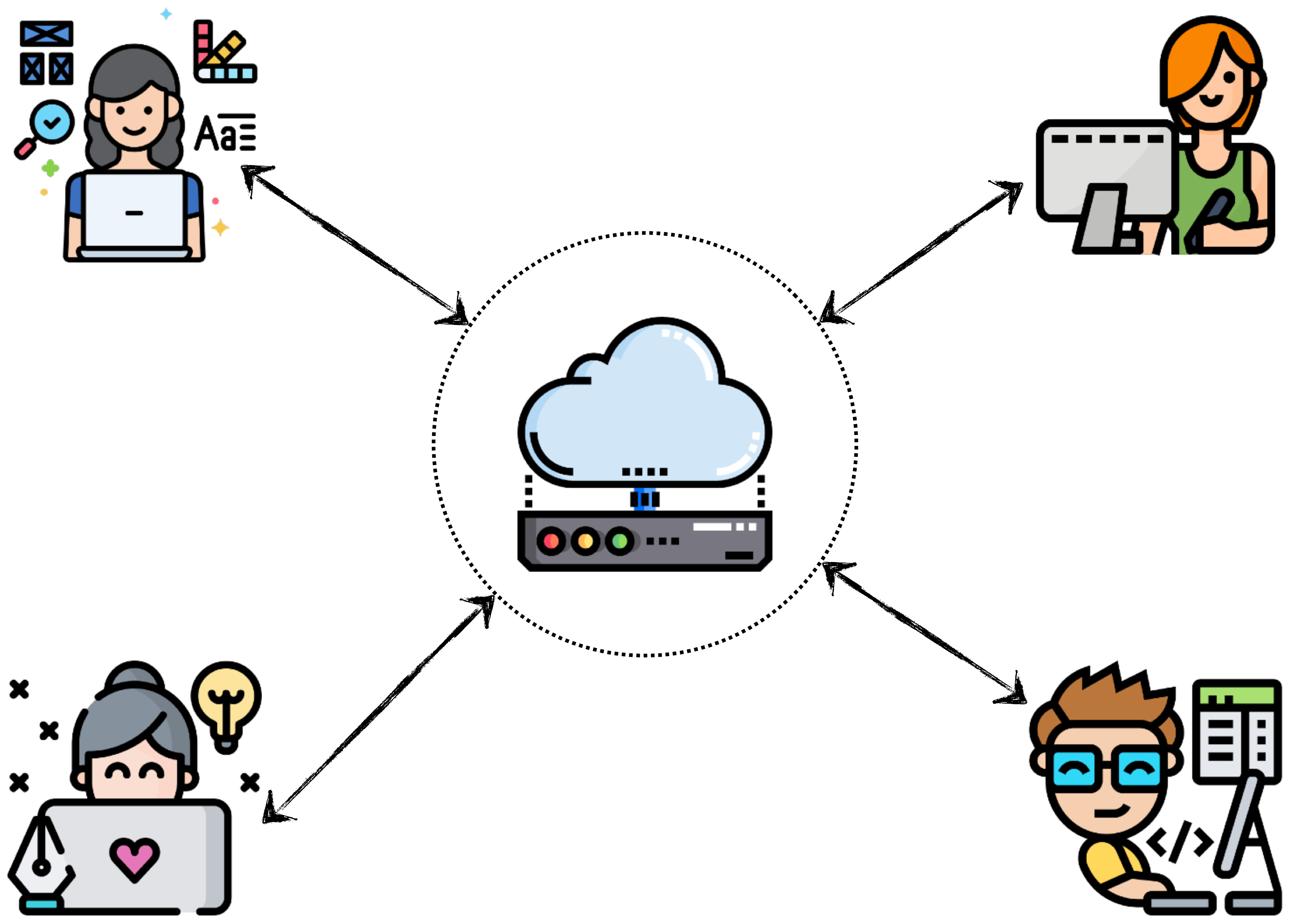
Joint work with Vimala Soundarapandian, Aseem Rastogi and Kartik Nagar

WG 2.1

Sep 8th to 12th, 2025

Collaborative Applications

Airtable Google Docs Figma Notion Overleaf



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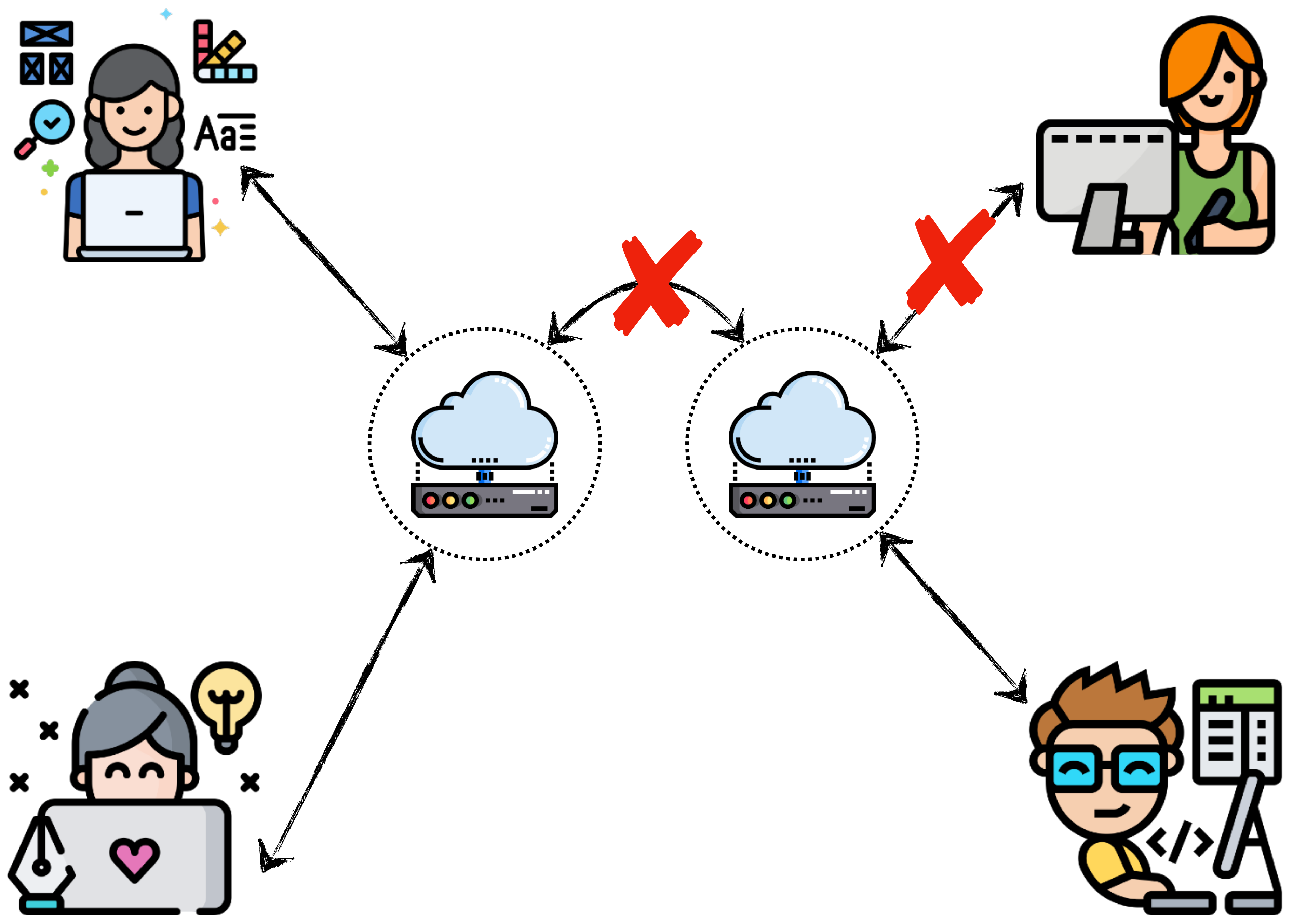
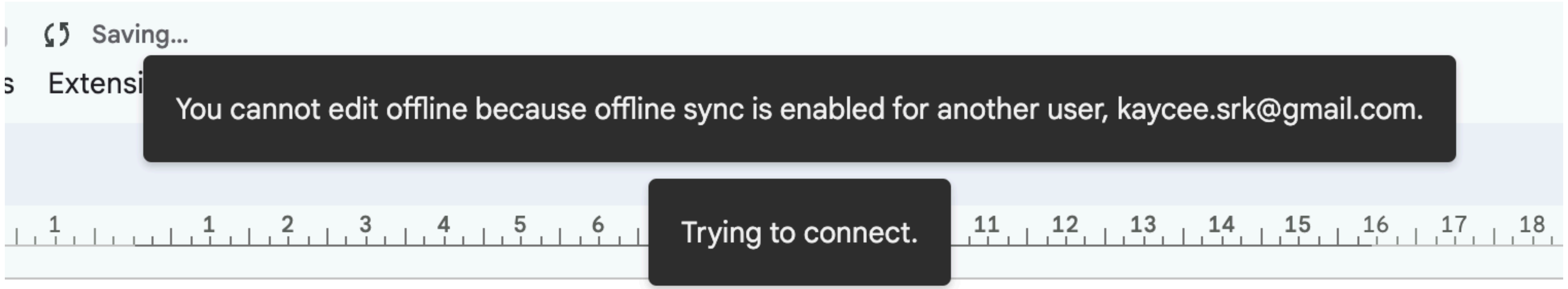


Network Partitions

- Centralised Apps provide limited support for offline editing



Enabling offline sync for one account prevents other accounts from working offline

Local-first software

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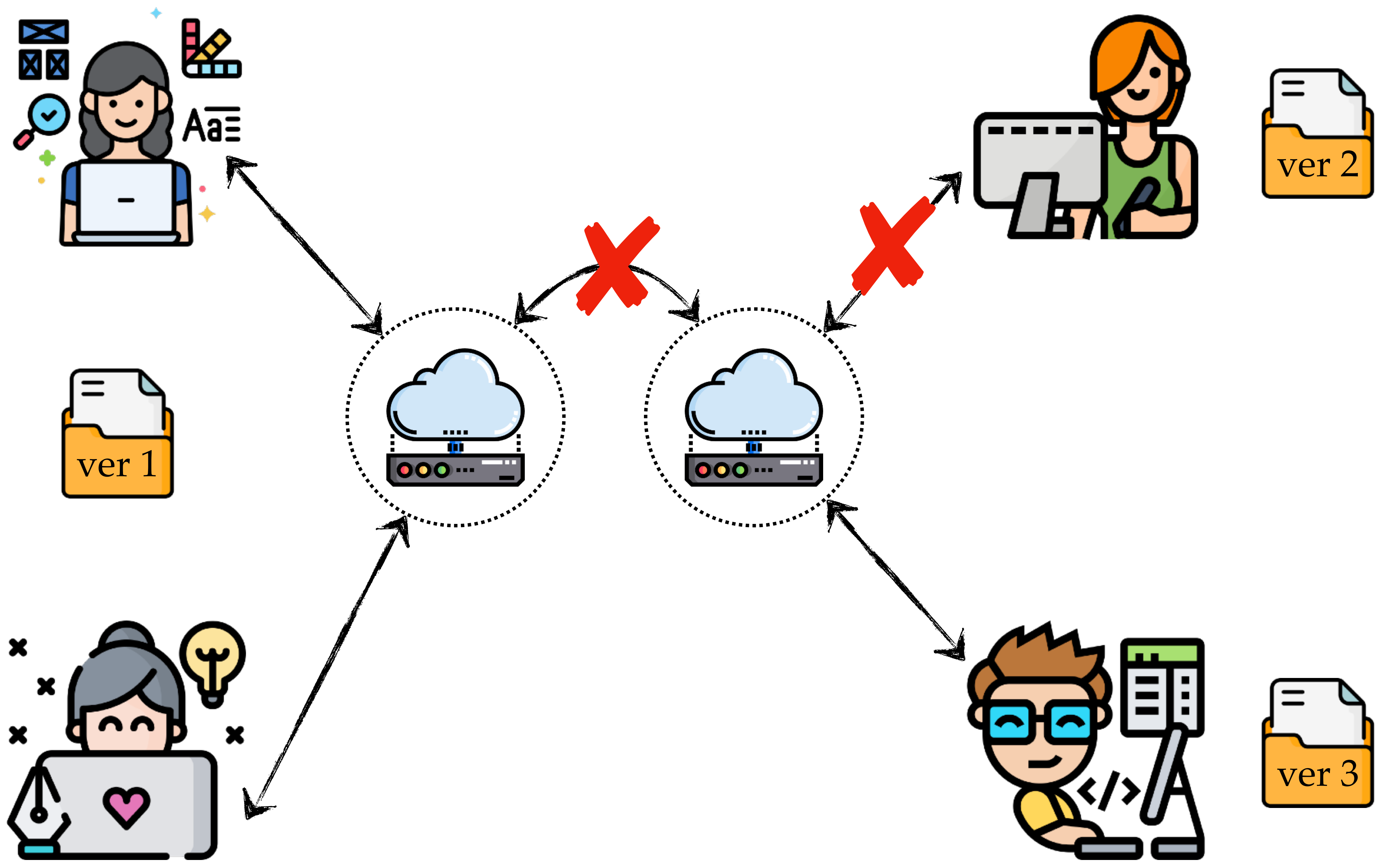

Local-first software

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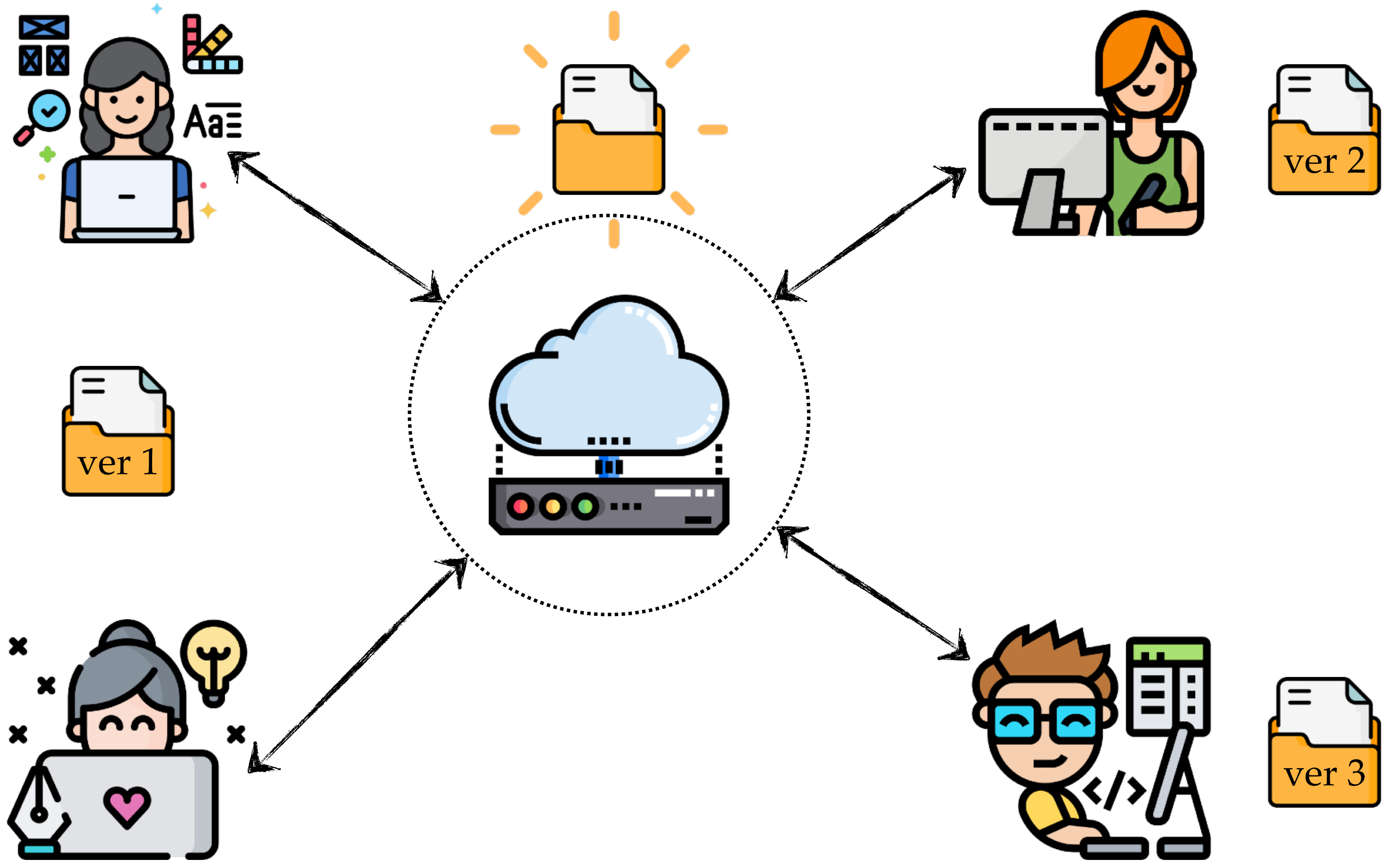


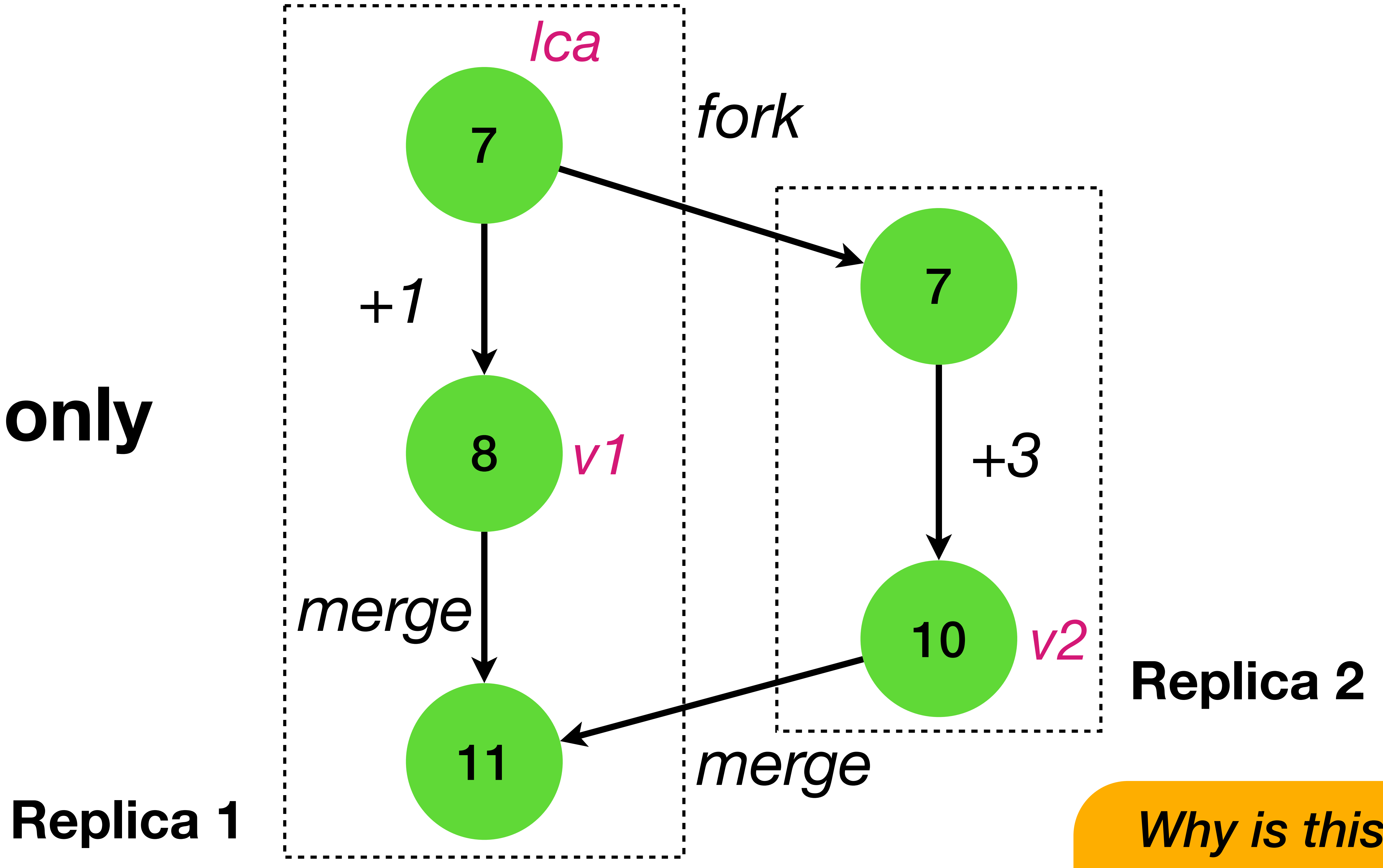
How do we build such applications?

Embed the notion of replication into the data types

Mergeable Replicated Data Types (MRDTs)

- MRDTs = Sequential data types + 3-way merge function à la Git

Increment-only counter




```
let merge lca v1 v2 =
  lca + (v1 - lca) + (v2 - lca)
```

Why is this correct?

How do we automatically verify it?

Verification using Algebraic Properties

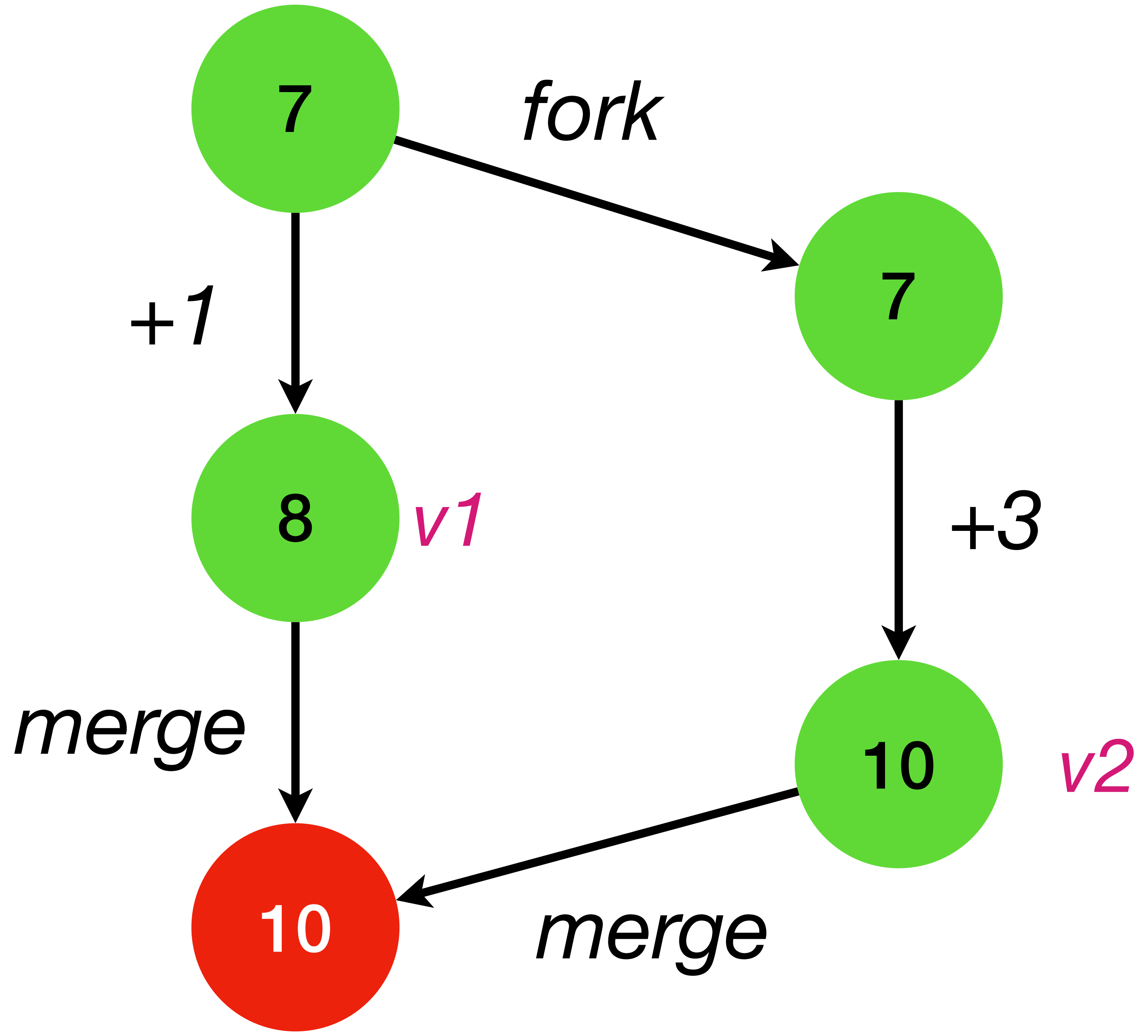
- State-based Convergent Replicated Data Types (CRDTs)
  - Merge is 2-way $-\mu(v_1, v_2)$
  - Verify algebraic properties of merge for *strong eventual consistency*

$\mu(a, b) = \mu(b, c)$	Commutativity
$\mu(a, a) = a$	Idempotence
$\mu(\mu(a, b), c) = \mu(a, \mu(b, c))$	Associativity

Satisfies algebraic properties

```
let merge v1 v2 = max v1 v2
```

Intent is not captured



Prior work: Capturing Intent through Axiomatic Spec

RESEARCH-ARTICLE | OPEN ACCESS

Certified mergeable replicated data types

Authors: Vimala Soundarapandian, Adharsh Kamath, Kartik Nagar, KC Sivaramakrishnan | Authors Info & Claims

PLDI 2022: Proceedings of the 43rd ACM SIGPLAN International Conference on Programming Language Design and Implementation
Pages 332 - 347 • https://doi.org/10.1145/3519939.3523735

- **Execution graph** = events + visibility (partial order)
- **Operations** = folds over execution graphs

Specification

Simulation relation

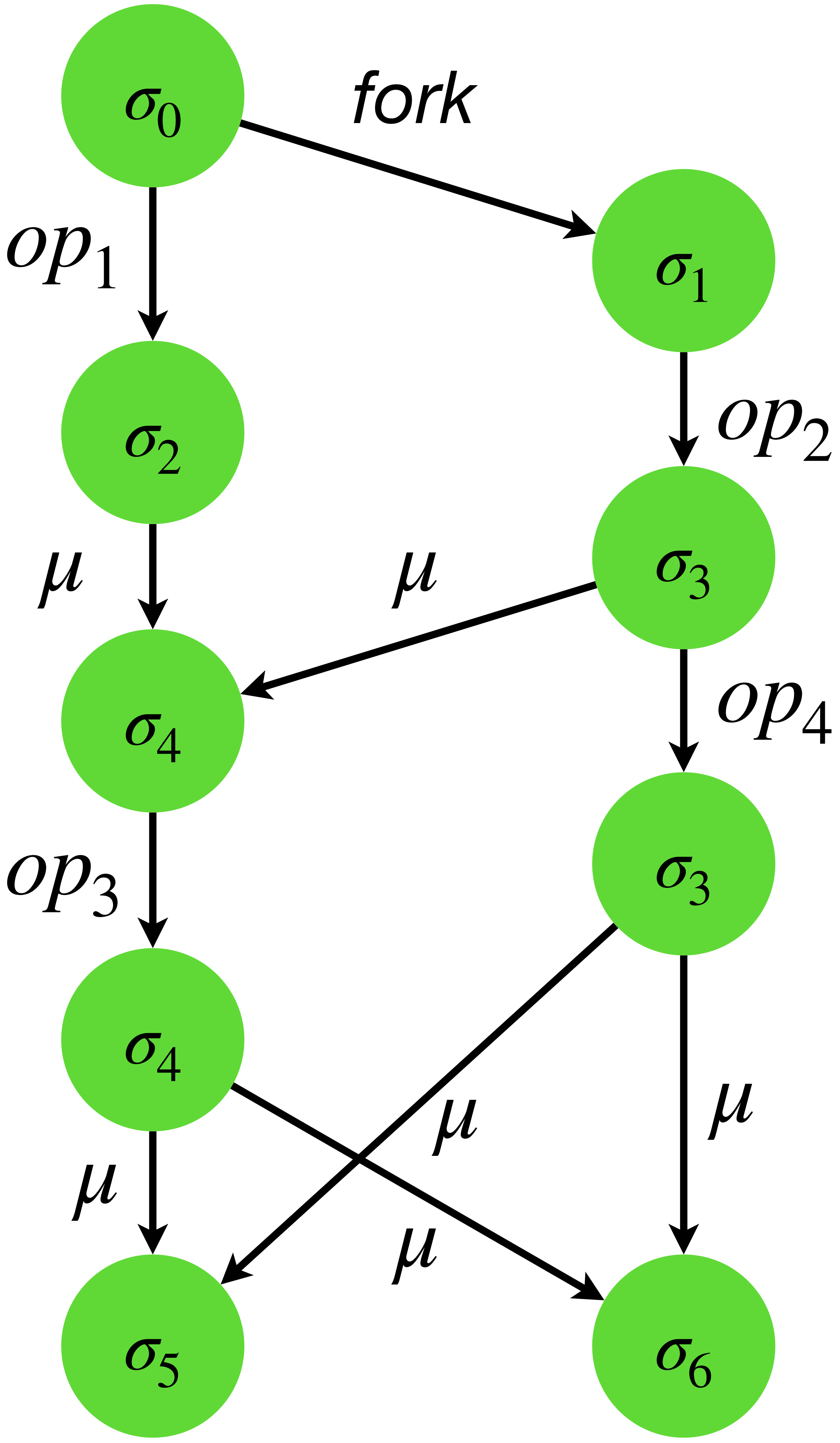
Sequential data type + operations + 3-way merge function

Implementation

Complex

Manual

Is there a more natural spec?



$$\sigma_5 = \sigma_6 = \text{linearization}(\{op_1, op_2, op_3, op_4\})\ \sigma_0$$

Replication-aware Linearizability

RESEARCH-ARTICLE

Replication-aware linearizability

Authors: Chao Wang, Constantin Enea, Suha Orhun Mutluergil, Gustavo Petri | Authors Info & Clai

PLDI 2019: Proceedings of the 40th ACM SIGPLAN Conference on Programming Language Design and Implementation
https://doi.org/10.1145/3314221.3314617

- Replica states should be a ***linearisation*** of observed ***update*** operations
  - Linearisation total order lo compatible with partially-ordered visibility relation vis
  - No real-time ordering requirement unlike traditional linearizability
- Payoff
  - If a replicated object is RA-linearizable, reason about it using sequential semantics

Add-wins set CRDT

- **Add-wins set**
  - A concurrent set where add-wins in a concurrent add(e) and rem(e)

$$(\Sigma_a, \Sigma_r) \xrightarrow{\text{add}(a)} (\Sigma_a \cup \{(a, id)\}, \Sigma_r) \quad \text{where id is fresh}$$

$$(\Sigma_a, \Sigma_r) \xrightarrow{\text{rem}(a)} (\Sigma_a, \Sigma_r \cup \{\ (a, id) \mid (a, id) \in \Sigma_a\ \}$$

$$(\Sigma_a, \Sigma_r) \xrightarrow{\text{read}()} \{a \mid (a, id) \in \Sigma_a \backslash \Sigma_r\}$$

Add-wins set *sequential* specification

- States are asynchronously broadcast to other replicas

$$(\Sigma_a, \Sigma_r) \xrightarrow{\text{merge}(\Sigma_a', \Sigma_r')} (\Sigma_a \cup \Sigma_a', \Sigma_r \cup \Sigma_r')$$

Replication-aware Linearizability

A history $h = (E, vis)$, $E \subseteq \text{Queries} \uplus \text{Updates}$, is RA-linearizable w.r.t. a sequential specification Spec if there exists a total order seq on E (same events) such that:

(i) $vis \cup seq$ is acyclic;

(ii) $seq\downarrow_{Updates} \in \text{Spec}$;

(iii) $\forall \ell_{qr} \in E, \left(seq\downarrow_{\text{vis}^{-1}(\ell_{qr}) \cap Updates}\right) \cdot \ell_{qr} \in \text{Spec}$.

- A CRDT is said to be RA-linearizable if every history *h* is RA-linearizable
- Add-wins set is RA-linearizable
- *RA-linearizability makes program reasoning easier!*

Using RA-linearizability for verification

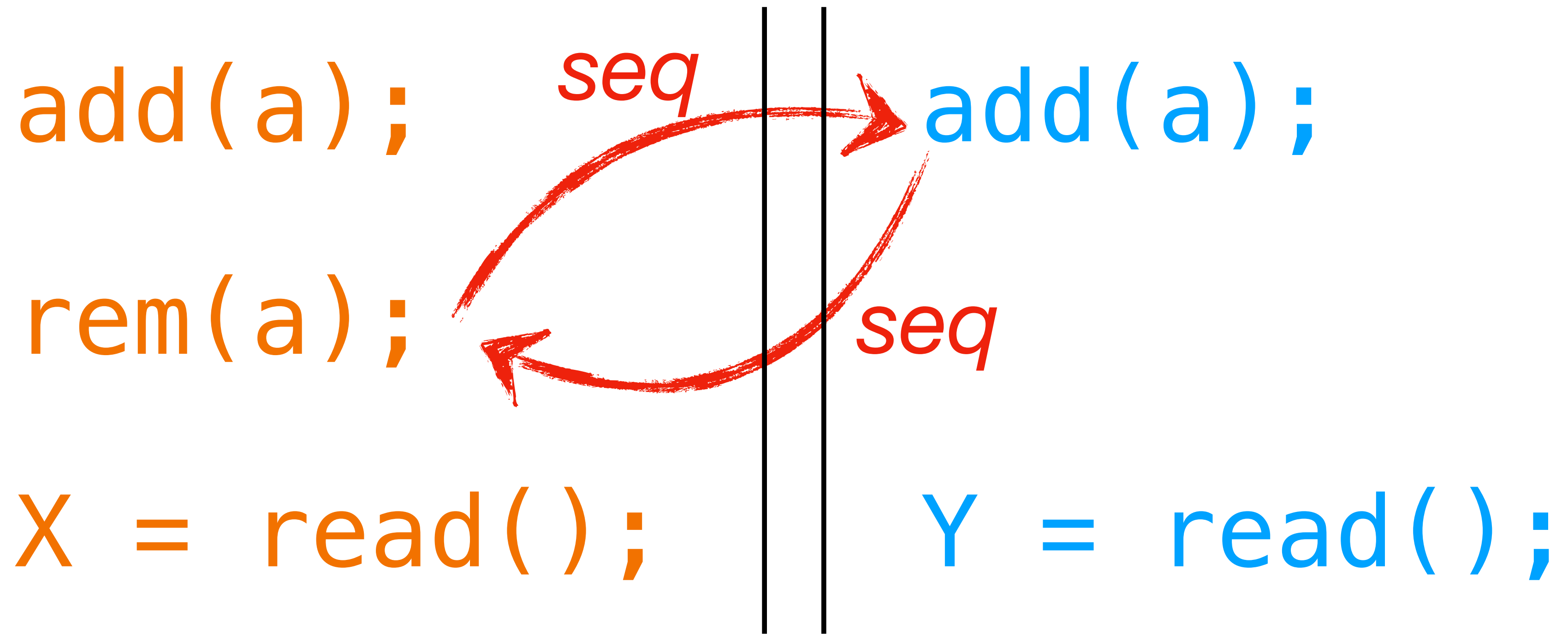
```
add(a);          ||  add(a);
rem(a);          ||
X = read();      ||  Y = read();
```

$$a \in X \implies a \in Y$$

- Since Add-wins set is RA-linearizable, you can use *totally ordered trace* and the *sequential spec* to reason about correctness

`add(a);`	`rem(a);`	`add(a);`	`X = read();`	`Y = read()`
$(\{a_1\},\{\})$	$(\{a_1\},\{a_1\})$	$(\{a_1,a_2\},\{a_1\})$	$X = \{a\}$	$Y = \{a\}$

Using RA-linearizability for verification



$$a \in X \implies a \in Y$$

- Let's try to make the statement false
    - Make $a \in X$ true and $a \in Y$ false

Replication-aware Linearizability

RESEARCH-ARTICLE

Replication-aware linearizability

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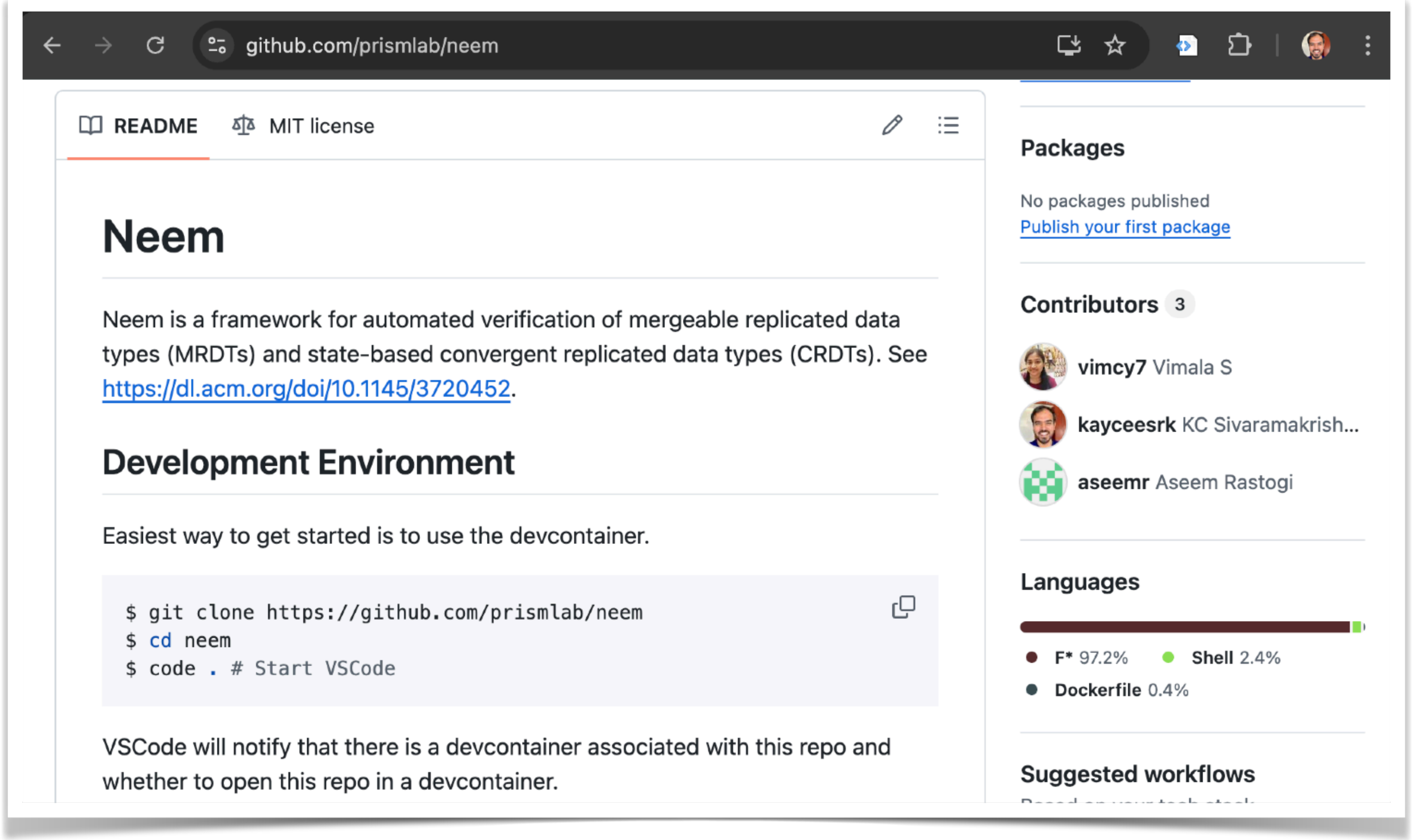
- Presented a proof methodology to show that a CRDT is linearisable
- Not automated or mechanised

Neem — Automatic verification of RDTs

- **What's in the box?**
  - Definition of RA-linearizability for MRDTs
  - A novel induction scheme for MRDTs and state-based CRDTs to ***automatically*** verify RA-linearizability
  - Implemented in F*



Resolving conflicts

- Not all operations commute
  - **Add-wins set** — `add(a)` and `rem(a)` do not commute
  - Specify ordering using the **Conflict Resolution** relation $rc = \{(rem_a, add_a) \mid a \in \mathbb{E}\}$
  - Linearization order lo must be compatible with rc for concurrent events
- Neem developers provide
  - MRDT = Sequential Data Type + 3-way merge
  - Conflict Resolution rc relation

Increment-only Counter

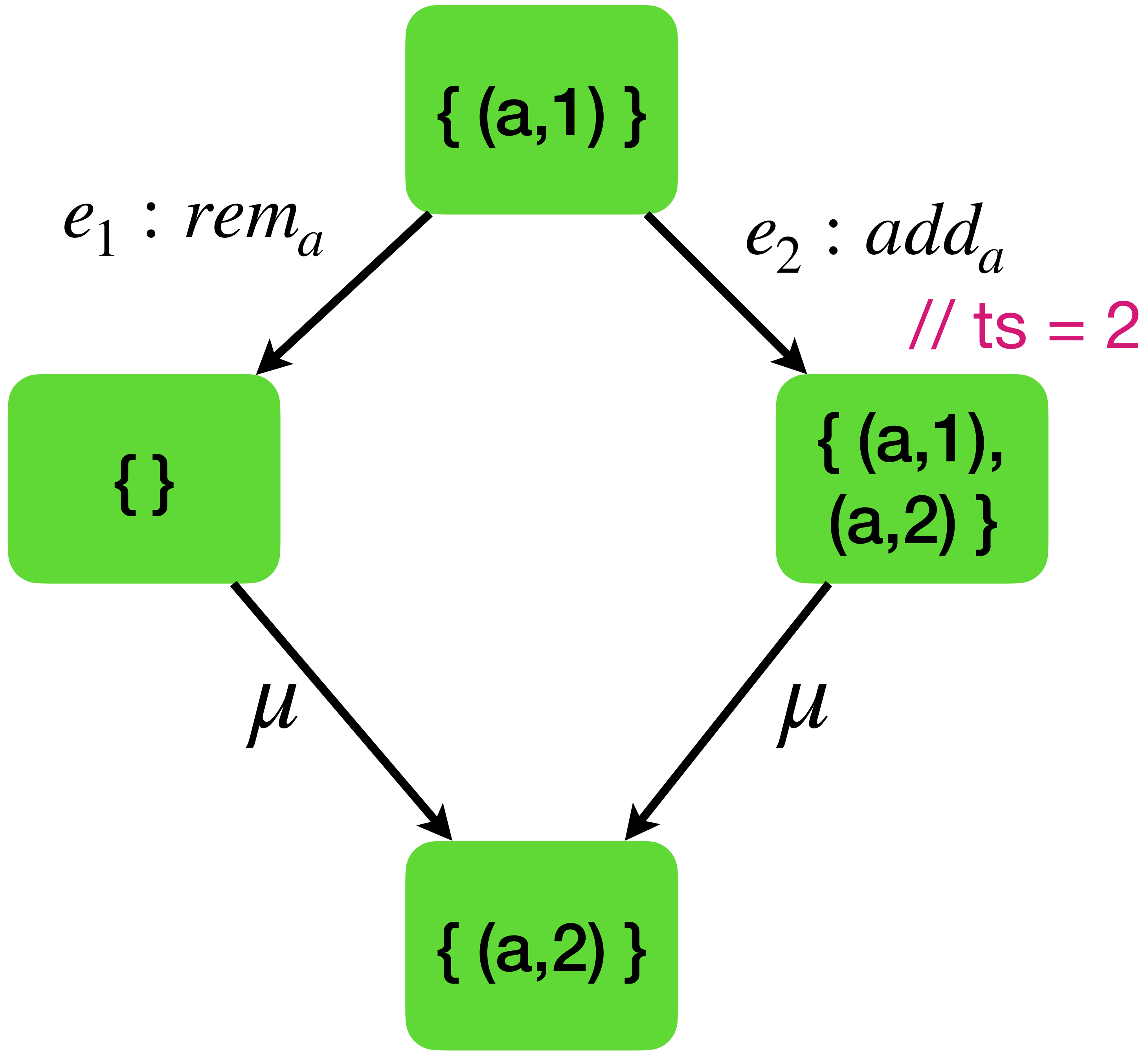
State	$\Sigma = \mathbb{N}$
Updates	$O = \{\mathrm{inc}\}$
Queries	$Q = \{\mathrm{rd}\}$
Init State	$\sigma_0 = 0$
Update behaviour	$\mathrm{do}(\sigma, \_, \_, \mathrm{inc}) = \sigma + 1$
Merge	$\mathrm{merge}(\sigma_\top, \sigma_1, \sigma_2) = \sigma_\top + (\sigma_1 - \sigma_\top) + (\sigma_2 - \sigma_\top)$
Query behaviour	$\mathrm{query}(\sigma, rd) = \sigma$
Resolve conflict	$\mathrm{rc} = \emptyset$

Unique timestamp

Replica ID

Add-wins Set

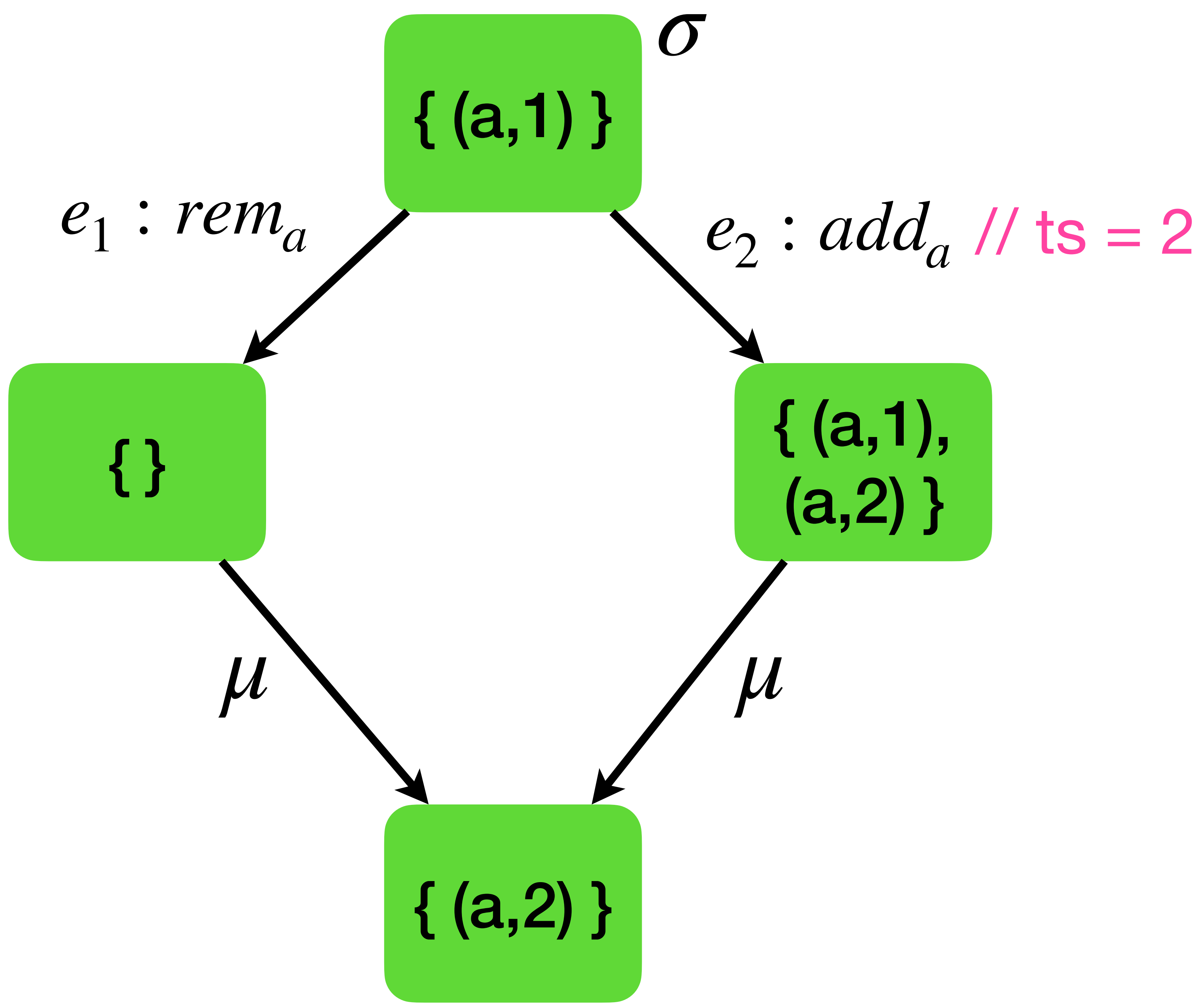
State	1: $\Sigma = \mathcal{P}(\mathbb{E} \times \mathcal{T})$
Updates	2: $O = \{\text{add}_a, \text{rem}_a \mid a \in \mathbb{E}\}$
Queries	3: $Q = \{\text{rd}\}$
Init State	4: $\sigma_0 = \{\}$
Update behaviour	5: $\text{do}(\sigma, t, \_, \text{add}_a) = \sigma \cup \{(a, t)\}$ 6: $\text{do}(\sigma, \_, \_, \text{rem}_a) = \sigma \setminus \{(a, i) \mid (a, i) \in \sigma\}$
Merge	7: $\text{merge}(\sigma_\top, \sigma_1, \sigma_2) = (\sigma_\top \cap \sigma_1 \cap \sigma_2) \cup (\sigma_1 \setminus \sigma_\top) \cup (\sigma_2 \setminus \sigma_\top)$
Query behaviour	8: $\text{query}(\sigma, rd) = \{a \mid (a, \_) \in \sigma\}$
Resolve conflict	9: $\text{rc} = \{(\text{rem}_a, \text{add}_a) \mid a \in \mathbb{E}\}$



$\{(a,2)\} = add_a(rem_a\{(a,1)\})$

Bottom up linearisation

$$rc = \{(rem_a, add_a) \mid a \in \mathbb{E}\}$$



To show

$$\mu(\sigma, e_1(\sigma), e_2(\sigma)) = e_2(e_1(\sigma))$$

[BottomUp-2-OP]

$$\frac{e_1 \neq e_2 \quad e_1 \xrightarrow{\mathsf{rc}} e_2 \vee e_1 \rightleftarrows e_2}{\mu(l, e_1(a), e_2(b)) = e_2(\mu(l, e_1(a), b))}$$

[BottomUp-1-OP]

$$\frac{(e_\top \neq \epsilon \wedge e_1 \neq e_\top) \vee (e_\top = \epsilon \wedge l = b)}{\mu(e_\top(l), e_1(a), e_\top(b)) = e_1(\mu(e_\top(l), a, e_\top(b)))}$$

[BottomUp-0-OP]

$$\mu(e_\top(l), e_\top(a), e_\top(b)) = e_\top(\mu(l, a, b))$$

[MergeIdempotence]

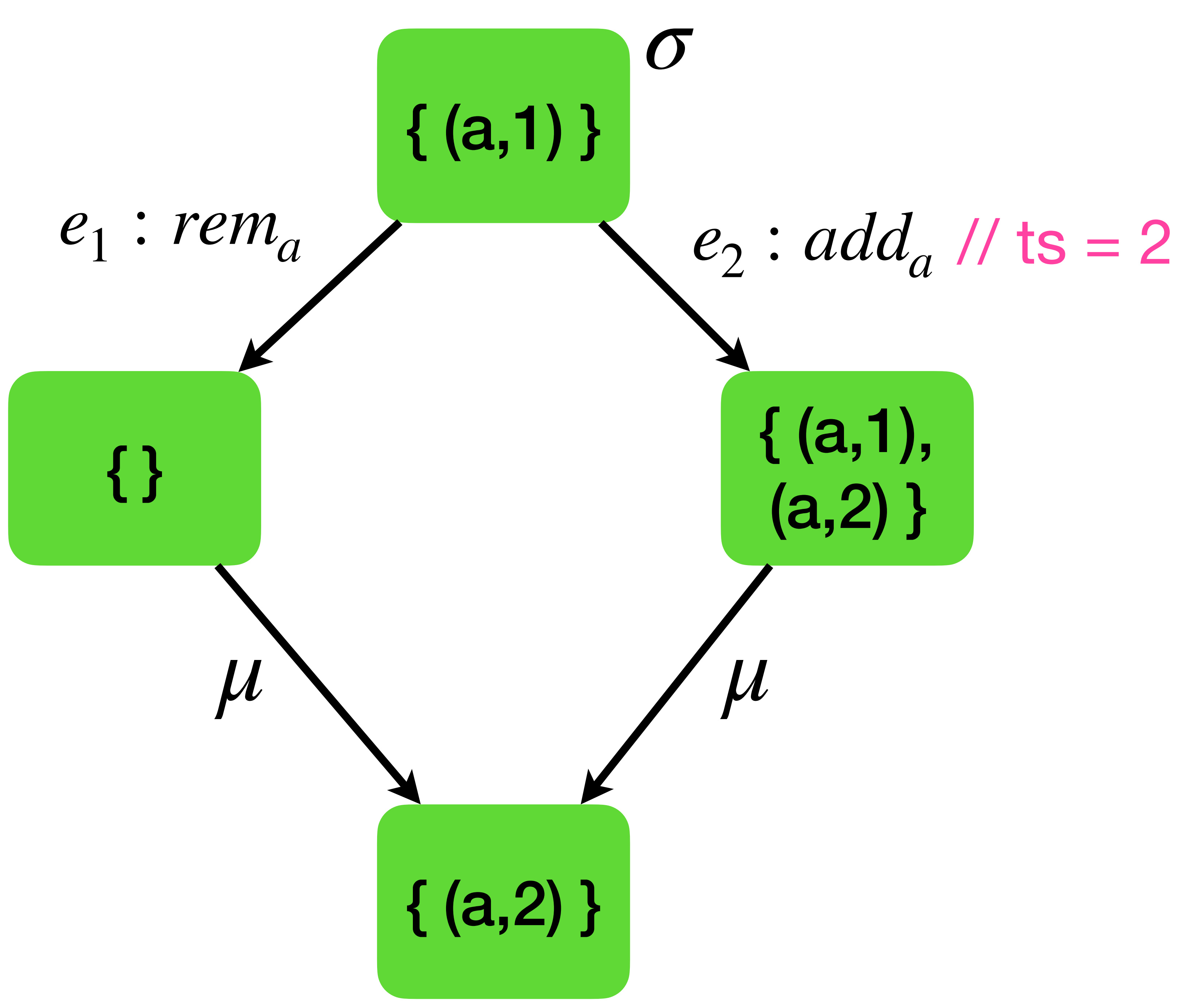
$$\mu(a, a, a) = a$$

[MergeCommutativity]

$$\mu(l, a, b) = \mu(l, b, a)$$

Bottom up linearisation

$rc = \{(rem_a, add_a) \,|\, a \in \mathbb{E}\}$



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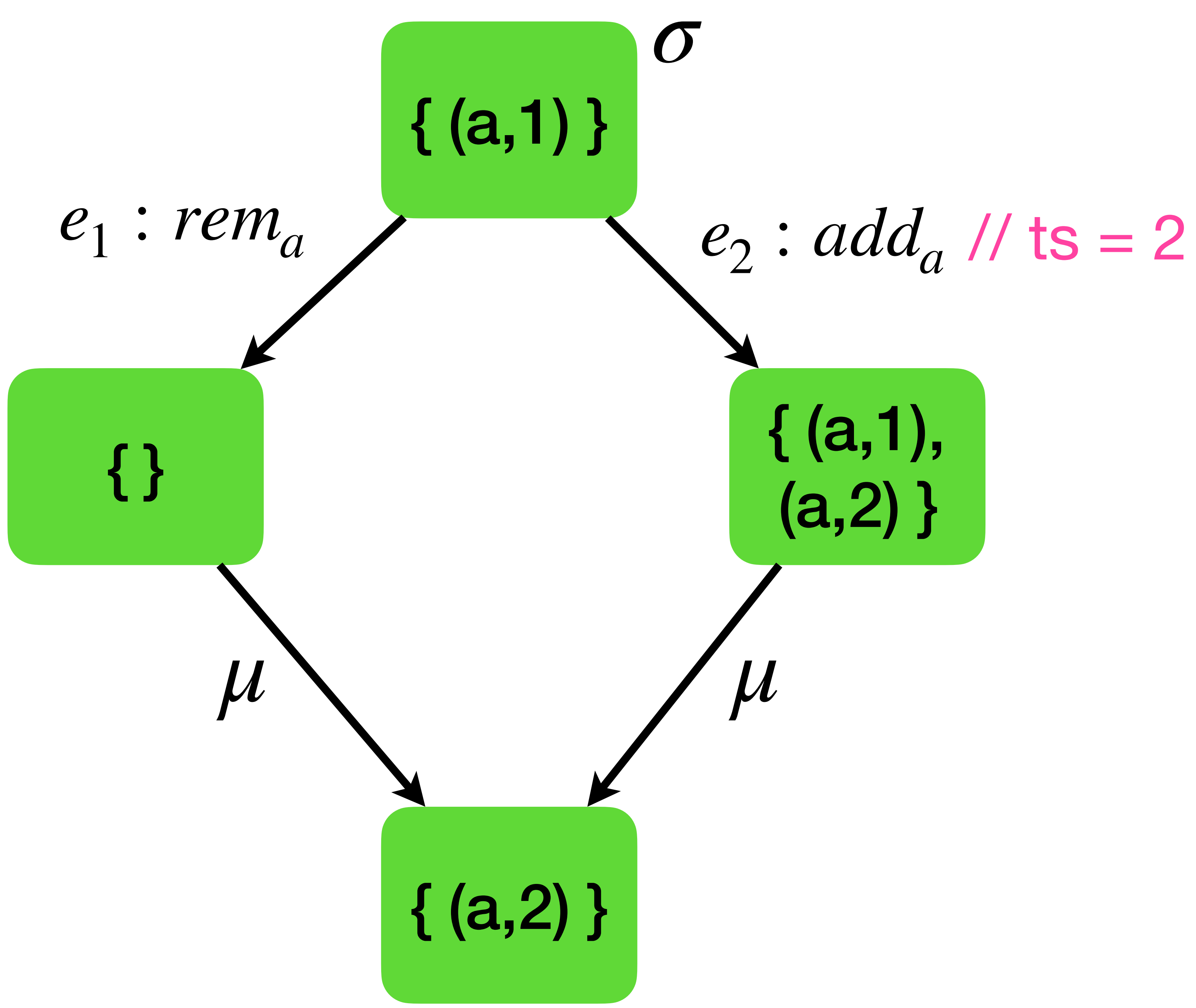

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$$e_2(\mu(\sigma, e_1(\sigma), \sigma)) = e_2(e_1(\sigma))$$

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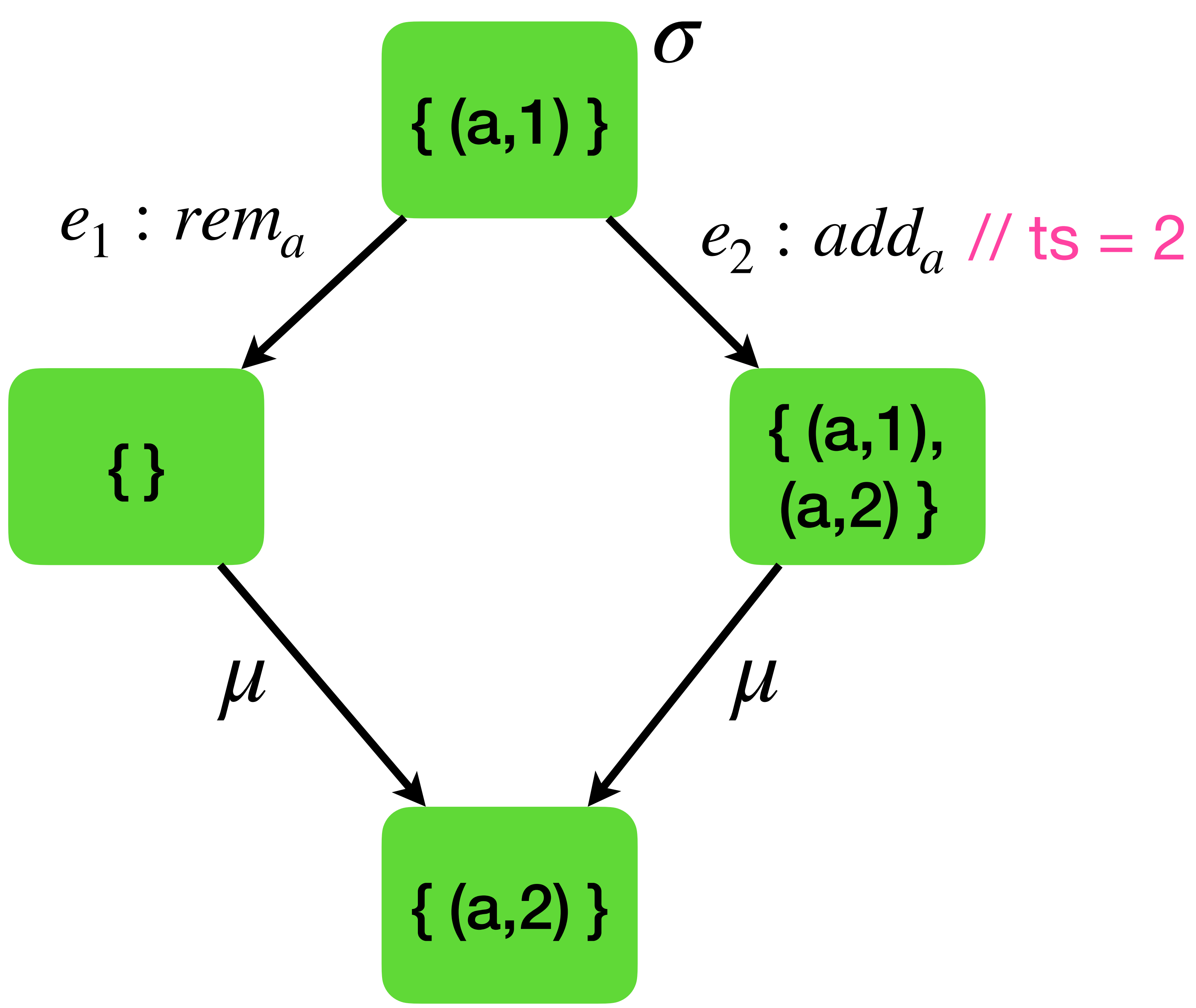

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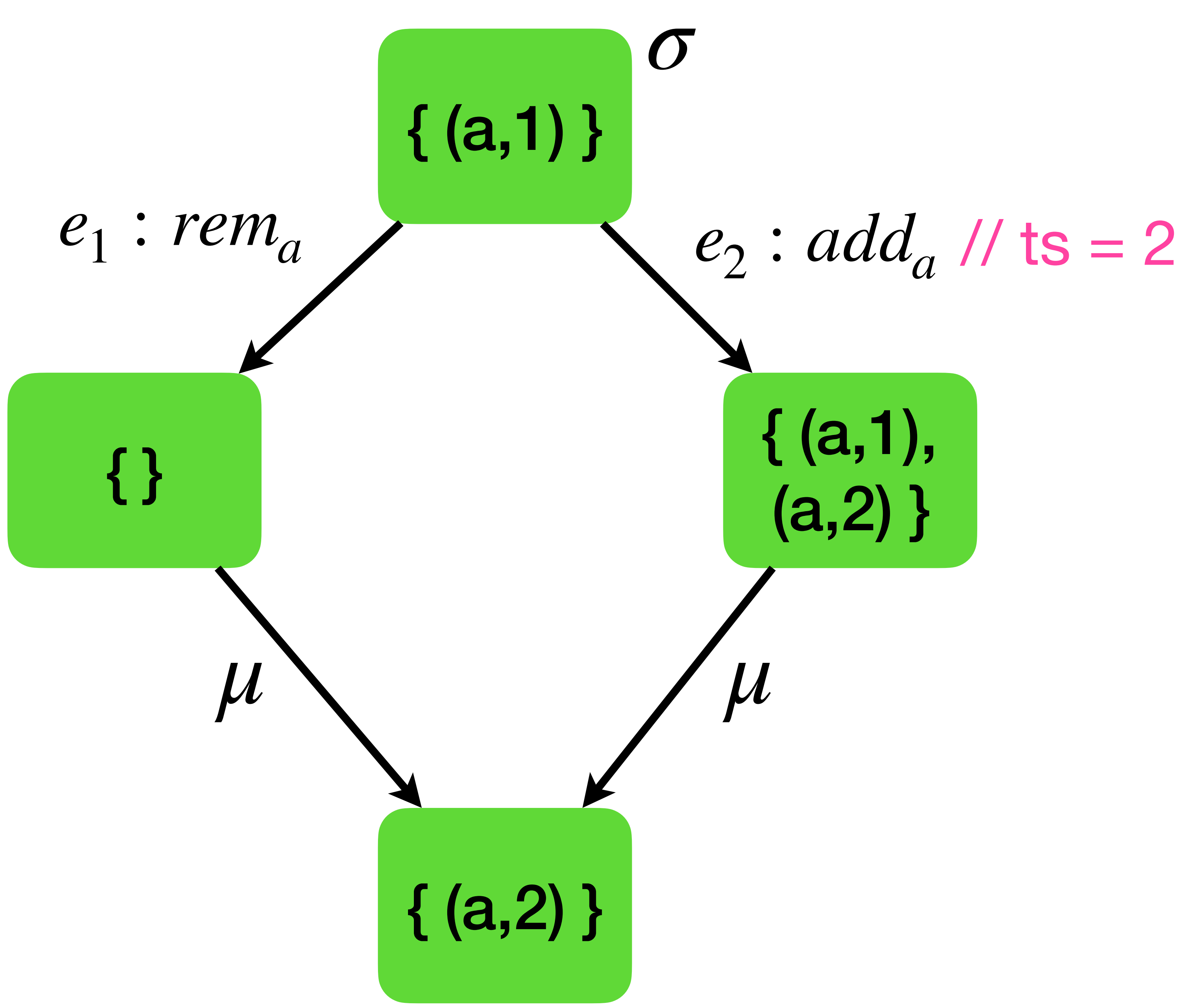

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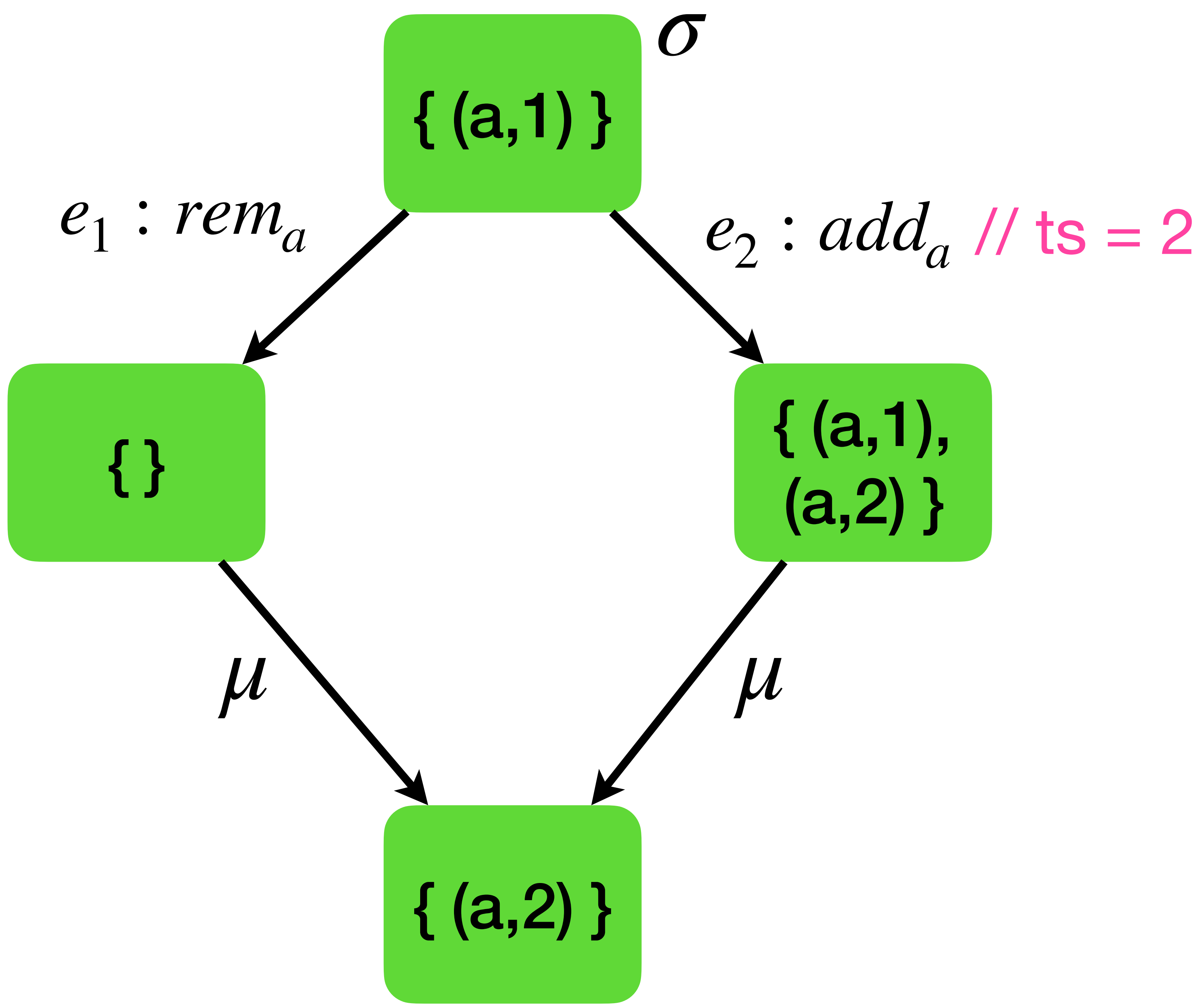
Bottom up linearisation

$$rc = \{(rem_a, add_a) \,|\, a \in \mathbb{E}\}$$



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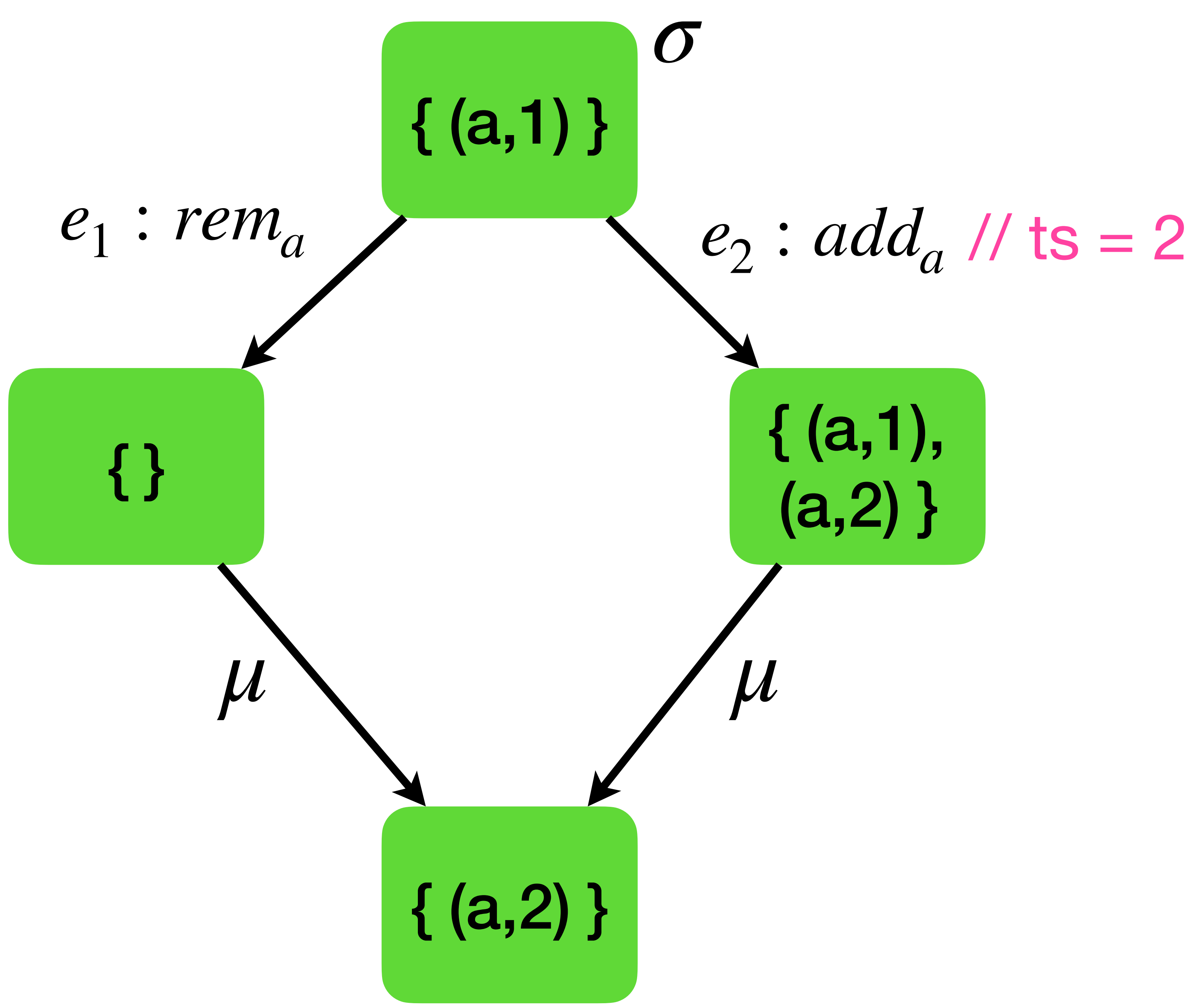

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[MergeIdempotence]

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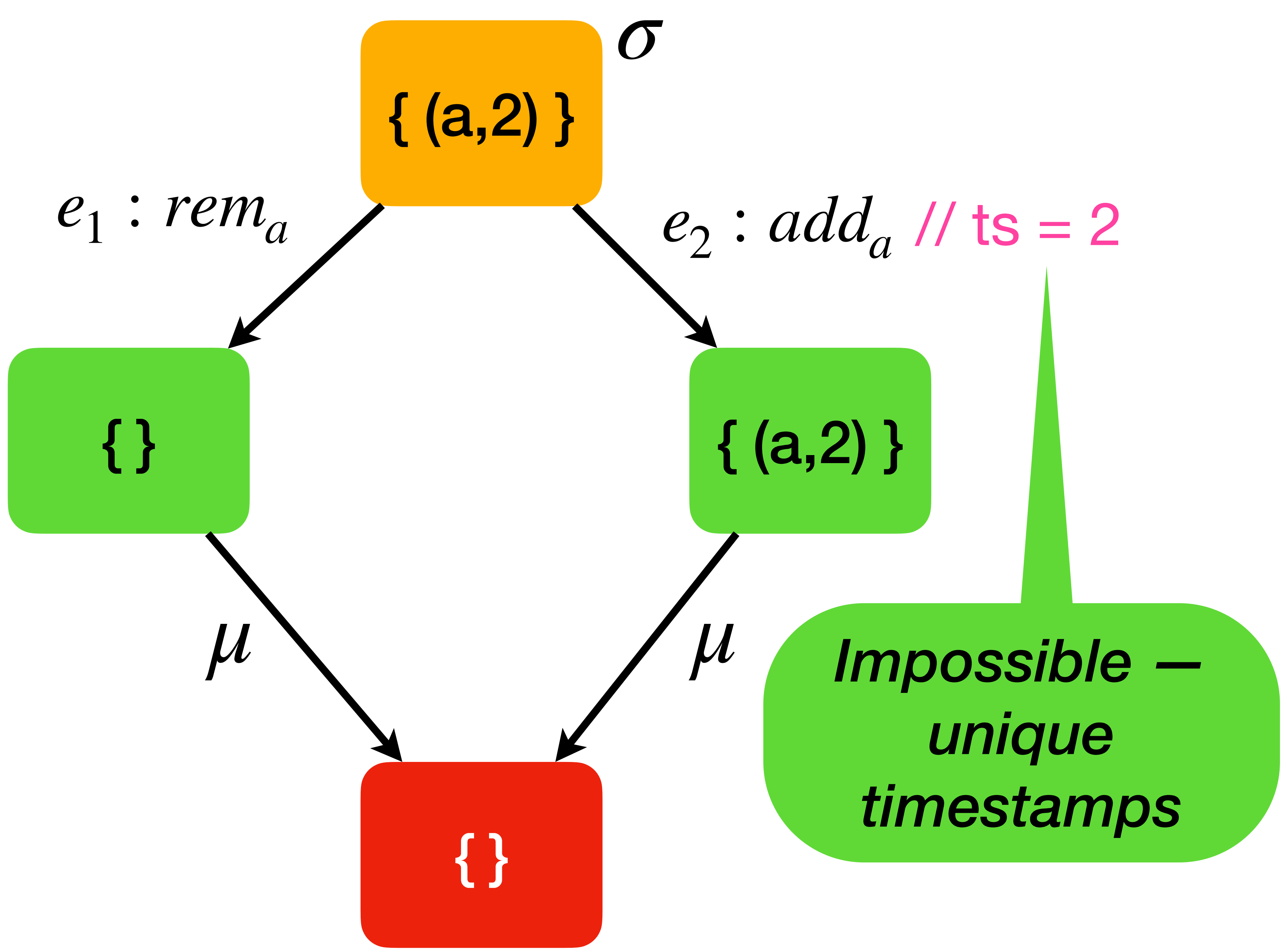
Making a good VC

$$\mathsf{rc} = \{(\mathsf{rem}_a, \mathsf{add}_a) \mid a \in \mathbb{E}\}$$



To show

$$\mu(\sigma, e_1(\sigma), e_2(\sigma)) = e_2(e_1(\sigma))$$

$$\{\} \neq \{(a,2)\}$$

[BottomUp-2-OP]

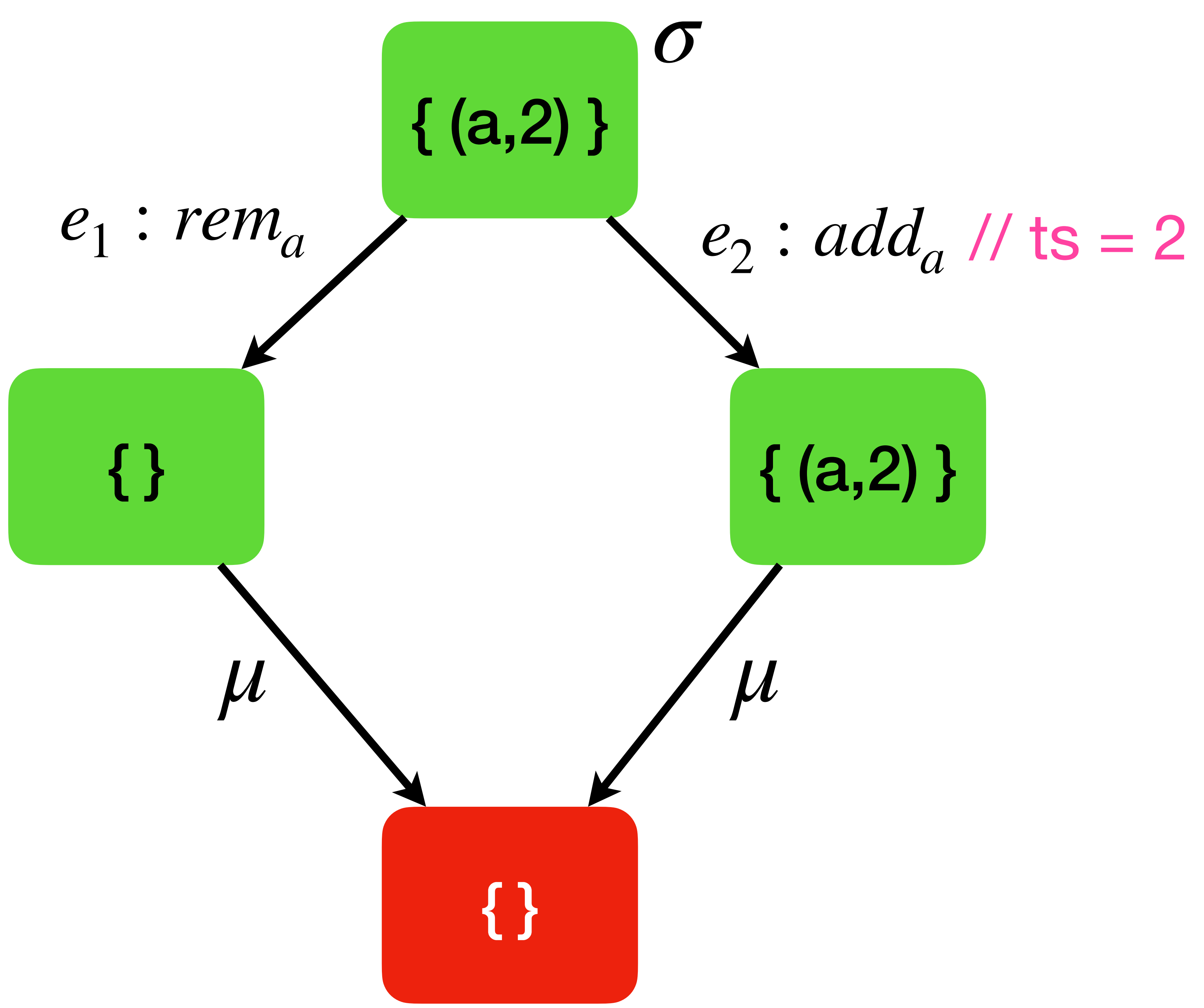
$$\frac{e_1 \neq e_2 \qquad e_1 \xrightarrow{\mathsf{rc}} e_2 \vee e_1 \rightleftarrows e_2}{\mu(l, e_1(a), e_2(b)) = e_2(\mu(l, e_1(a), b))}$$

Cannot prove for an arbitrary l

l must be a ***feasible state***, obtained by application of updates on the initial state

Induction over event sequences

$$\mathsf{rc} = \{(\mathsf{rem}_a, \mathsf{add}_a) \mid a \in \mathbb{E}\}$$



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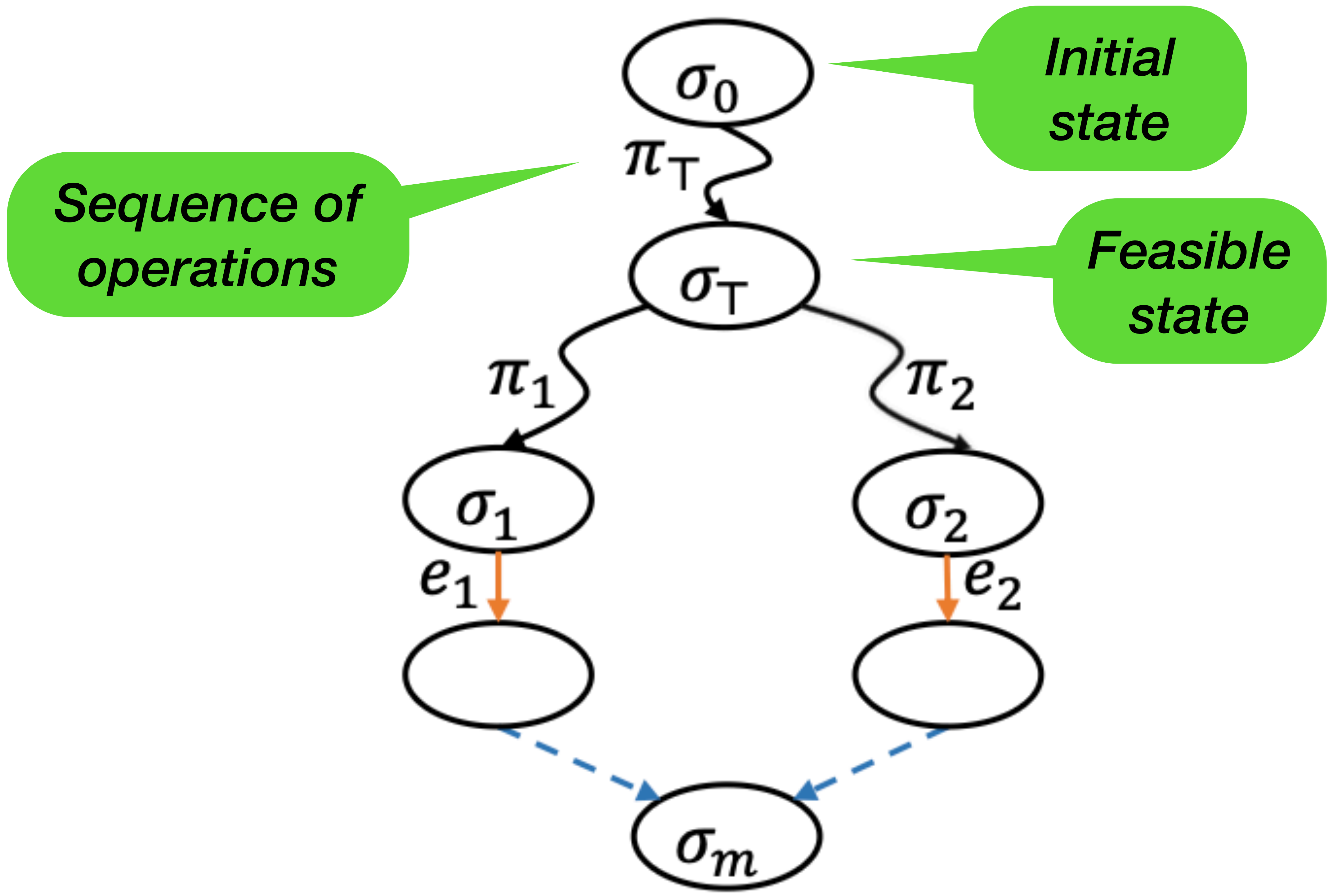
$$\frac{e_1 \neq e_2 \qquad e_1 \xrightarrow{\mathsf{rc}} e_2 \lor e_1 \rightleftarrows e_2}{\mu(l, e_1(a), e_2(b)) = e_2(\mu(l, e_1(a), b))}$$



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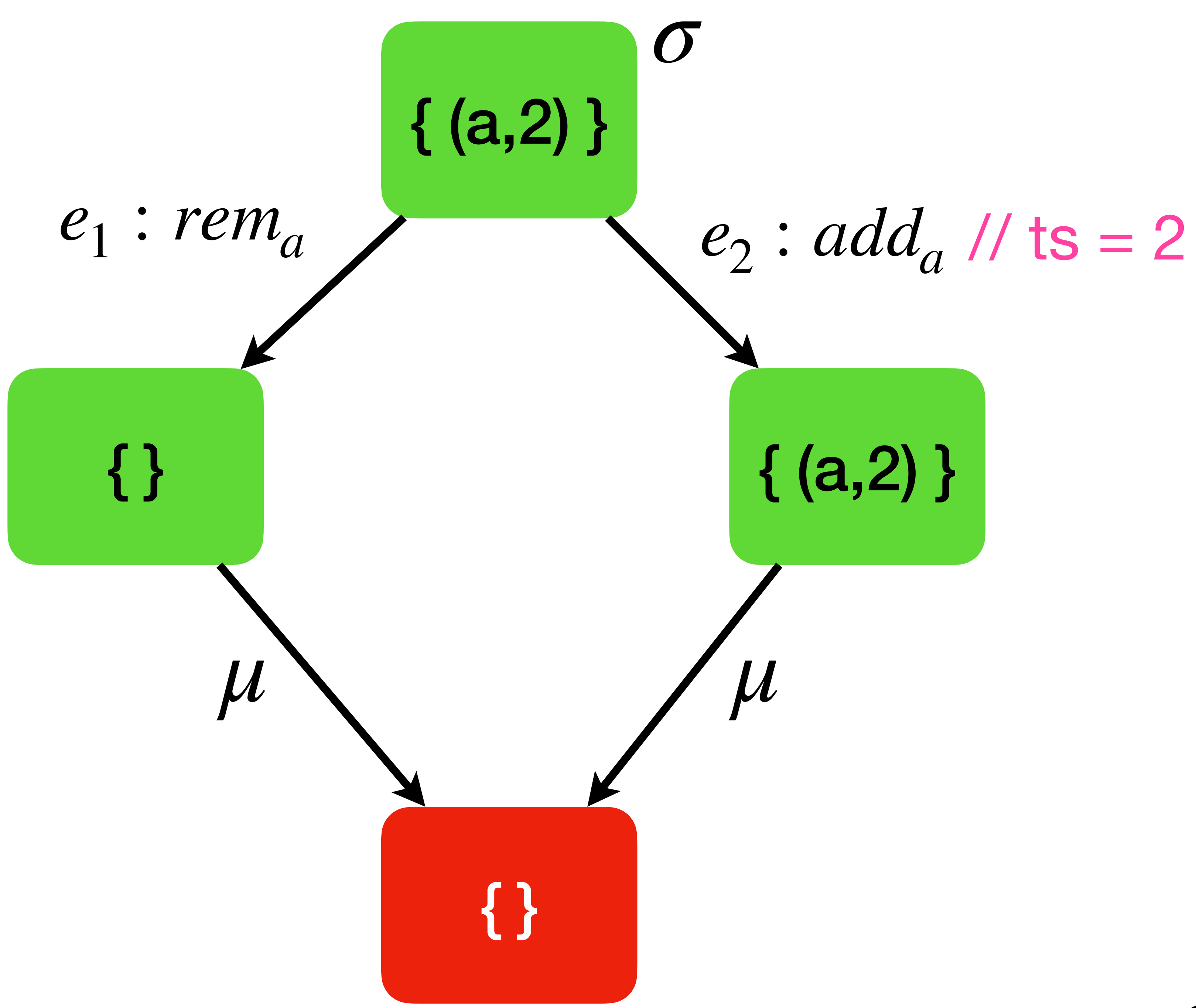

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Induction on $\pi_\top$

Base case

$$\frac{< pre >}{\mu(\sigma_0, e_1(\sigma_0), e_2(\sigma_0)) = e_2(\mu(\sigma_0, e_1(\sigma_0), \sigma_0))}$$

$(a,2) \notin \sigma_0$

Inductive case

$$\frac{< pre > \qquad \mu(\sigma_t, e_1(\sigma_t), e_2(\sigma_t)) = e_2(\mu(\sigma_t, e_1(\sigma_t), \sigma_t)) \qquad \sigma'_t = e(\sigma_t)}{\mu(\sigma'_t, e_1(\sigma'_t), e_2(\sigma'_t)) = e_2(\mu(\sigma'_t, e_1(\sigma'_t), \sigma'_t))}$$

Timestamps are unique

Linearizable MRDTs

THEOREM 4.7. *If an MRDT $\mathcal{D}$ satisfies the VCs ψ^*(*BOTTOMUP-2-OP*), ψ^*(*BOTTOMUP-1-OP*), ψ^*(*BOTTOMUP-0-OP*), MERGEIDEMPOTENCE *and* MERGECOMMUTATIVITY*, then $\mathcal{D}$ is linearizable.*

An MRDT that satisfies the algebraic properties is RA-linearizable

LEMMA 3.10. *If MRDT $\mathcal{D}$ is RA-linearizable, then for all executions $\tau \in [\![\mathcal{S}_{\mathcal{D}}]\!]$, for all transitions $C \xrightarrow{query(r,q,a)} C'$ in τ where $C = \langle N, H, L, G, vis \rangle$, there exists a sequence π consisting of all events in $L(H(r))$ such that $\mathsf{lo}(C)_{|L(H(r))} \subseteq \pi$ and $a = \mathsf{query}(\pi(\sigma_0), q)$.*

RA-linearizable MRDT query results match those obtained on the linearised updates applied to the initial state

Verified MRDTs

MRDT	rc **Policy**	**#LOC**	**Verification Time (s)**
Increment-only counter [12]	none	6	0.72
PN counter [23]	none	10	1.64
Enable-wins flag*	$\text{disable} \xrightarrow{\text{rc}} \text{enable}$	30	29.80
Disable-wins flag*	$\text{enable} \xrightarrow{\text{rc}} \text{disable}$	30	37.91
Grows-only set [12]	none	6	0.45
Grows-only map [23]	none	11	4.65
OR-set [23]	$\text{rem}_a \xrightarrow{\text{rc}} \text{add}_a$	20	4.53
OR-set (efficient)*	$\text{rem}_a \xrightarrow{\text{rc}} \text{add}_a$	34	660.00
Remove-wins set*	$\text{add}_a \xrightarrow{\text{rc}} \text{rem}_a$	22	9.60
Set-wins map*	$\text{del}_k \xrightarrow{\text{rc}} \text{set}_k$	20	5.06
Replicated Growable Array [1]	none	13	1.51
Optional register*	$\text{unset} \xrightarrow{\text{rc}} \text{set}$	35	200.00
Multi-valued Register*	none	7	0.65
JSON-style MRDT*	Fig. 13	26	148.84

Neem also supports verification of RA-linearizability of state-based CRDTs

https://github.com/prismlab/neem

Limitations

- Automated verification returns yes / no / ¯_(ツ)_/¯
  - Not pleasant for engineering
  - No counterexamples!
- **Current work**
  - Optimal bounded model checking of MRDTs against RA-linearizability
    - Standard DPOR fails optimality
  - Moving to Lean — ITP with SMT backend, proof reconstruction, “Loom”, etc.

Neem — Automatic verification of RDTs

- **What's in the box?**
  - Definition of RA-linearizability for MRDTs
  - A novel induction scheme for MRDTs and state-based CRDTs to ***automatically*** verify RA-linearizability
  - Implemented in F*