Certified Mergeable Replicated Data Types

“KC” Sivaramakrishnan

joint work with

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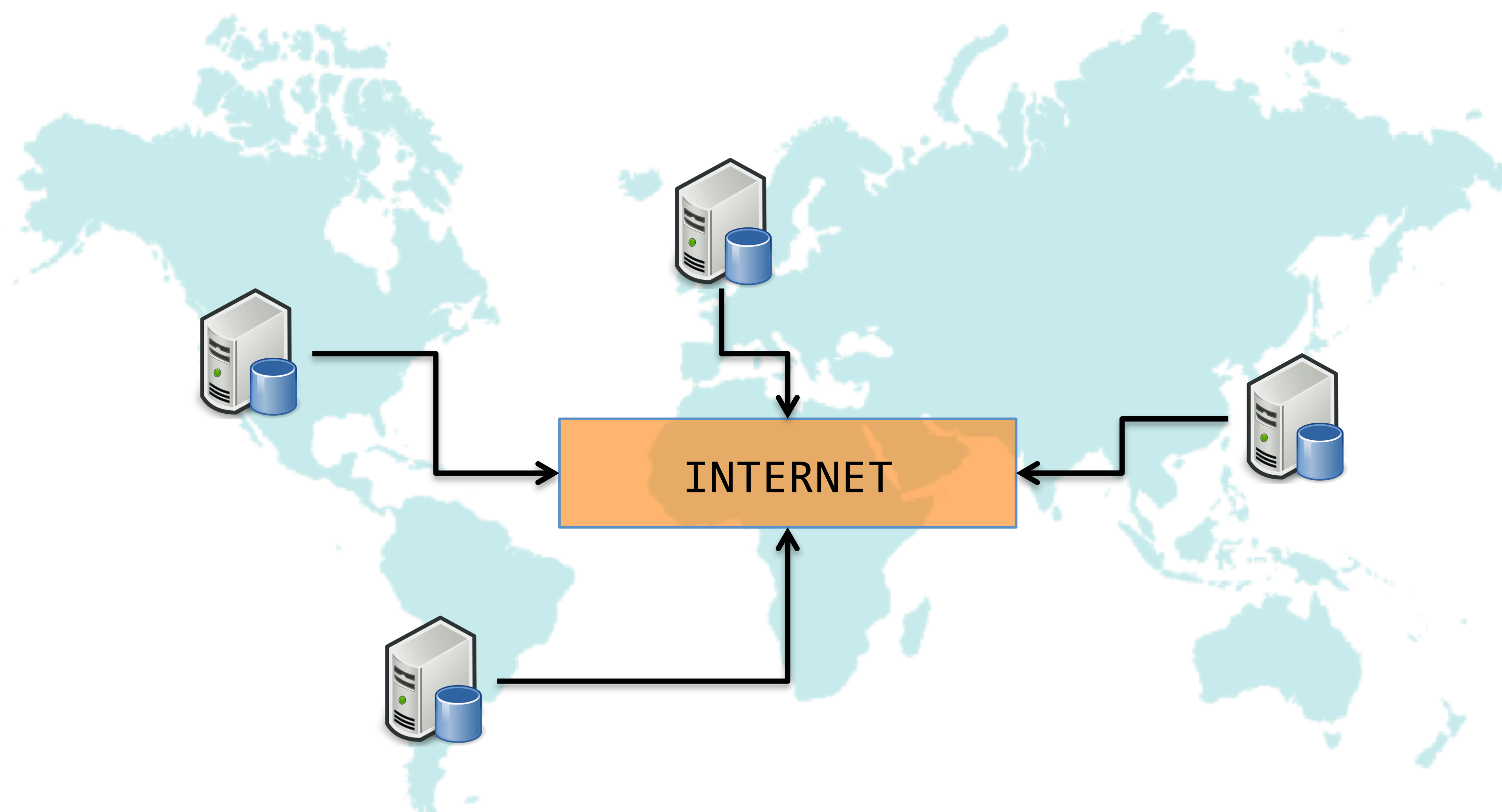
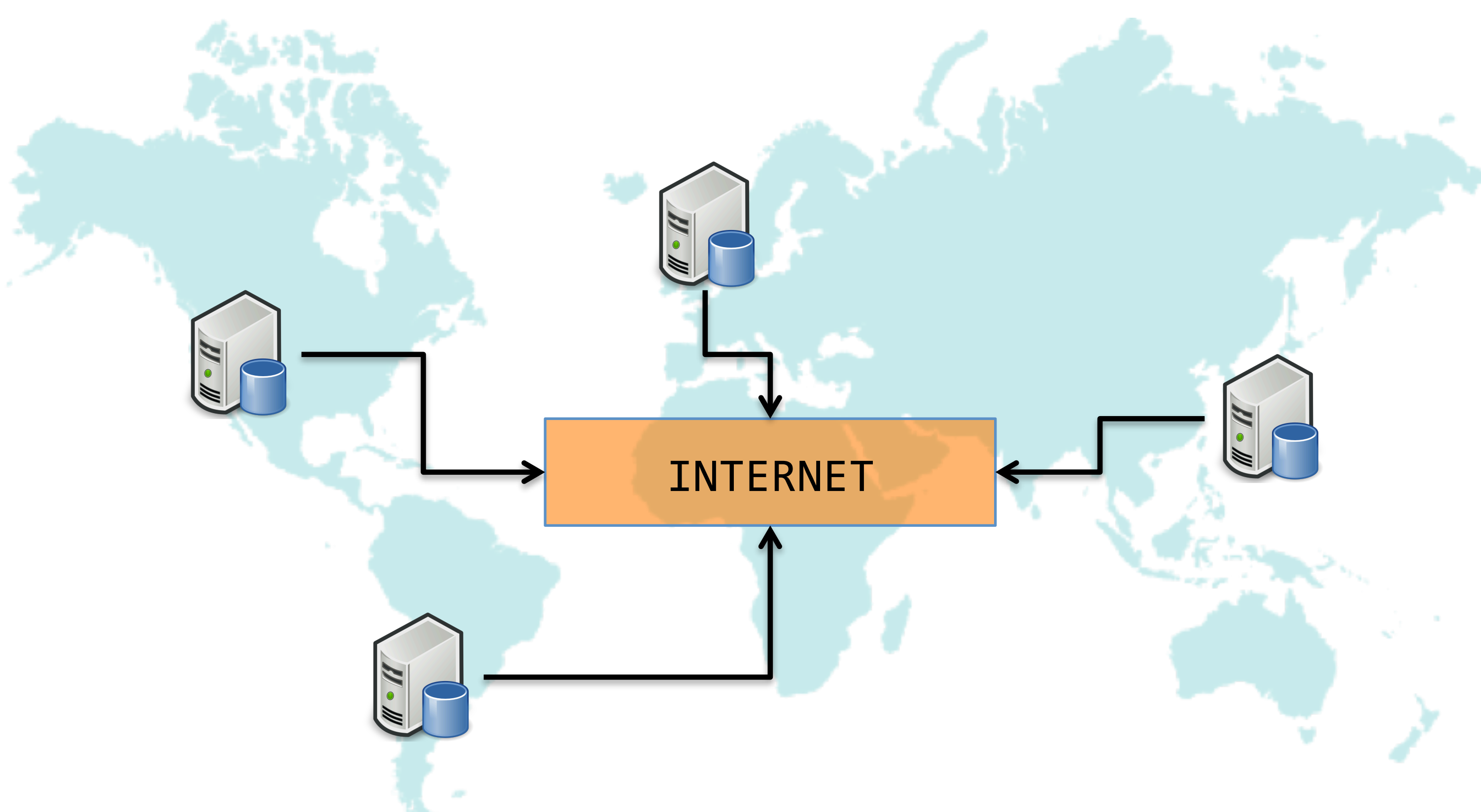
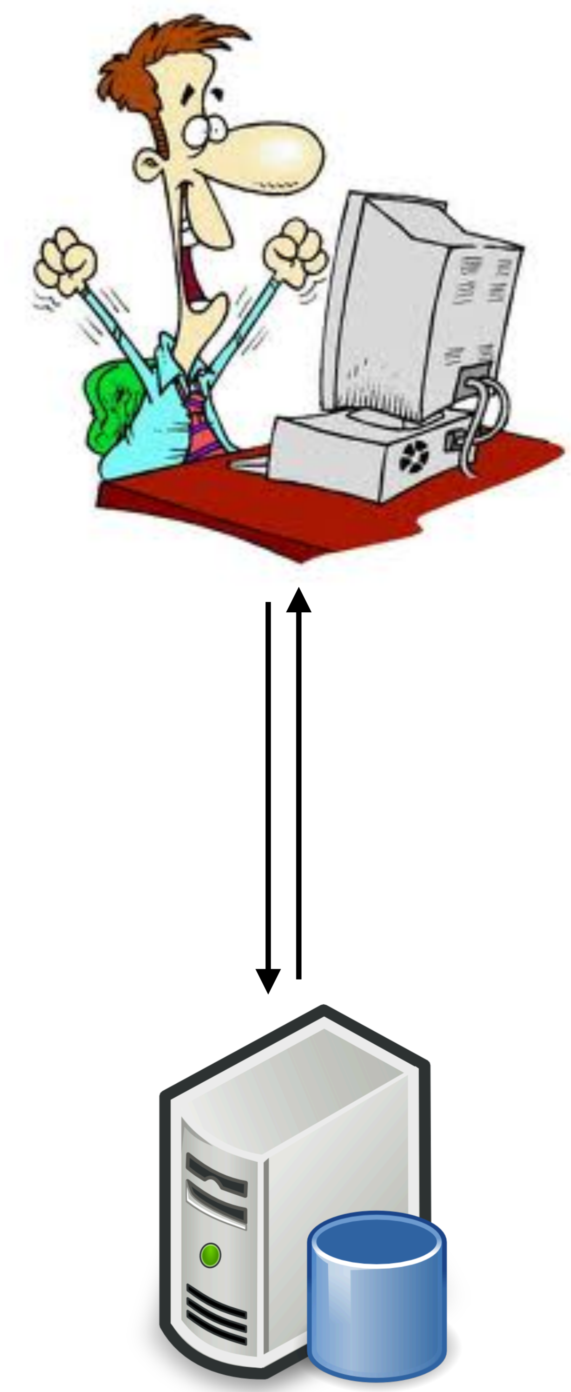
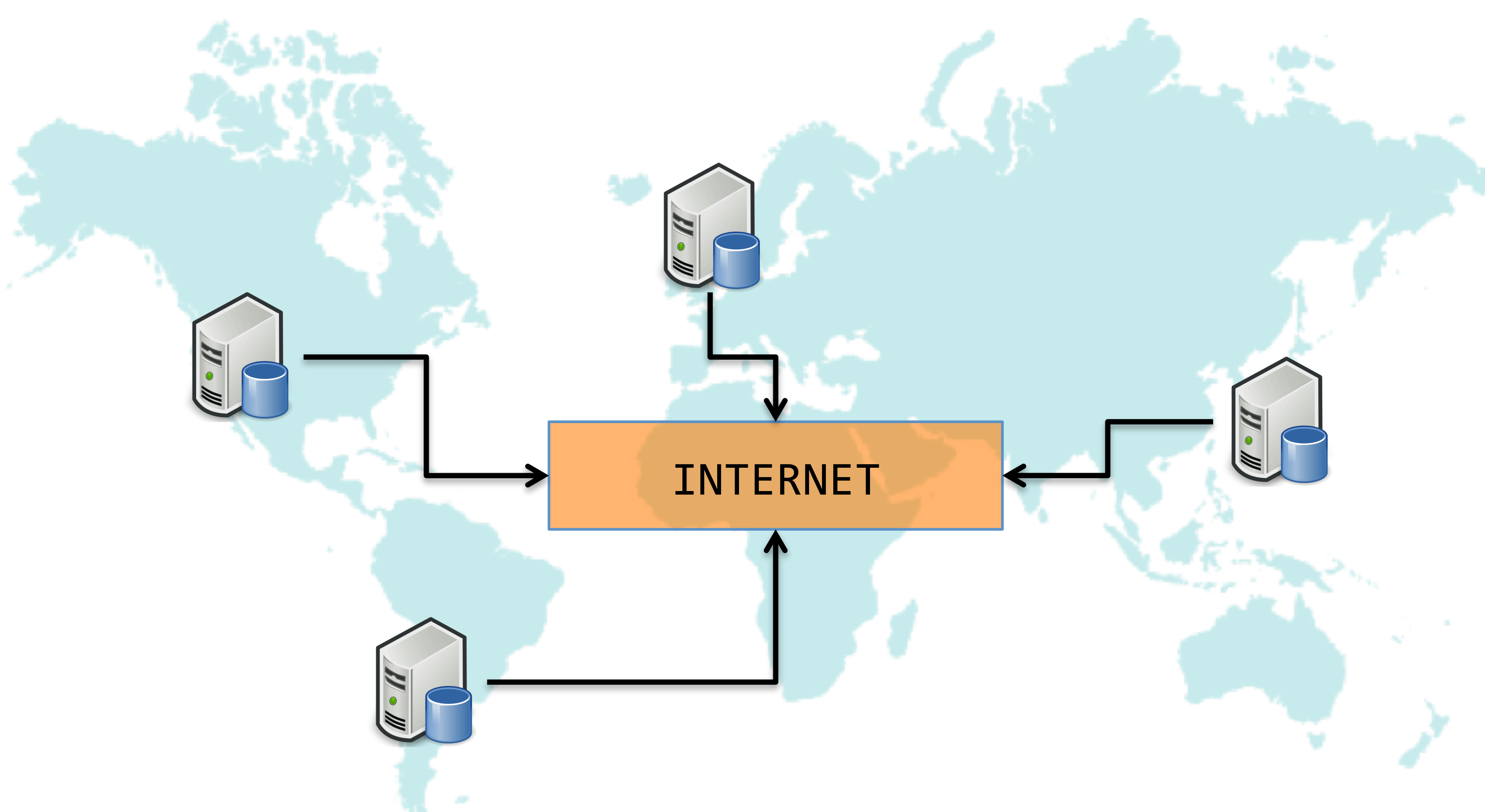



INTERNET




INTERNET

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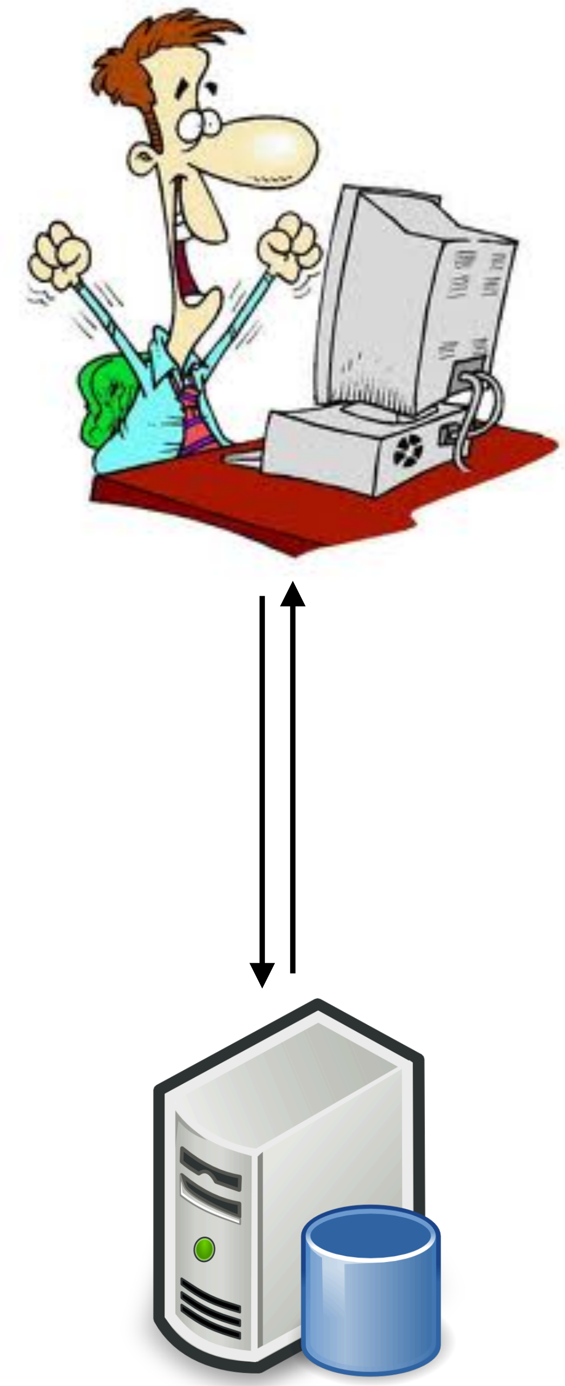

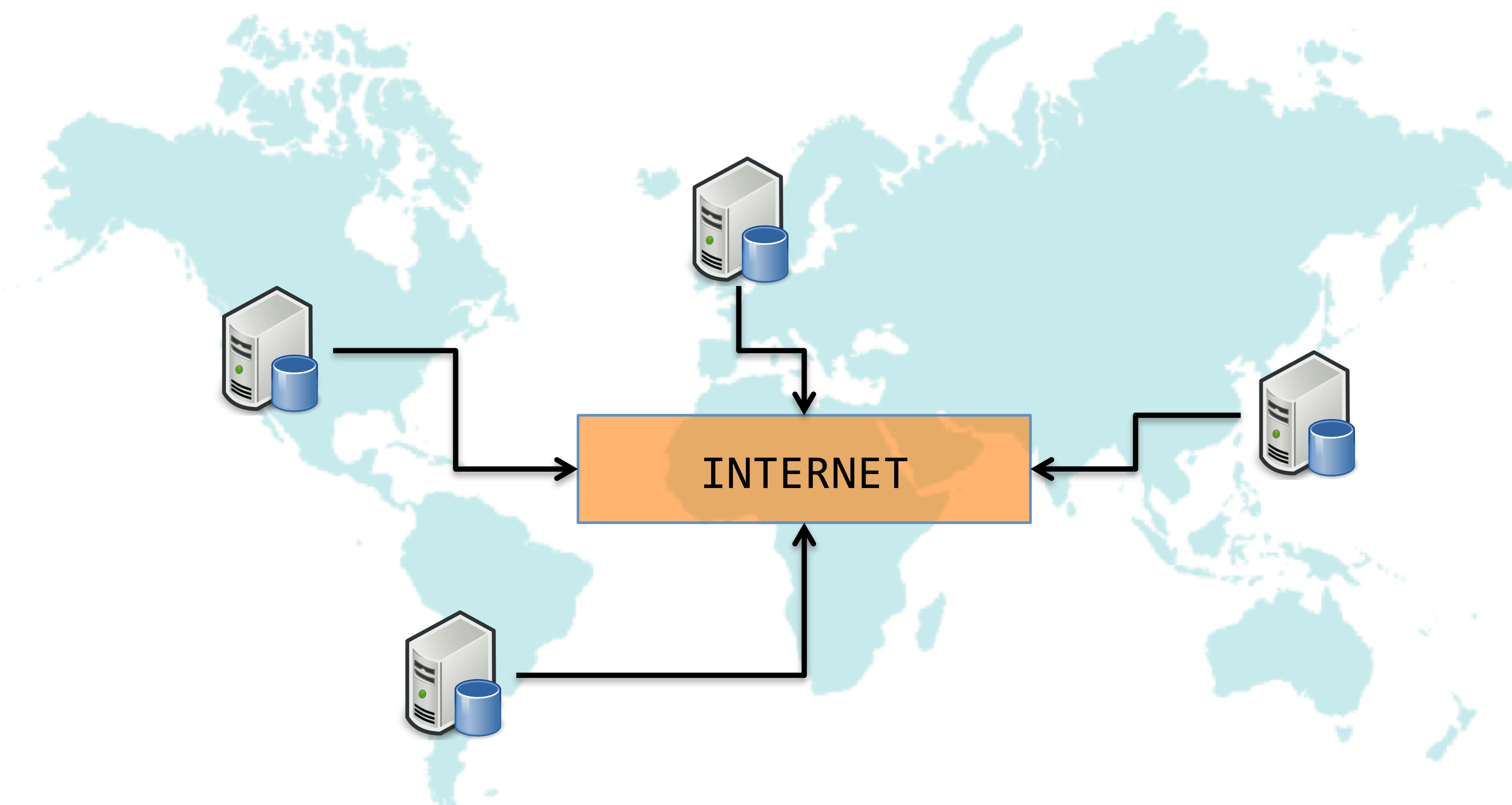

• Weak Consistency & Isolation

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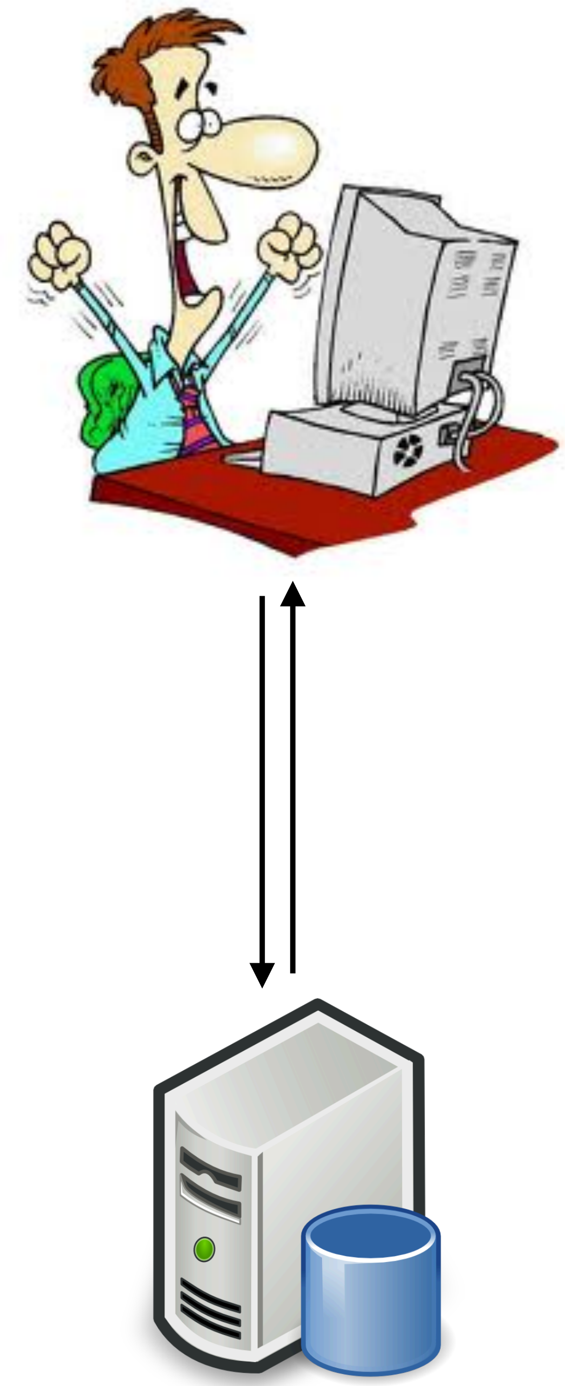
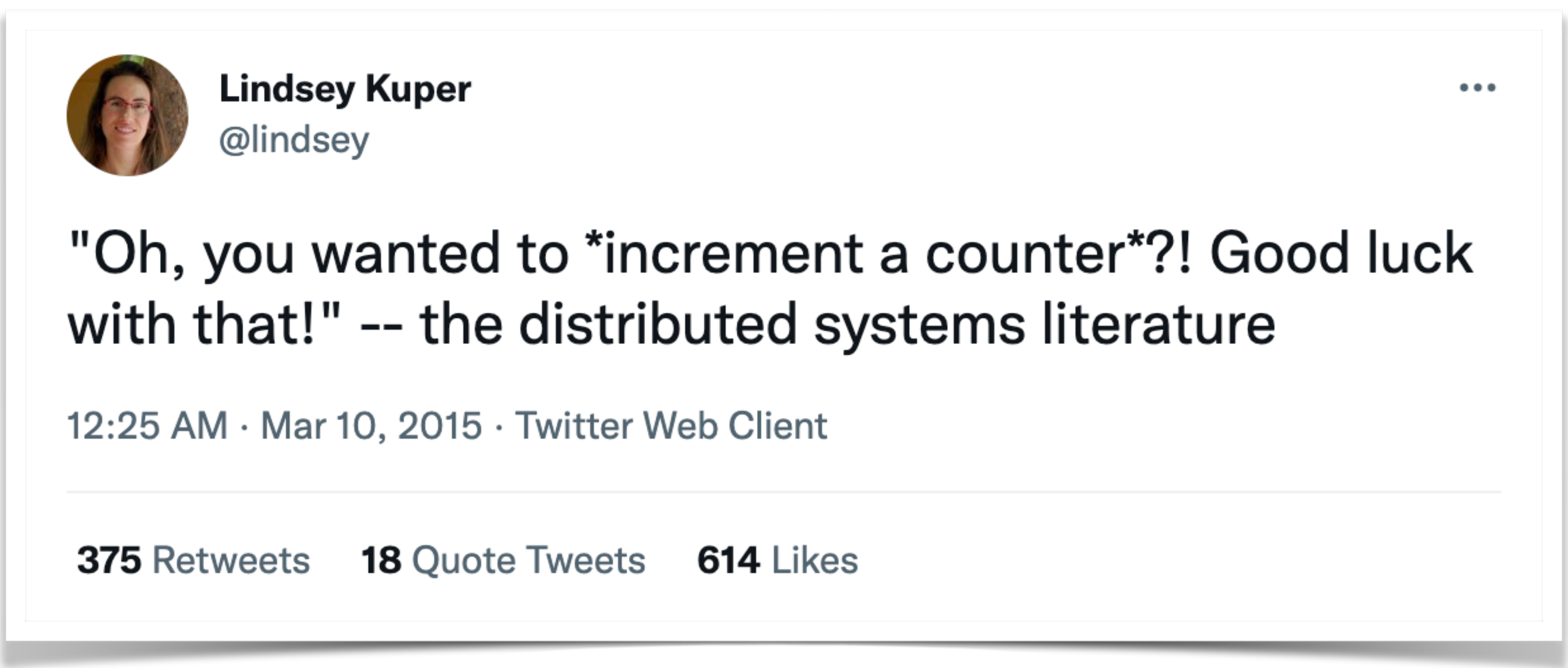


• Serializability
• Linearizability

Even *simple* data structures attract enormous *complexity* when made *distributed*



Sequential Counter

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module Counter : sig
  type t
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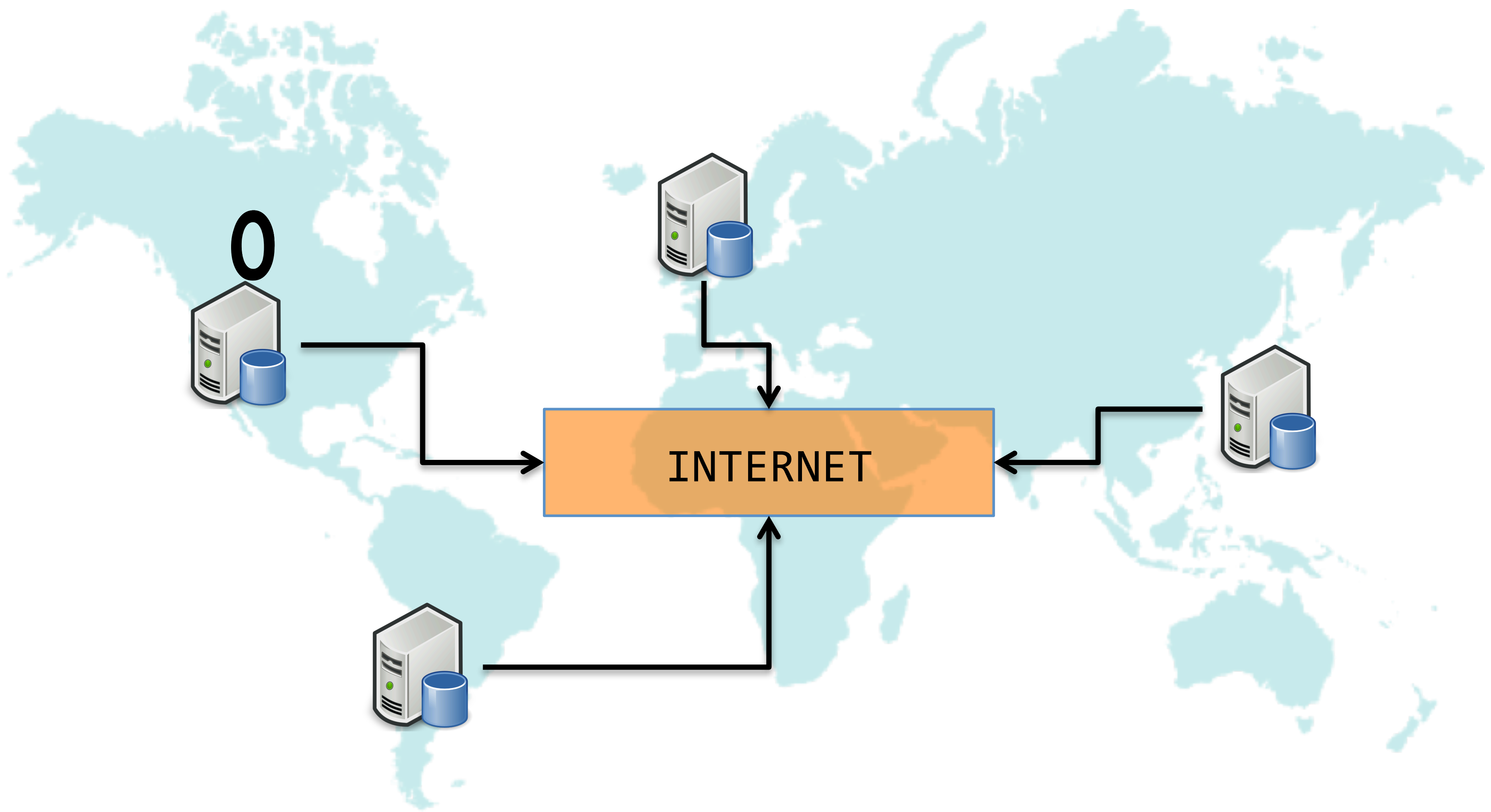
- Written in idiomatic style
- Composable

```
type counter_list = Counter.t list
```

Replicated Counter



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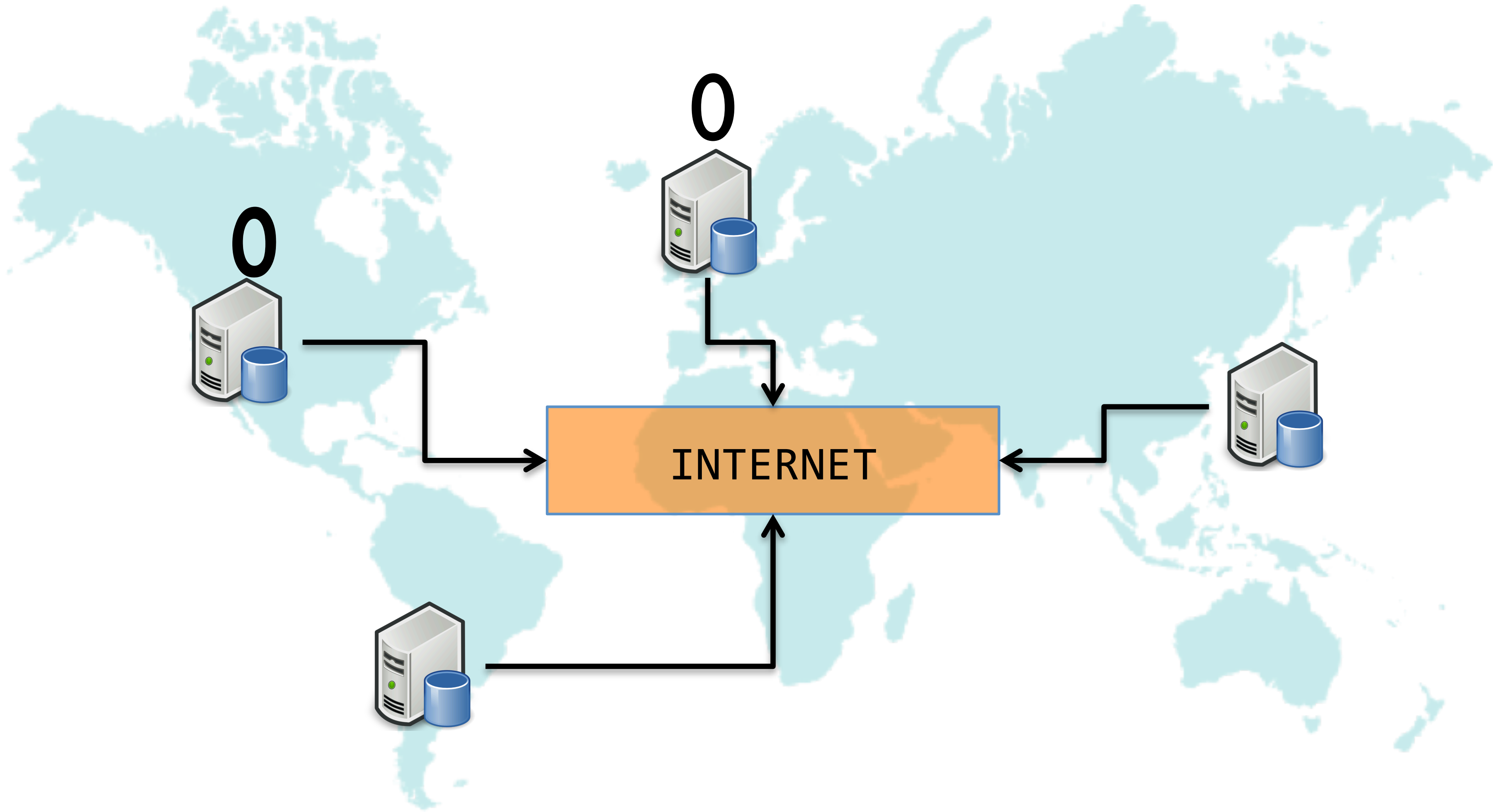

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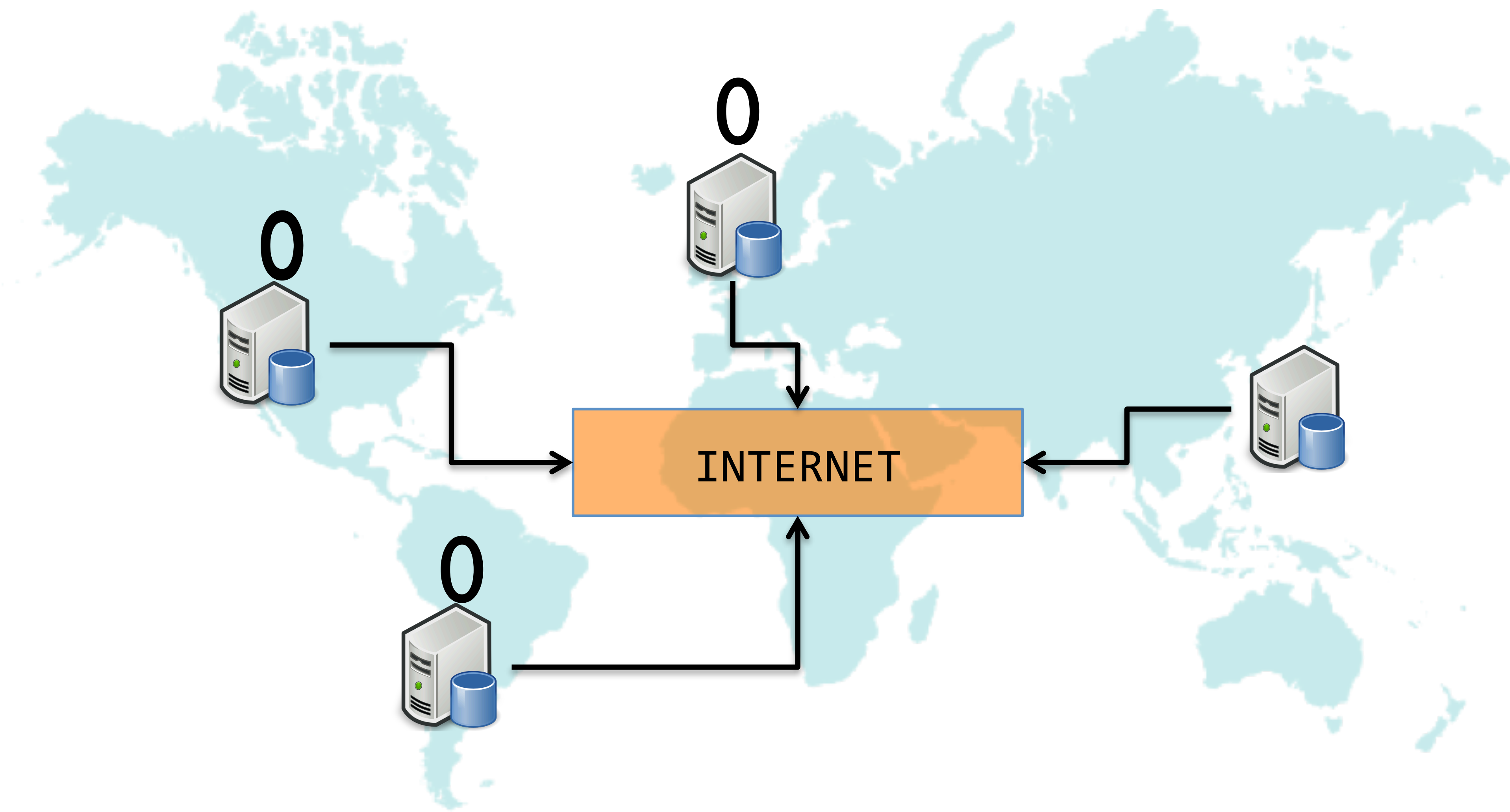

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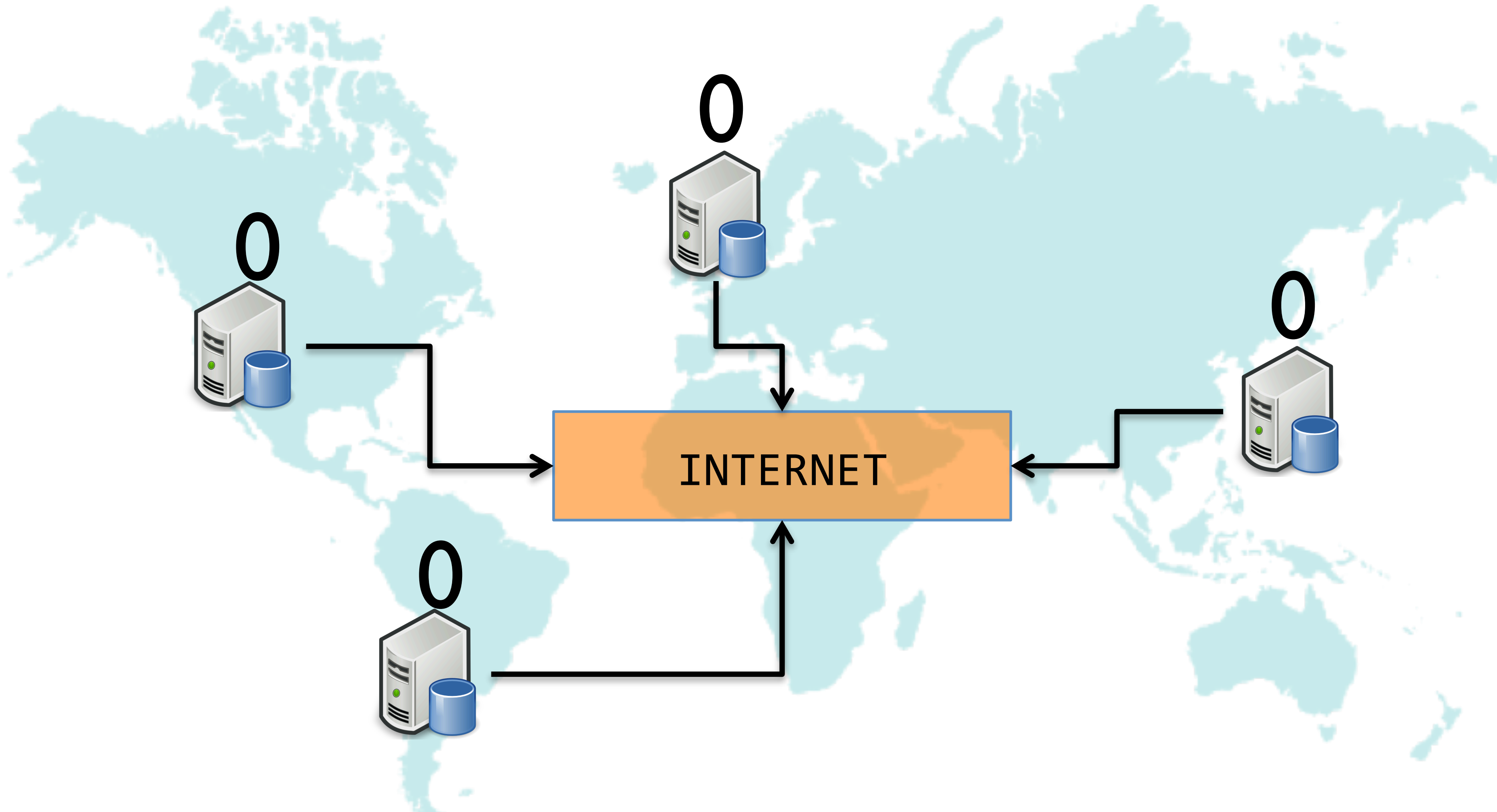

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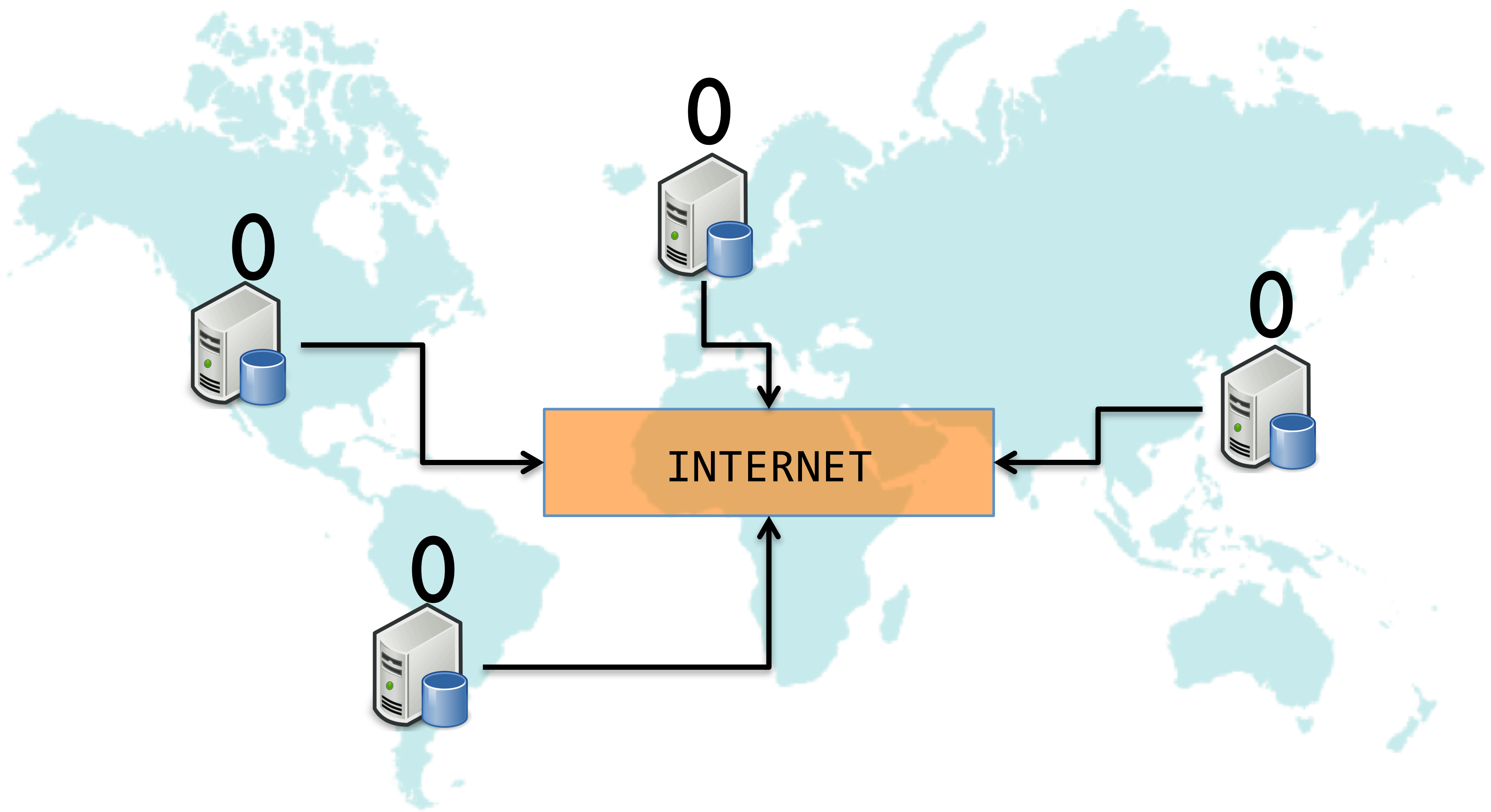

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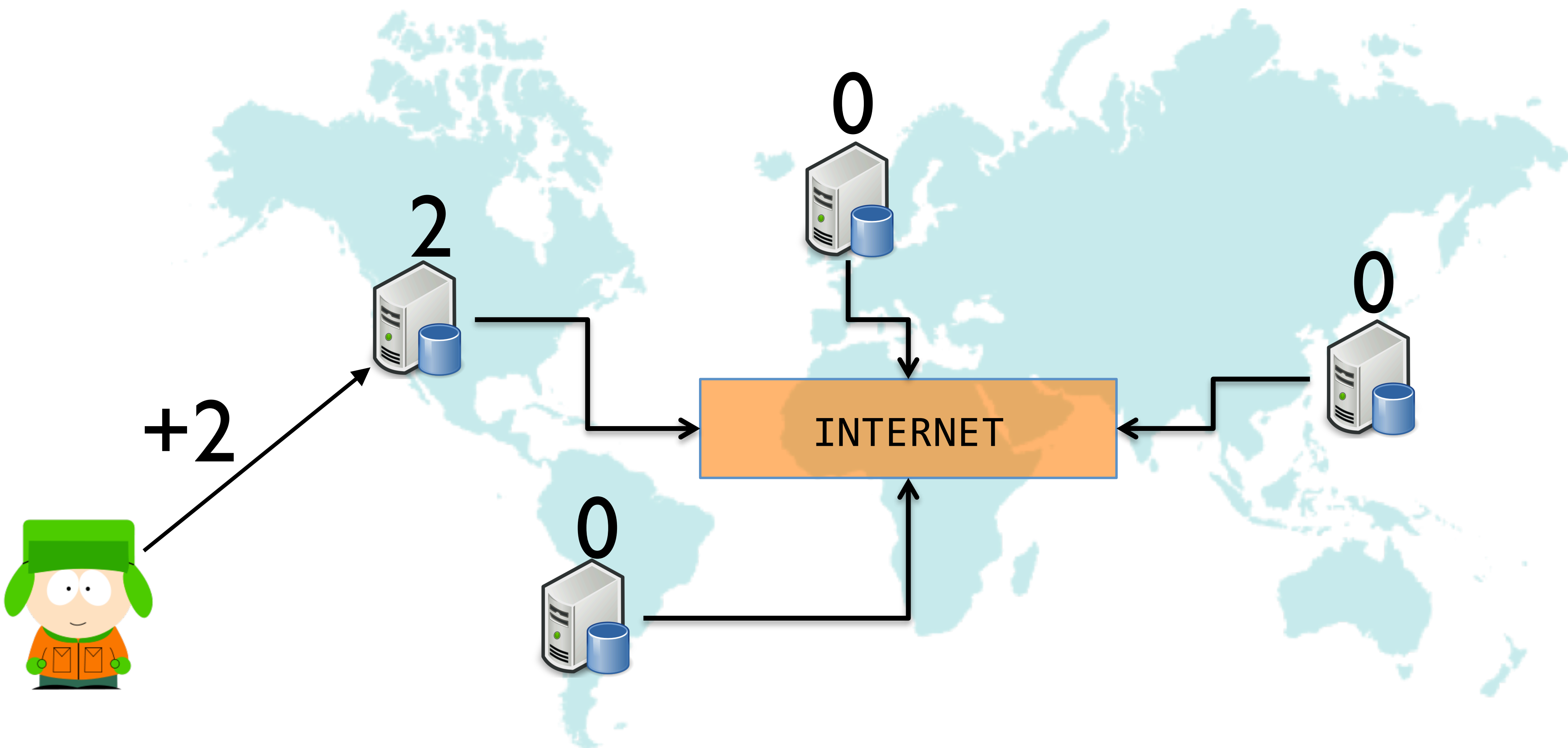

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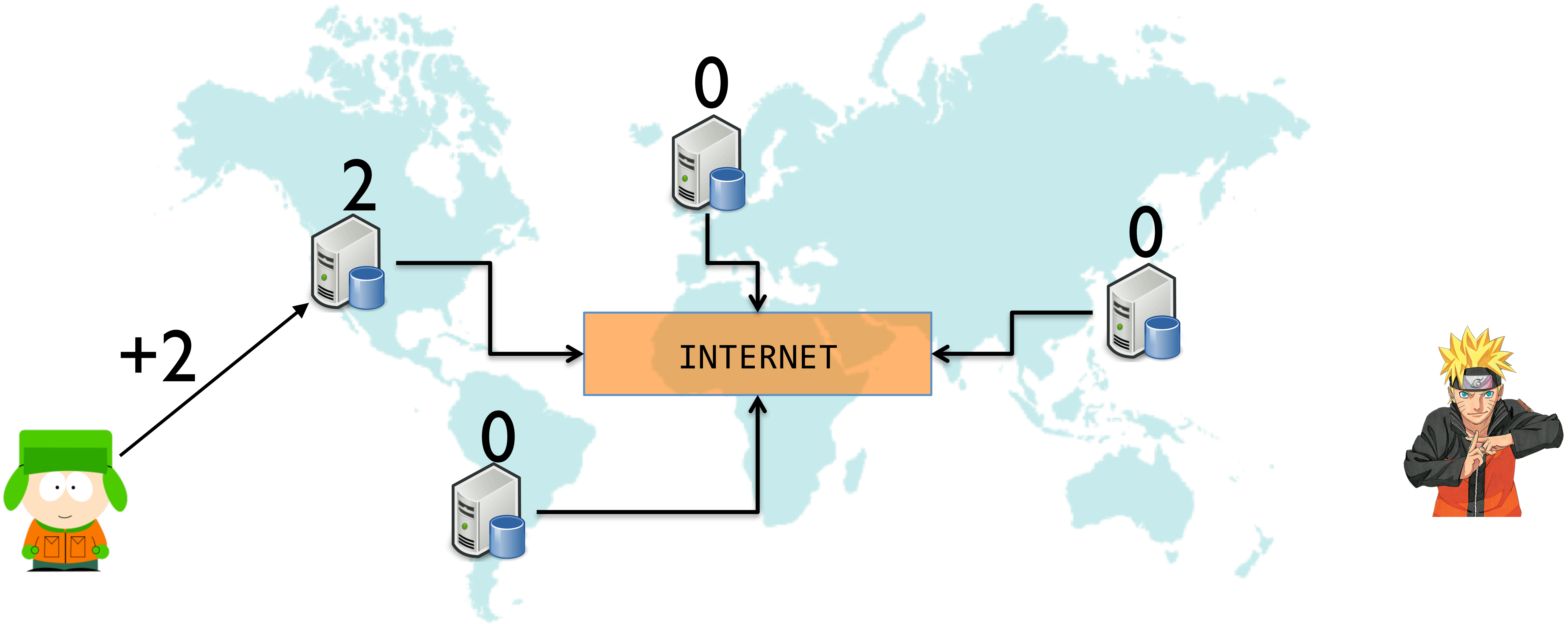

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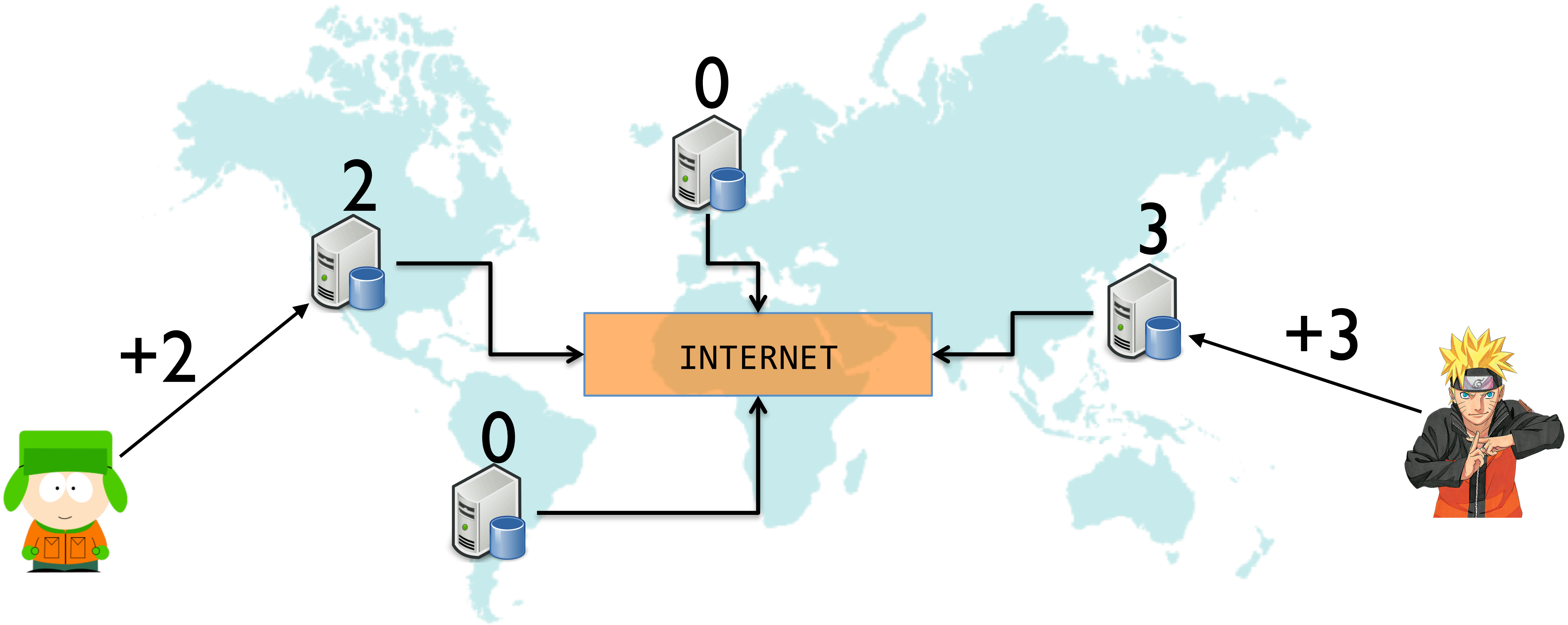

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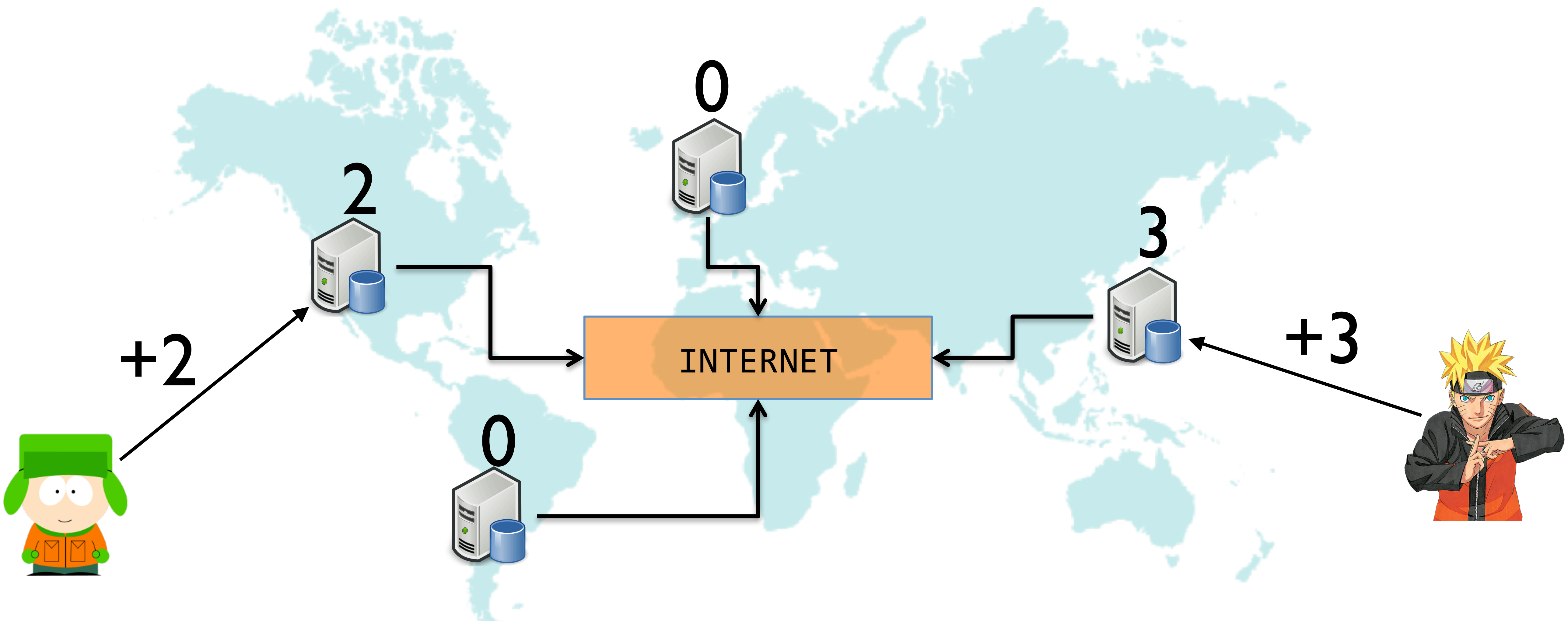
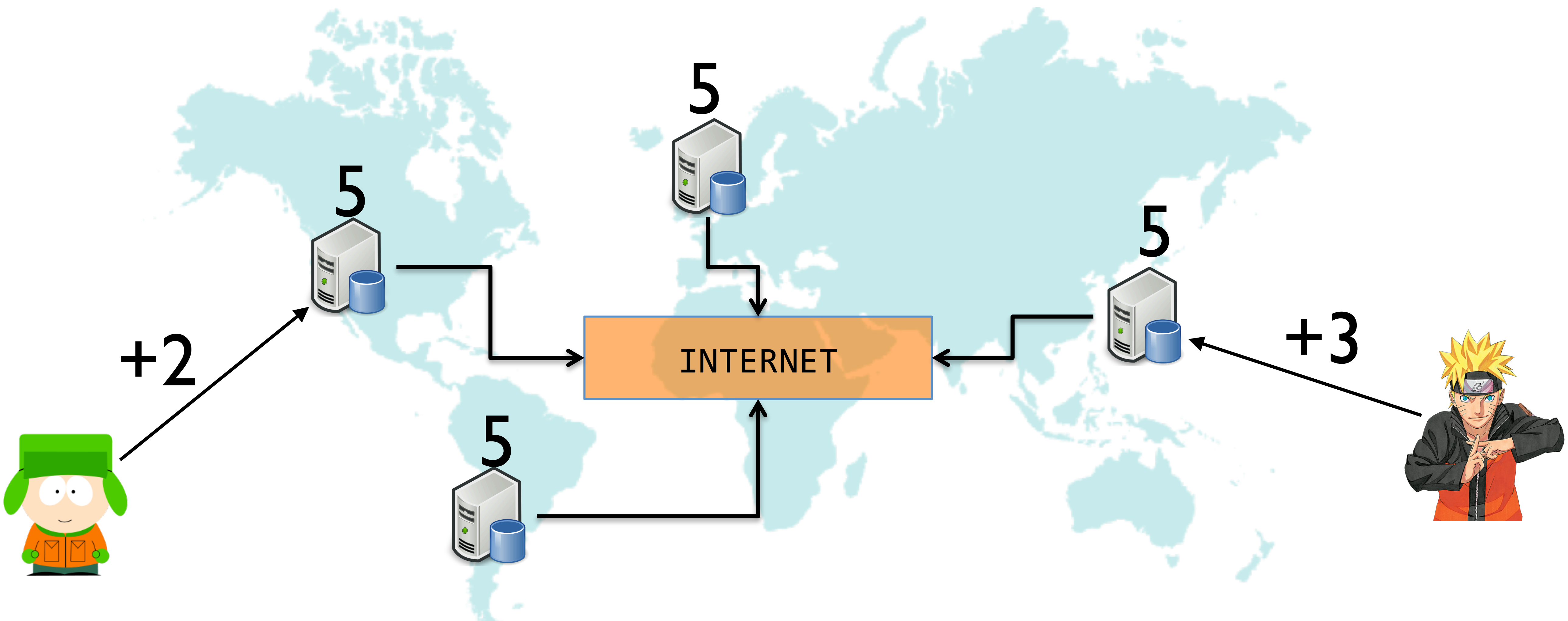


- ***Idea:*** Apply the local operations at all replicas

Replicated Counter



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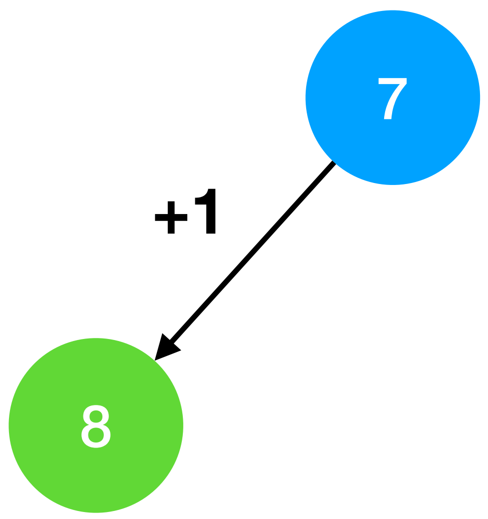

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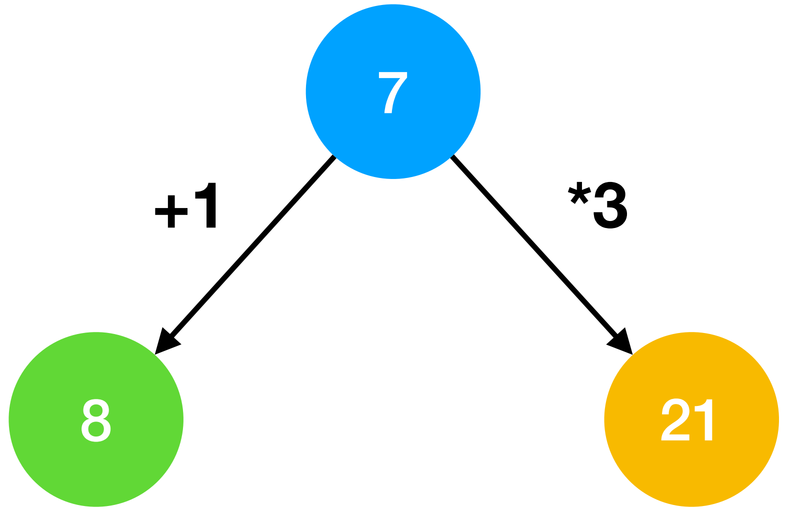

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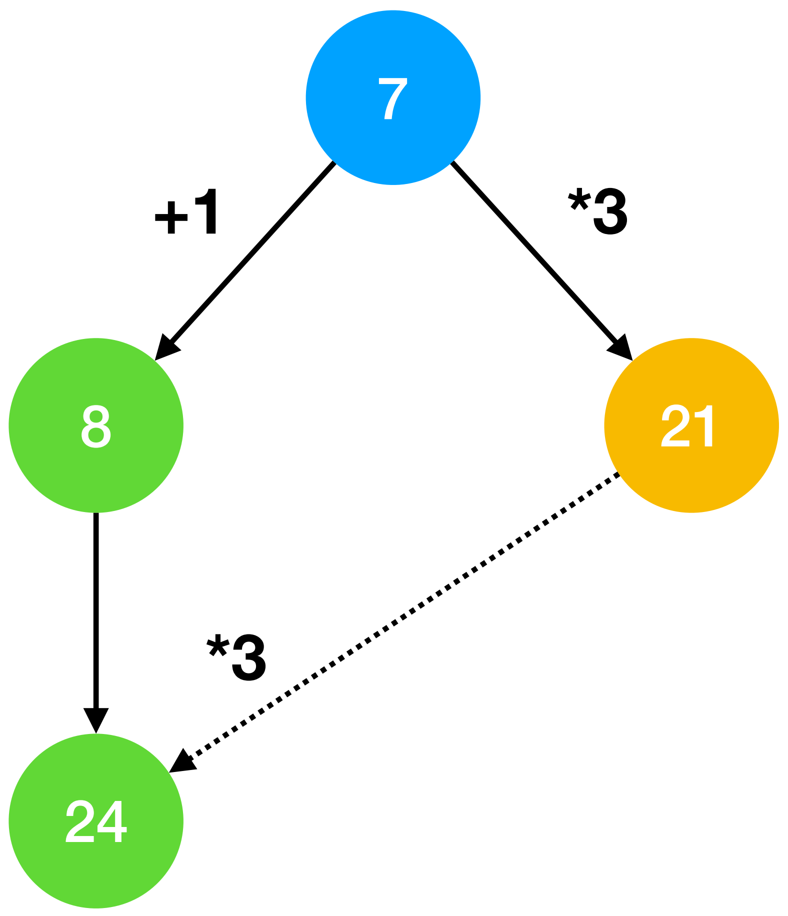

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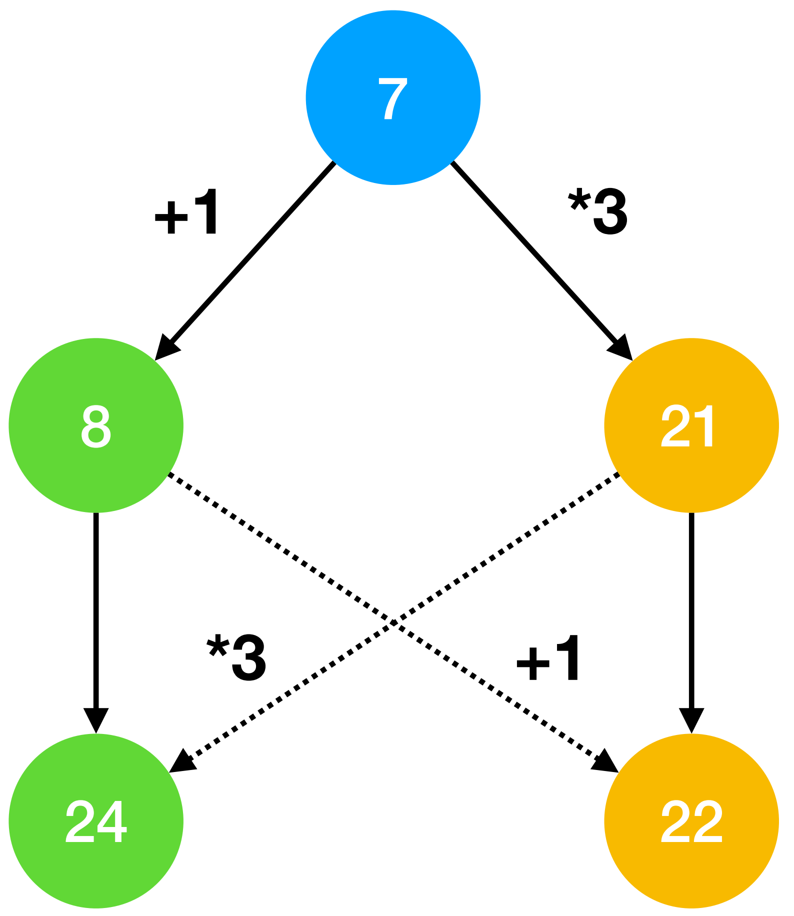

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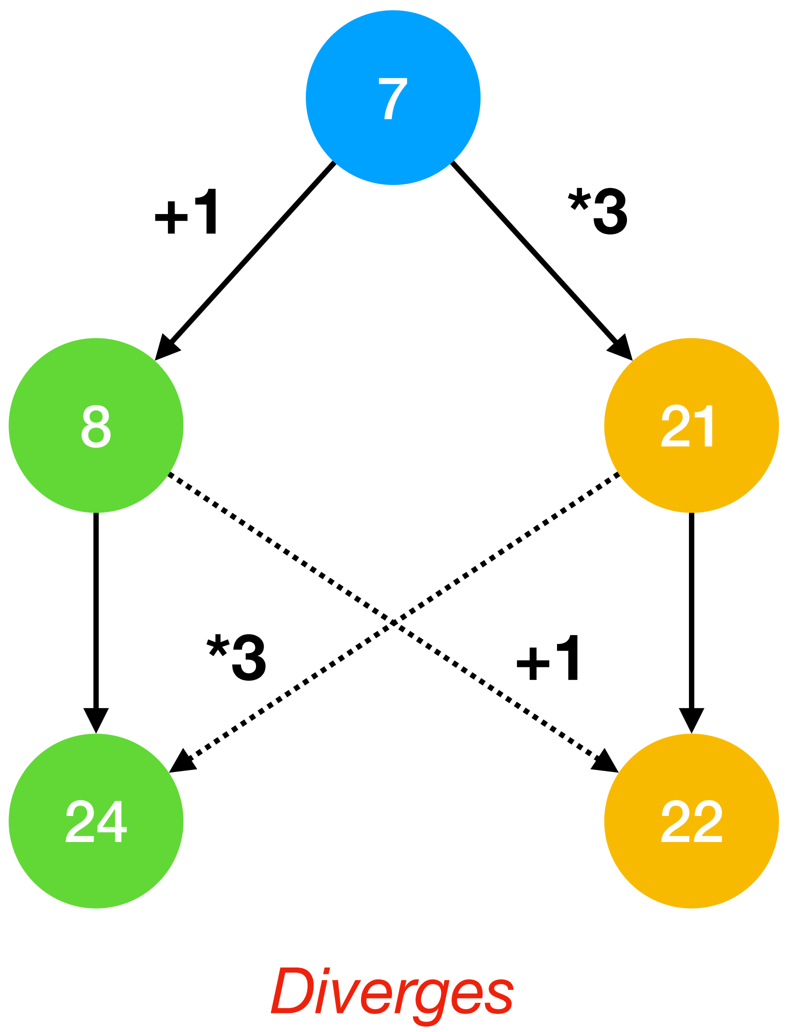

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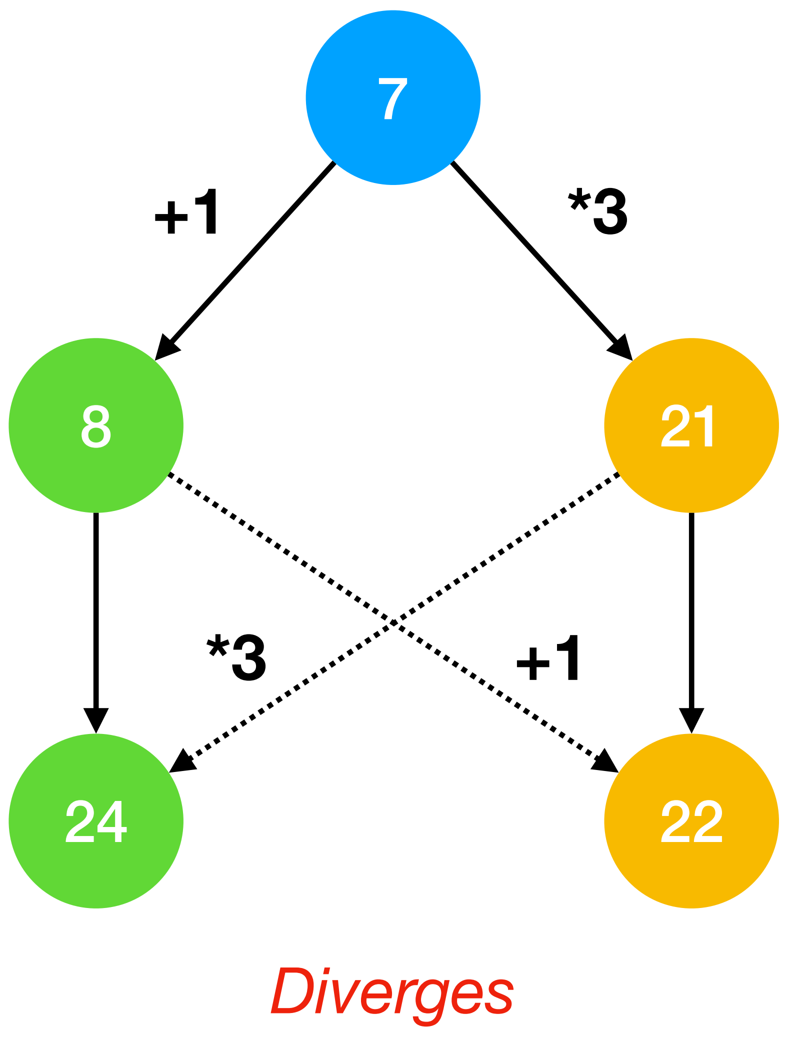

Addition and multiplication do not commute

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- ***Idea:*** Capture the effect of multiplication through the *commutative* addition operation

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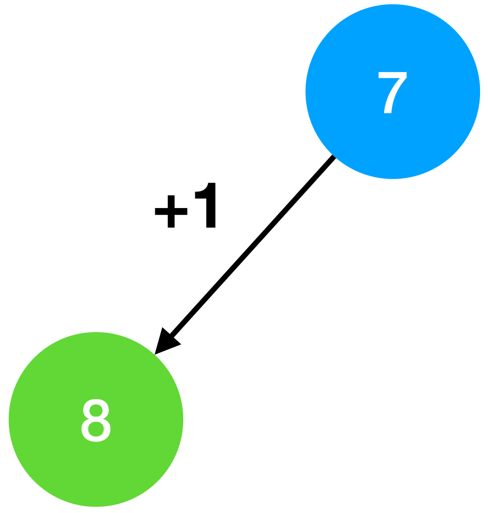

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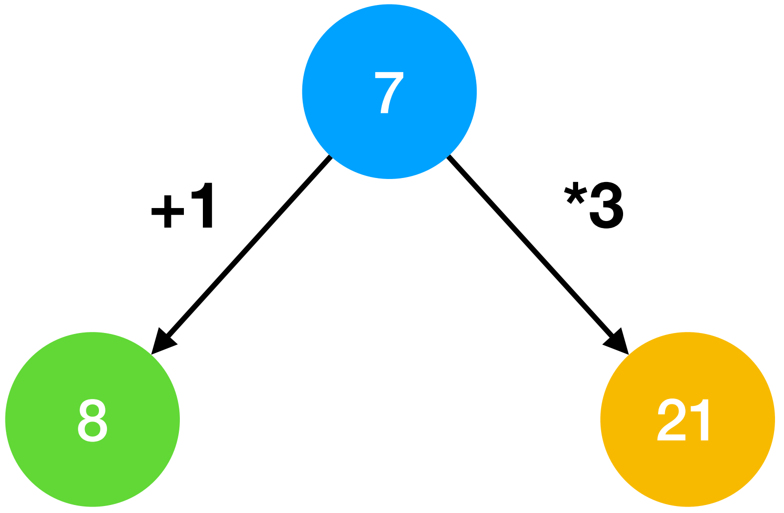

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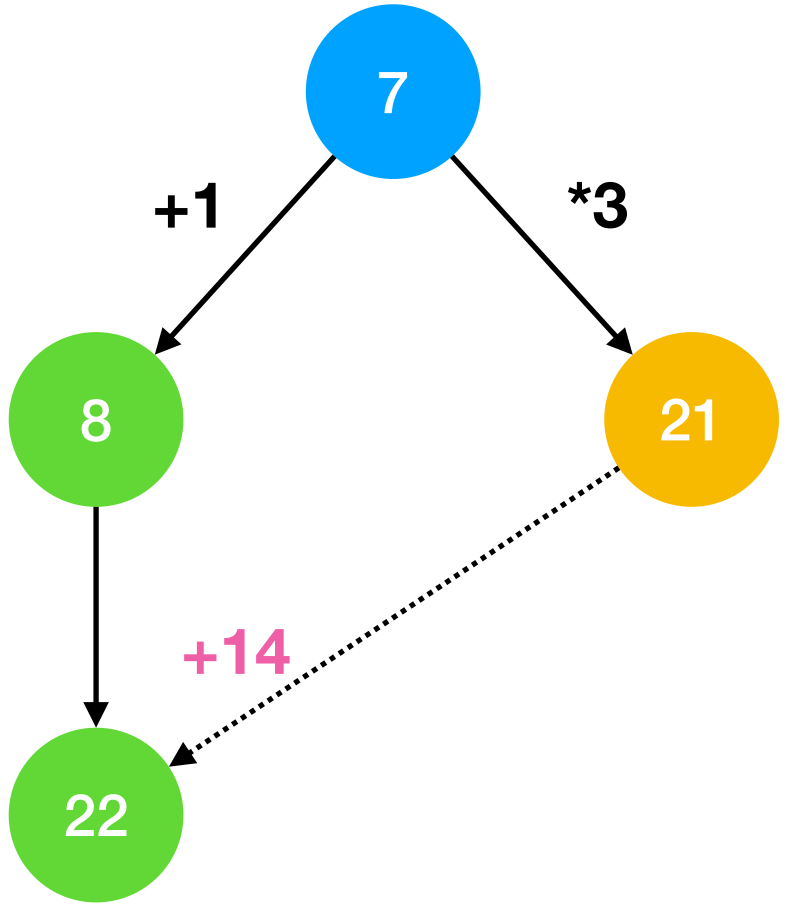

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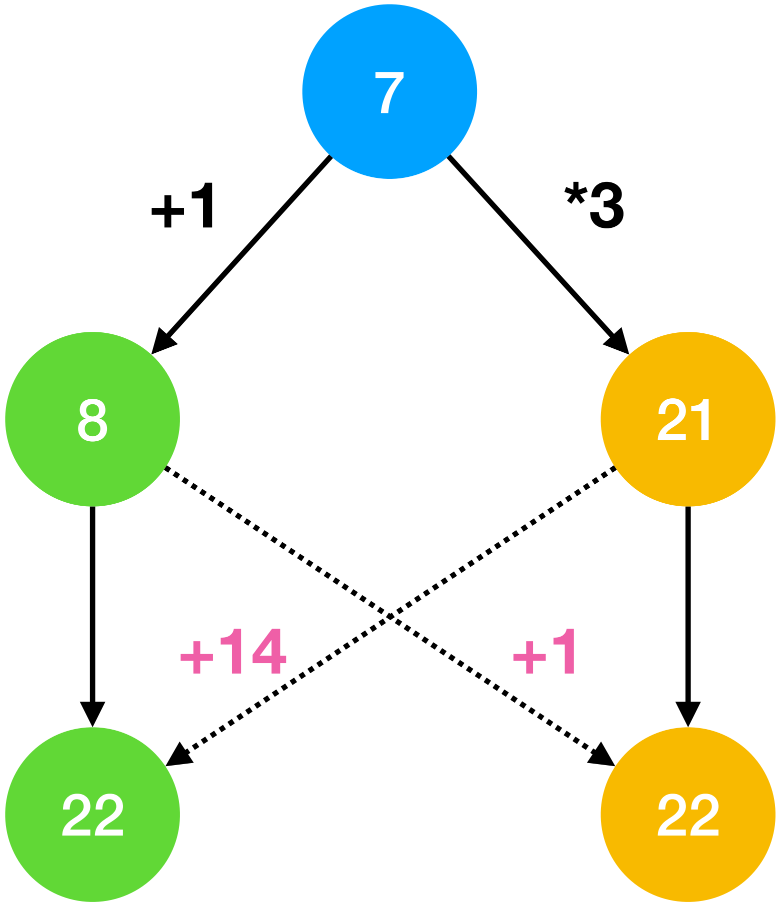

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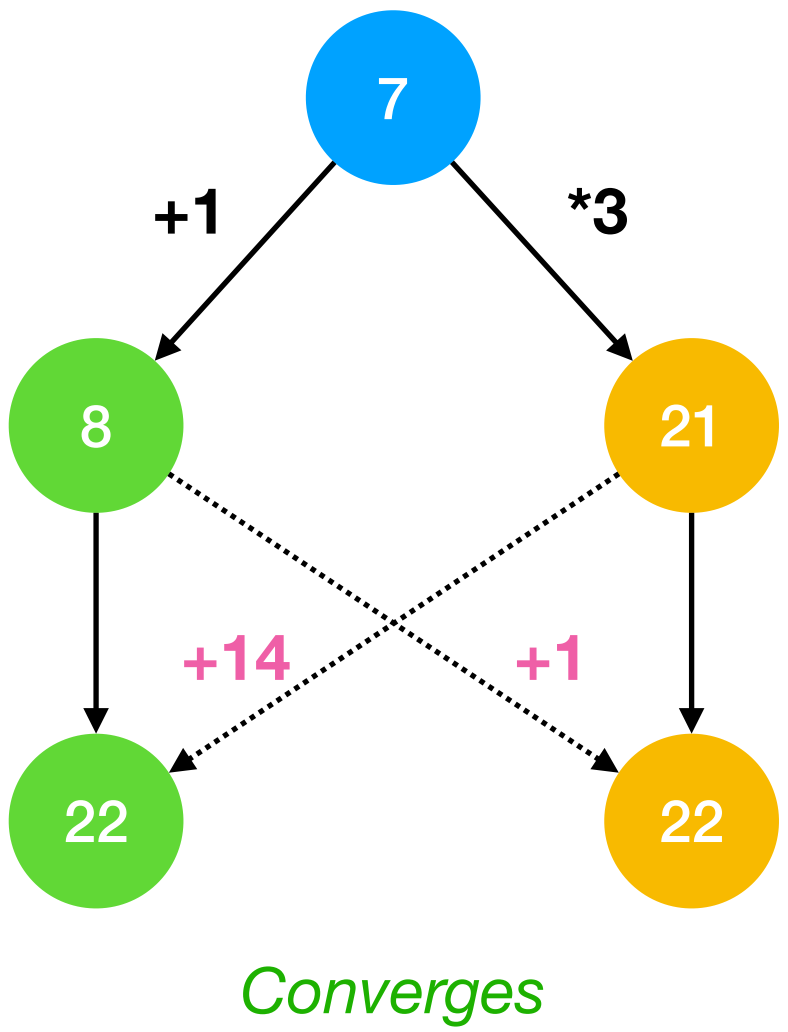

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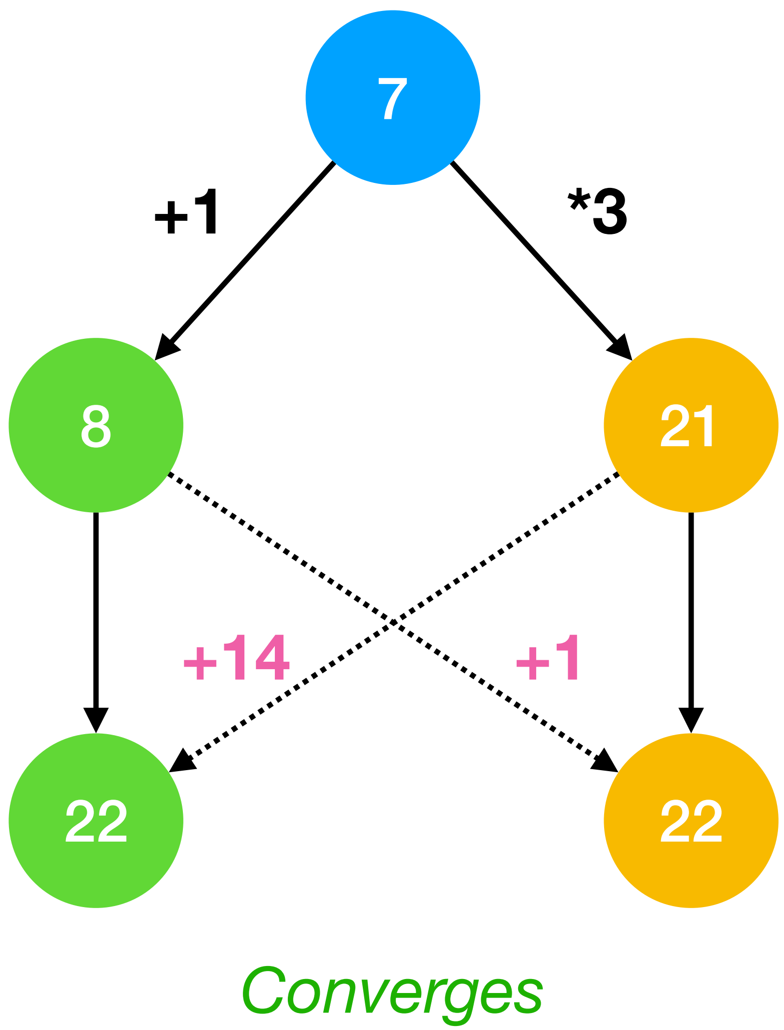

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- *CRDTs*

Convergent Replicated Data Types (CRDT)

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- CRDT is guaranteed to ensure *strong eventual consistency (SEC)*
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  - ★ Simple interface for the clients of CRDTs

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- CRDT is guaranteed to ensure *strong eventual consistency (SEC)*
  - ★ G-counters, PN-counters, OR-Sets, Graphs, Ropes, docs, sheets
  - ★ Simple interface for the clients of CRDTs
- Need to reengineer every datatype to ensure SEC (commutativity)
  - ★ Do not mirror sequential counter parts => implementation & proof burden
  - ★ Do not compose!
    - ✦ `counter set` is not a composition of `counter` and `set` CRDTs

Can we *program & reason about* replicated data types as an extension of their sequential counterparts?

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MRDT

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end = struct
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  let merge ~lca ~v1 ~v2 =
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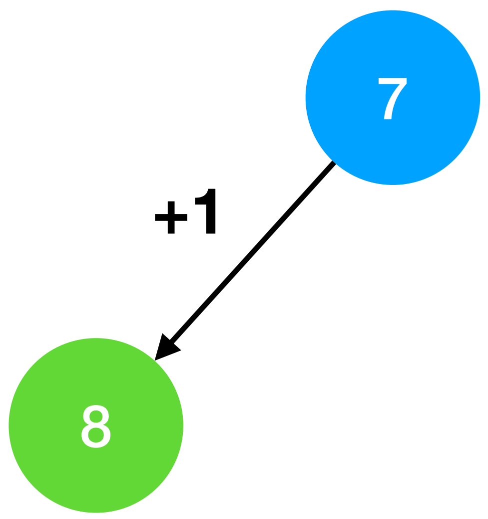

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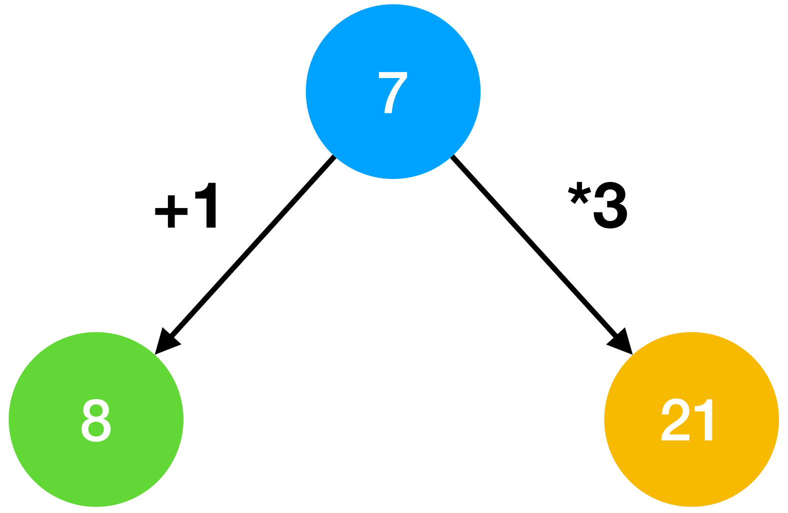

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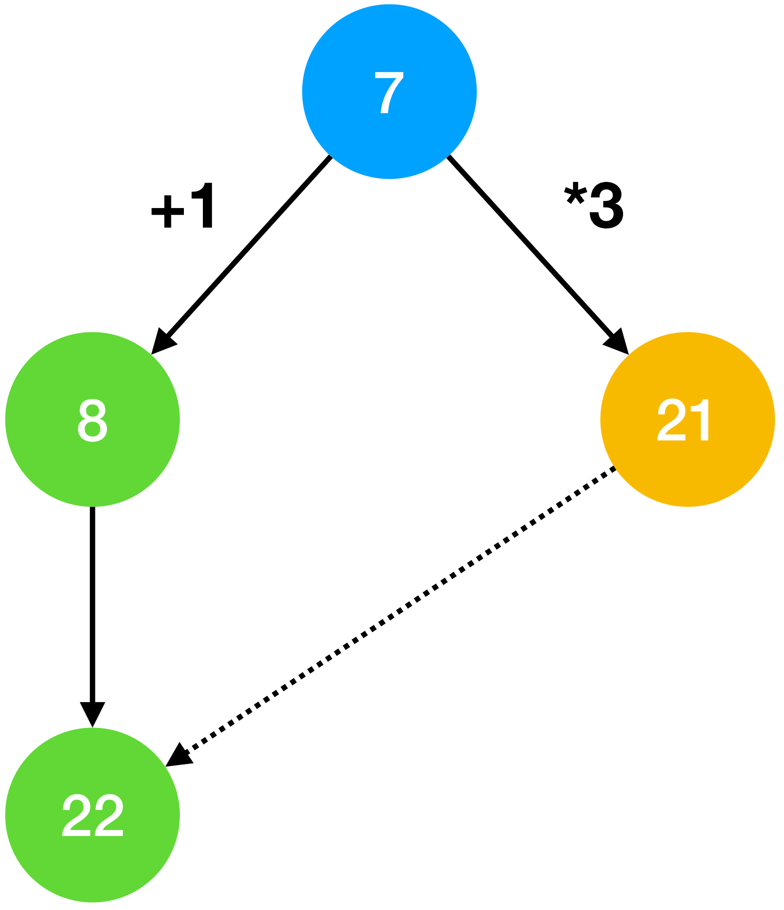

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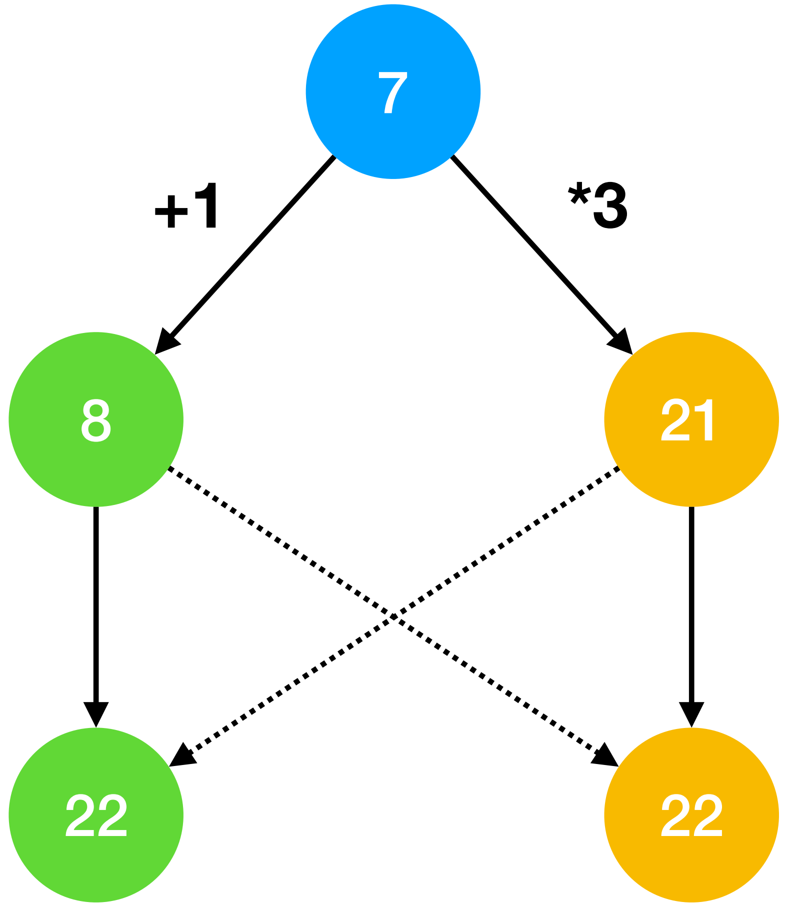

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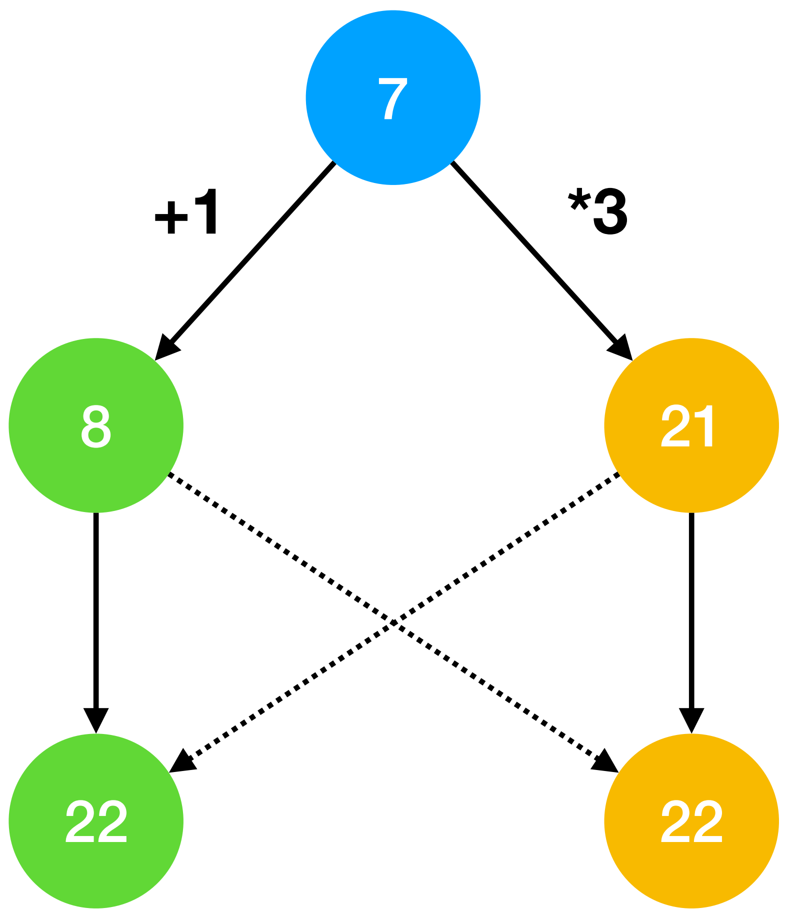

22 = 7 + (8-1) + (21 -7)

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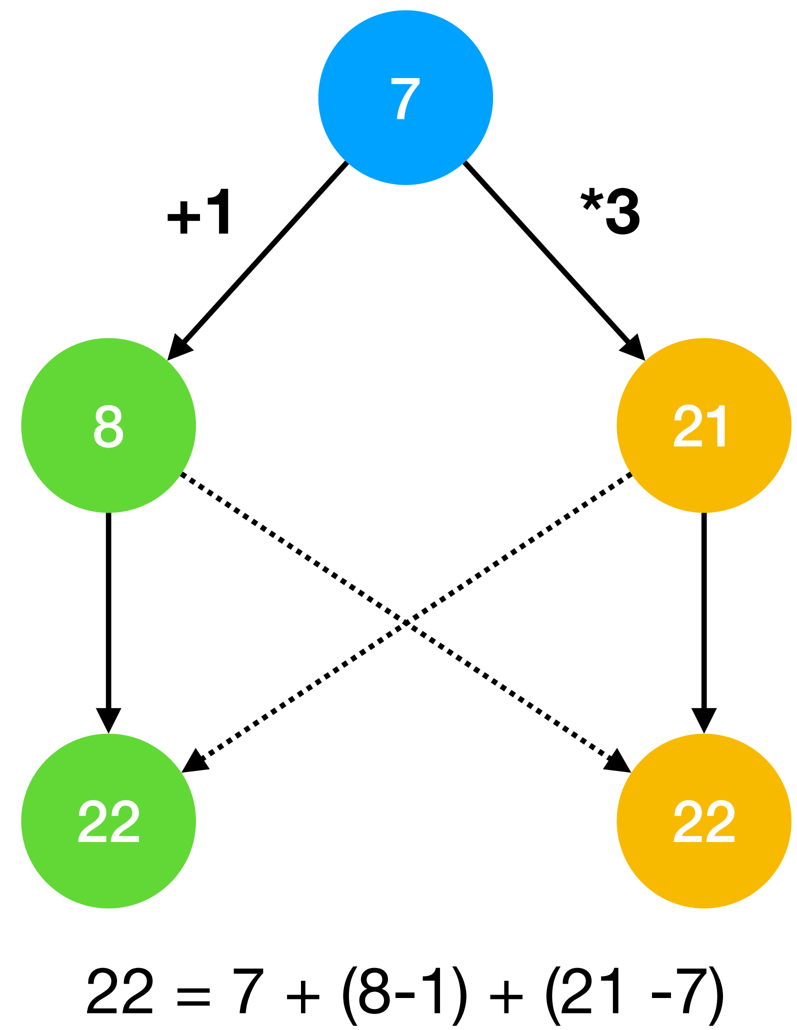

- 3-way merge function makes the counter suitable for distribution

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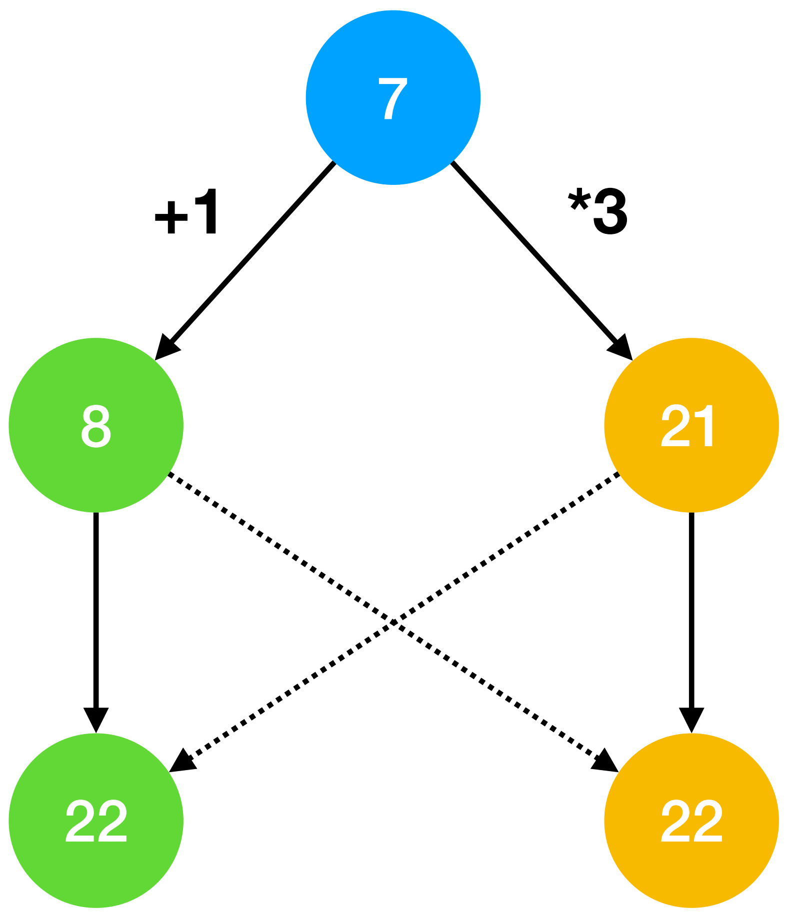
  val mult : t -> int -> t
  val merge : lca:t -> v1:t -> v2:t -> t
end = struct
  type t = int
  let read x = x
  let add x d = x + d
  let sub x d = x - d
  let mult x n = x * n
  let merge ~lca ~v1 ~v2 =
    lca + (v1 - lca) + (v2 - lca)
end
```



22 = 7 + (8-1) + (21 -7)

- 3-way merge function makes the counter suitable for distribution
- Does not appeal to individual operations => independently extend data-type

Systems ➞ PL

Systems → PL

- CRDTs need to take care of systems level concerns such as *message loss, duplication and reordering*

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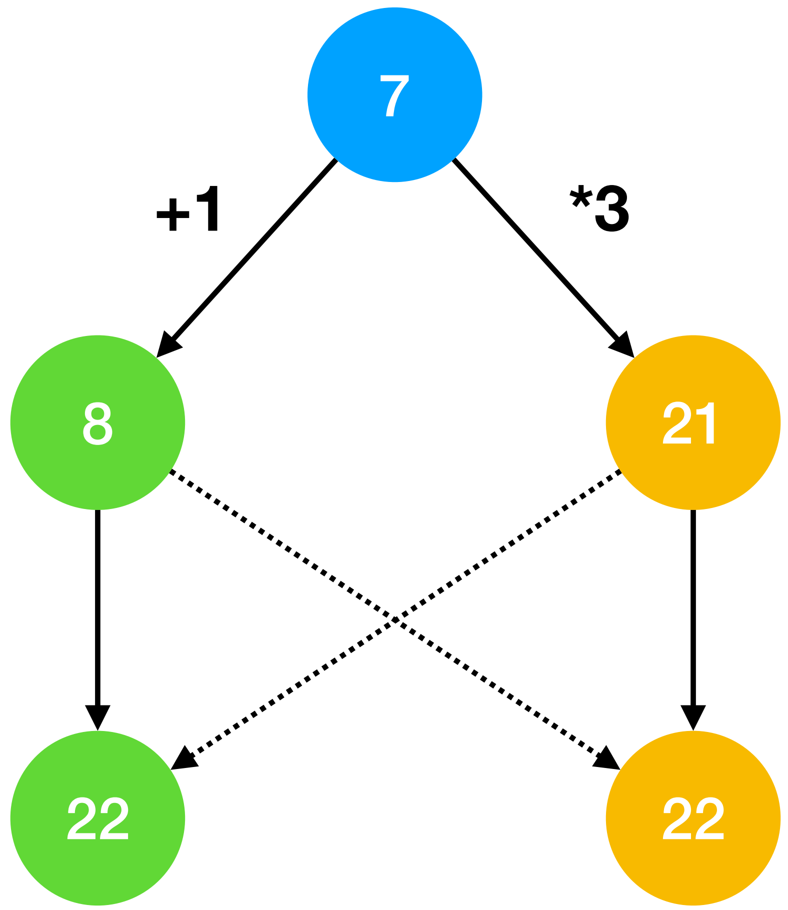

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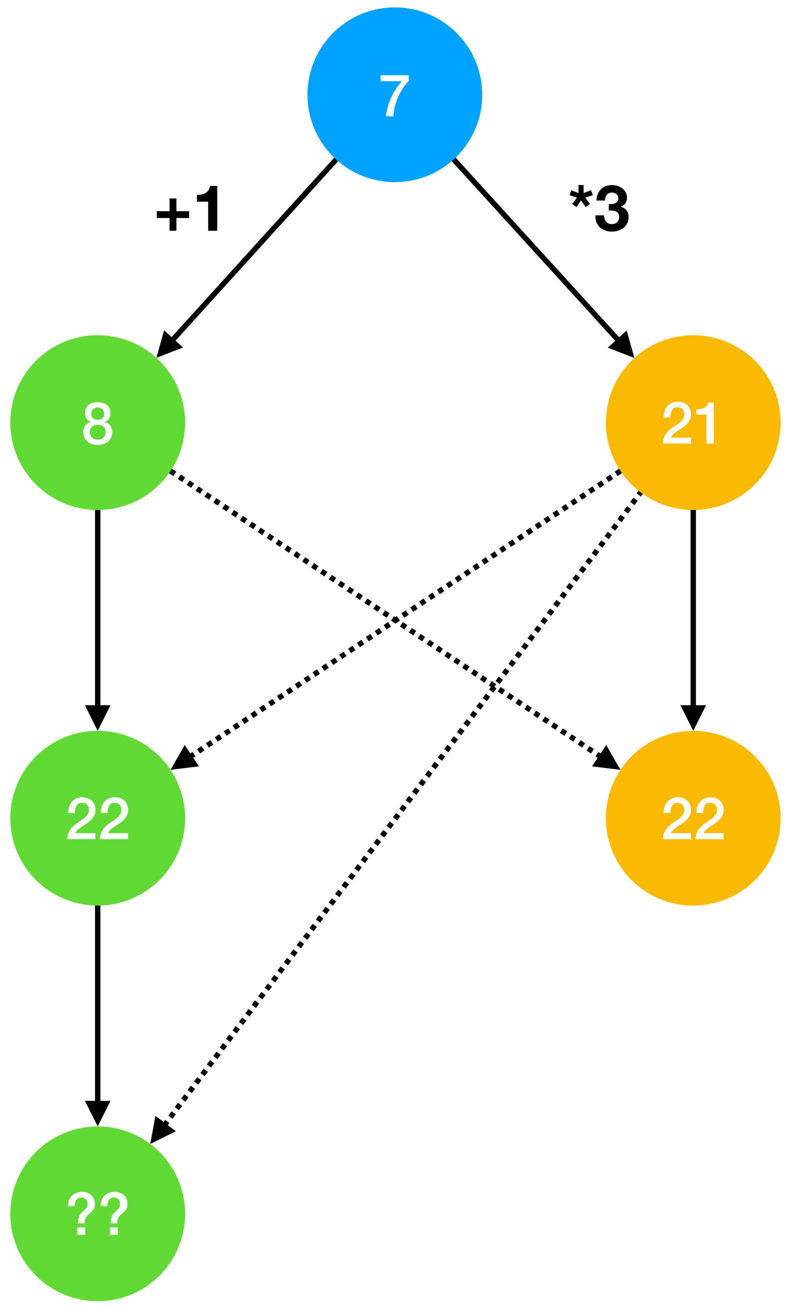

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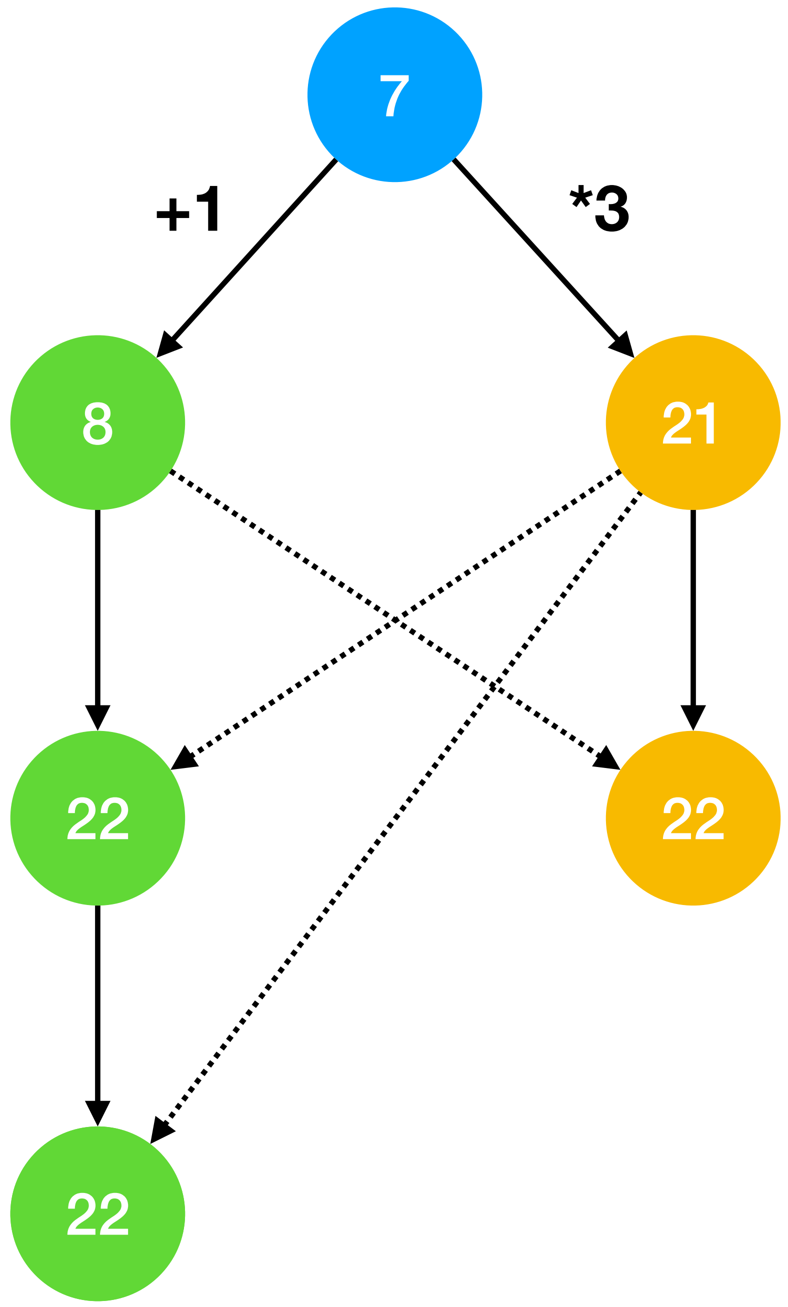

22 = 21 + (21-21) + (22 -21)

Does the 3-way merge idea generalise?

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Sort of

Observed-Removed Set

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let merge ~lca ~v1 ~v2 =
  (lca ∩ v1 ∩ v2) (* unmodified elements *)
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Kaki et al. "Mergeable Replicated Data Types", OOPSLA 2019

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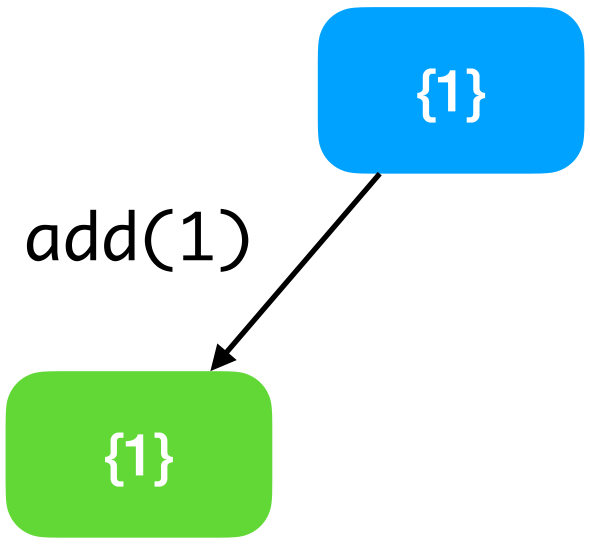

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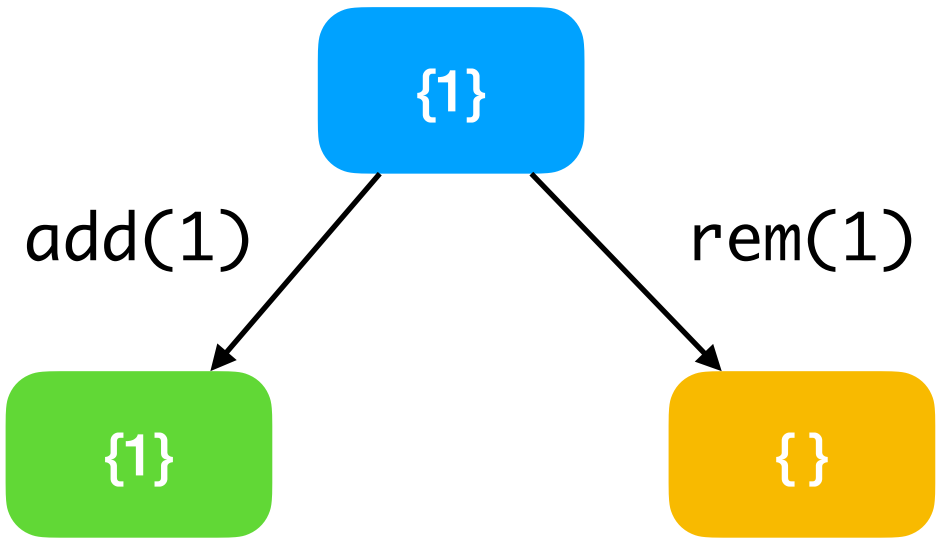

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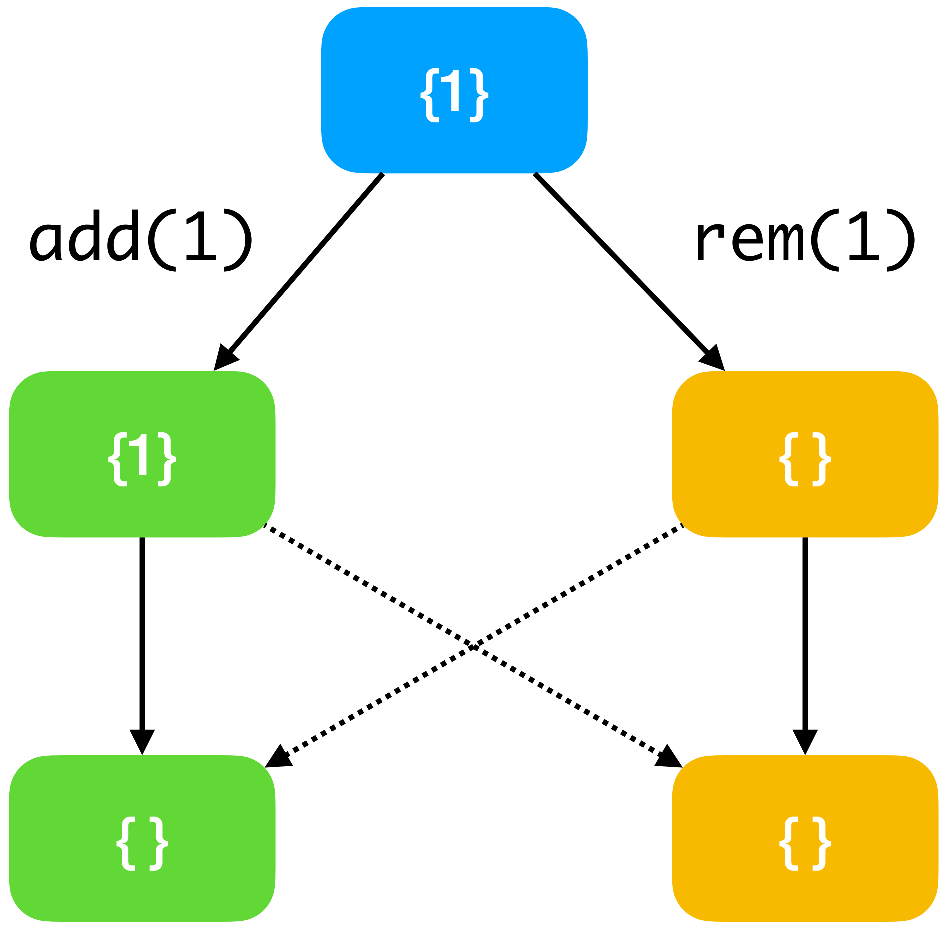

$$
\begin{aligned}
& \{\,\} \cup (\{1\} - \{1\}) \cup (\{\,\} - \{1\}) \\
= & \{\,\} \cup \{\,\} \cup \{\,\} \\
= & \{\,\} \ \textit{(expected \{1\})}
\end{aligned}
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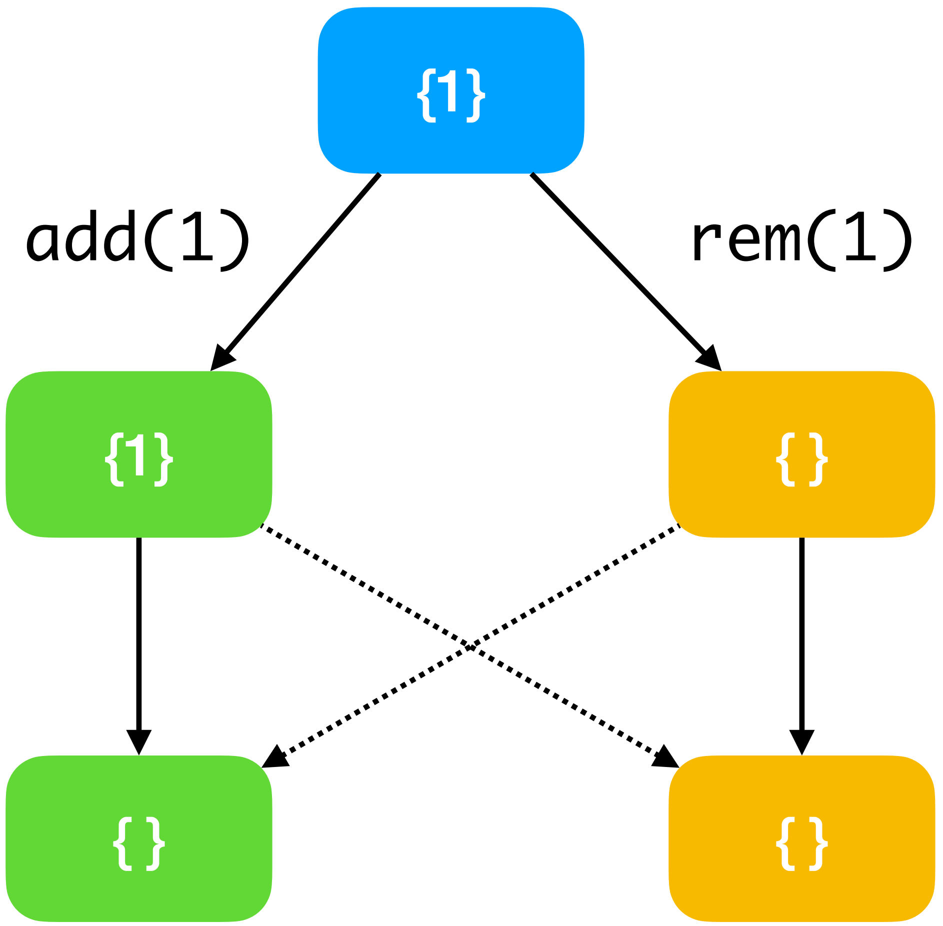


$\{\} \cup (\{1\} - \{1\}) \cup (\{\} - \{1\})$

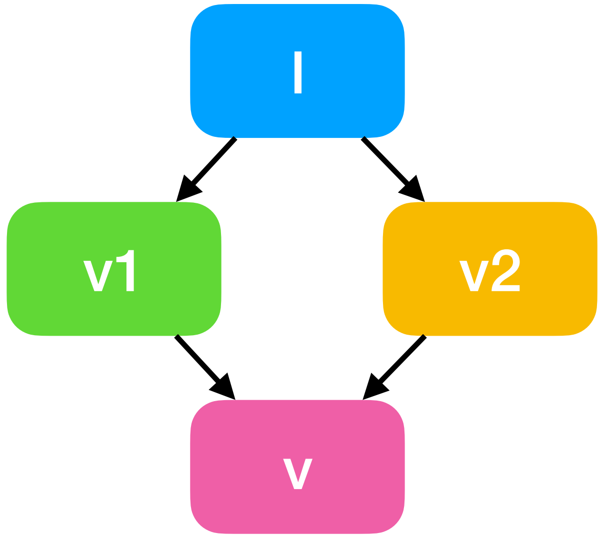
$= \{\} \cup \{\} \cup \{\}$

$= \{\}$ *(expected {1})*

- Convergence is not sufficient; *Intent* is not preserved

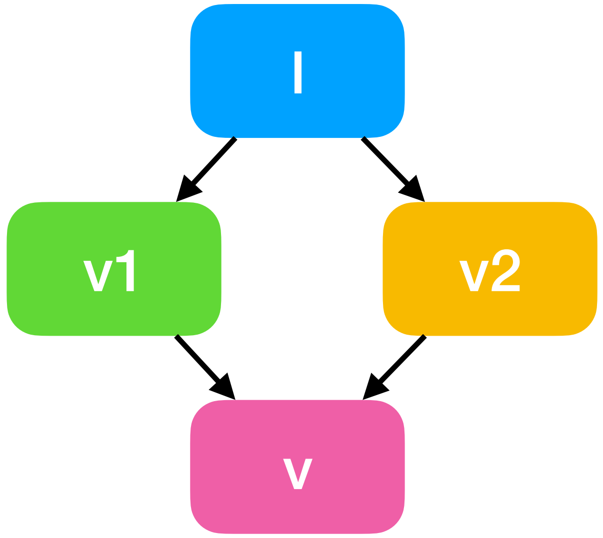
Concretising Intent

- Intent is a woolly term
  - ★ *How can we formalise the intent of operations on a data structure?*



Concretising Intent

- Intent is a woolly term
  - ★ *How can we formalise the intent of operations on a data structure?*
- We need
  - ★ A *formal language* to specify the *intent* of an RDT
  - ★ *Mechanization* to bridge the air gap between specification and implementation due to distributed system complexity



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- *Replication-aware simulation* to connect *specification* with *implementation*
- Composition of MRDTs and their proofs!
- Extracted RDTs are compatible with Irmin — a Git-like distributed database

Fixing OR-Set

- Discriminate duplicate additions by associating a unique id

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{ (a,1) }

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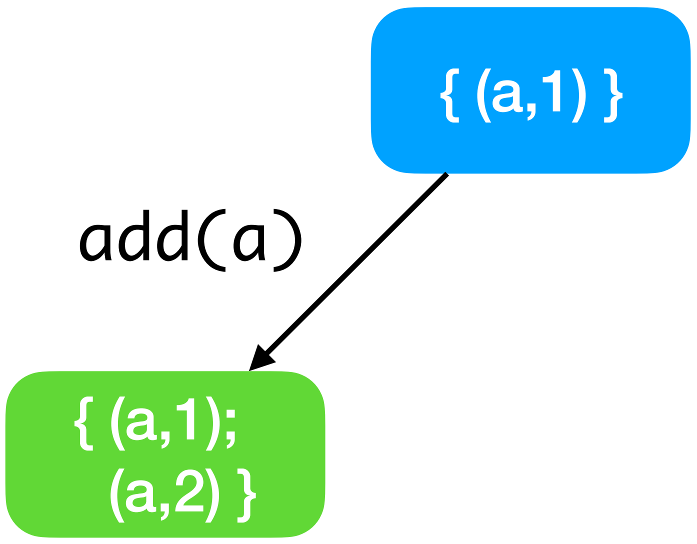

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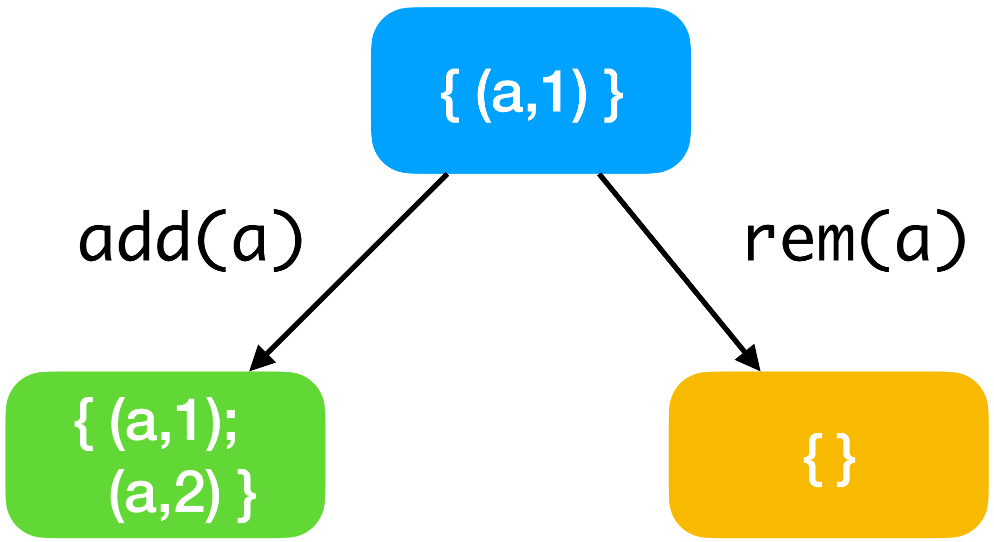

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$\{\ \} \cup (\{ (a,1); (a,2) \} - \{ (a,1) \}) \cup (\{\ \} - \{ (a,1) \})$

$= \{\ \} \cup \{ (a,2) \} \cup \{\ \}$

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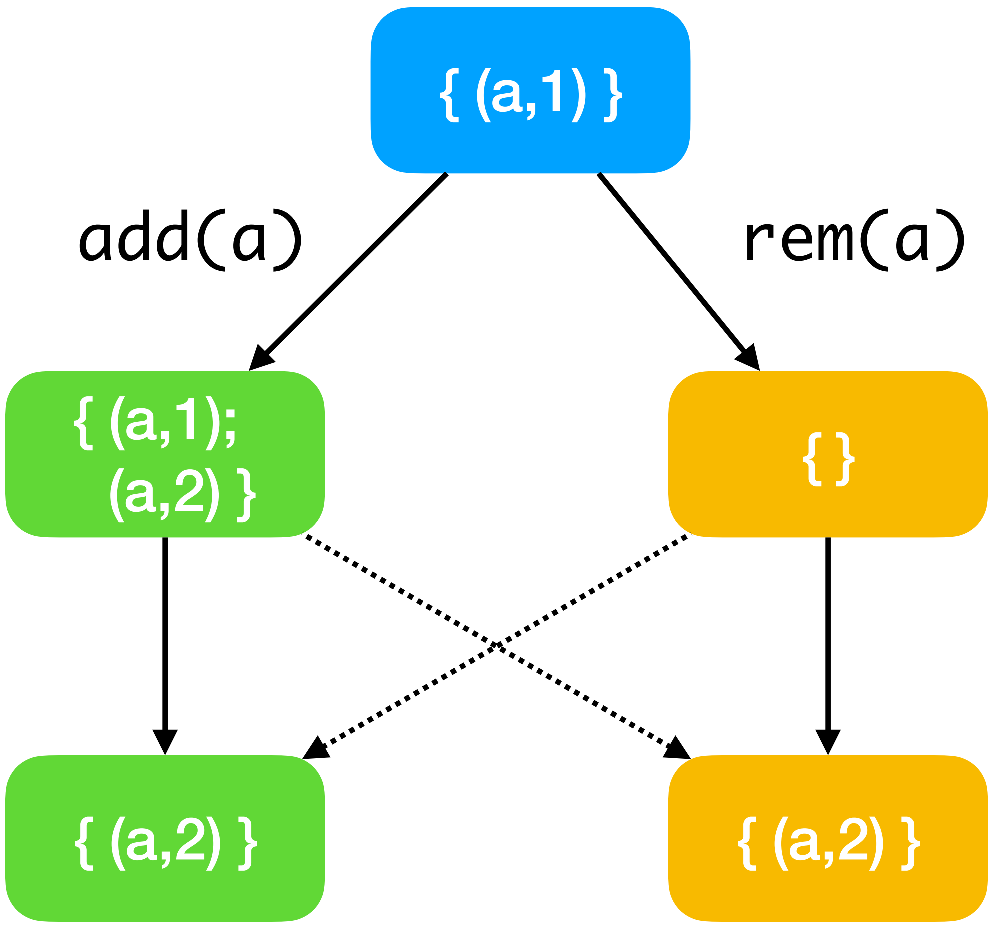

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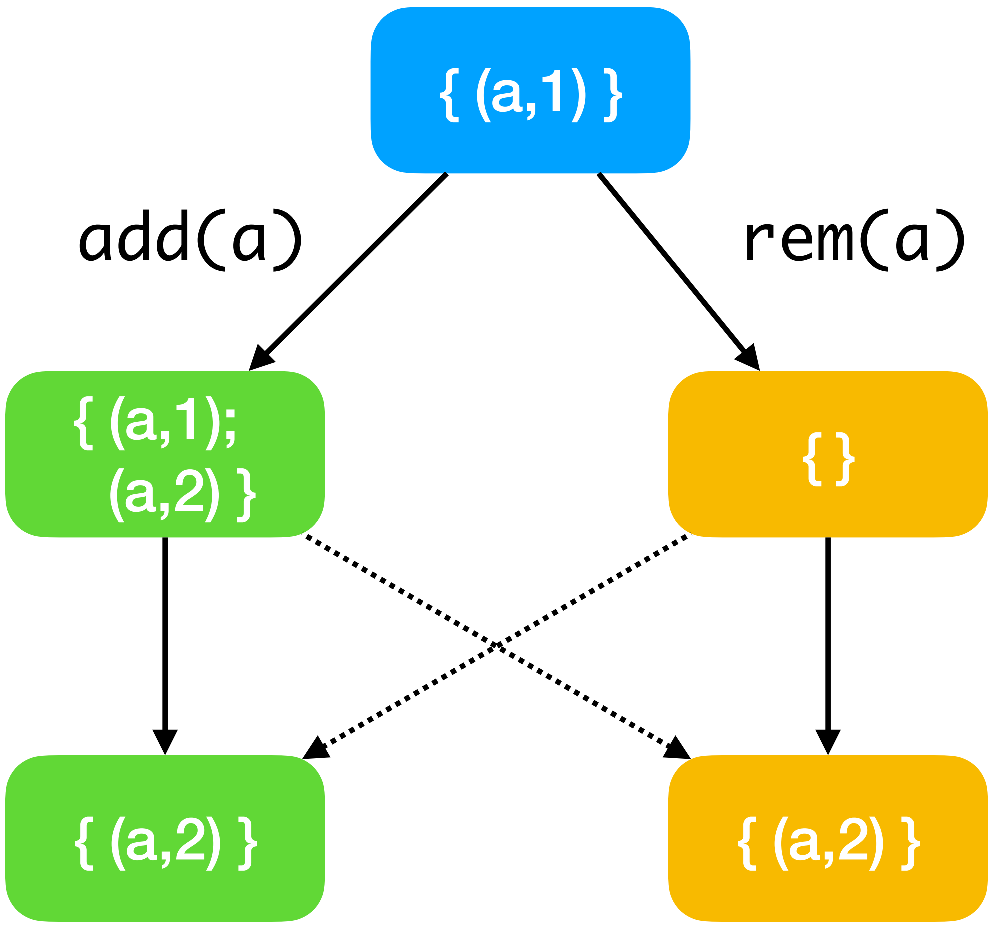
- MRDT implementation

$$D_\tau = (\Sigma, \sigma_0, do, merge)$$



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$$D_\tau = (\Sigma, \sigma_0, do, merge)$$

1: $\Sigma = \mathcal{P}(\mathbb{N} \times \mathbb{N})$

2: $\sigma_0 = \{\}$

3: $do(rd, \sigma, t) = (\sigma, \{a \mid (a,t) \in \sigma\})$

4: $do(add(a), \sigma, t) = (\sigma \cup \{(a,t)\}, \bot)$

5: $do(remove(a), \sigma, t) = (\{e \in \sigma \mid fst(e) \neq a\}, \bot)$

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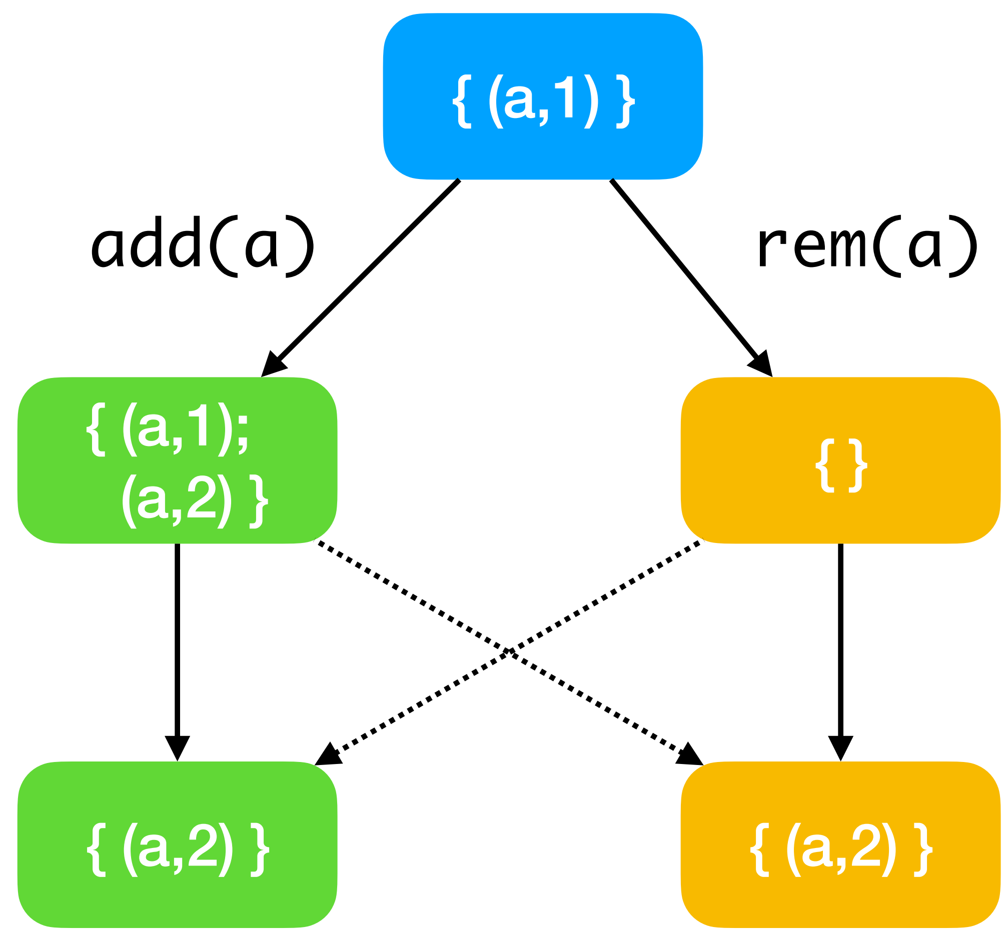

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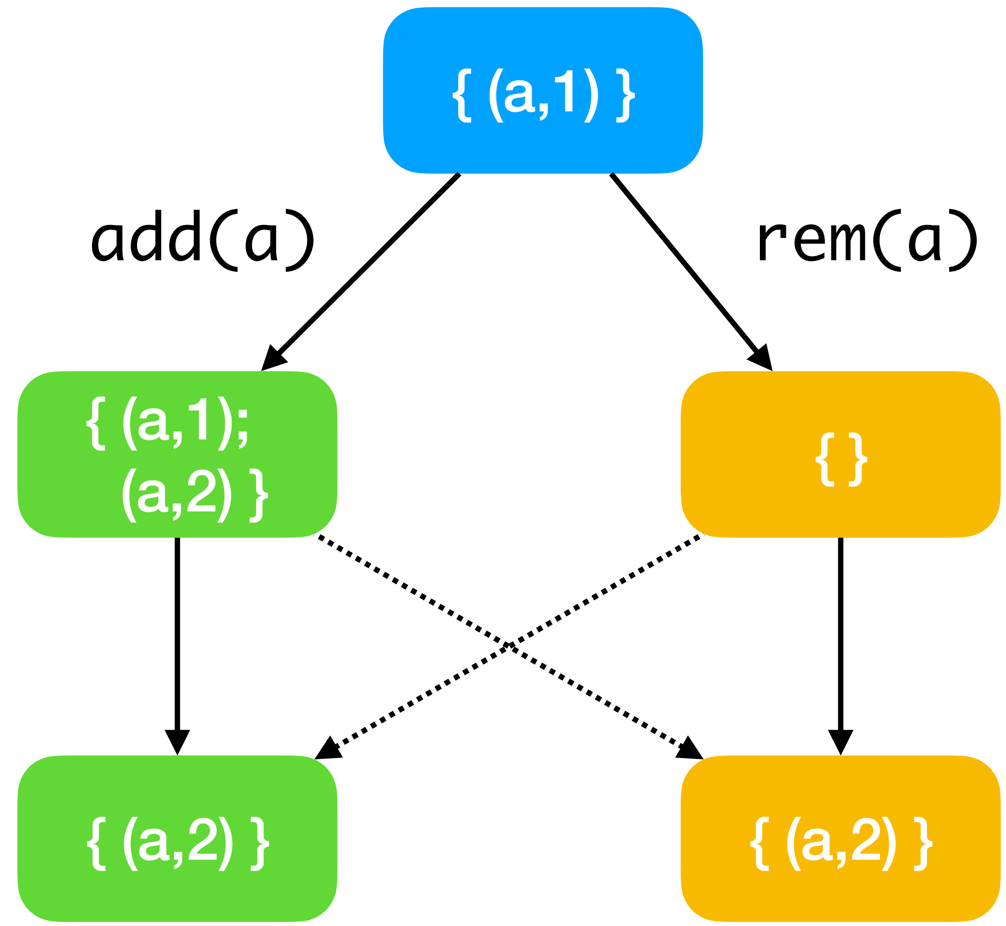

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Unique Lamport Timestamps

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Abstract state $I = \langle E, oper, rval, time, vis \rangle$

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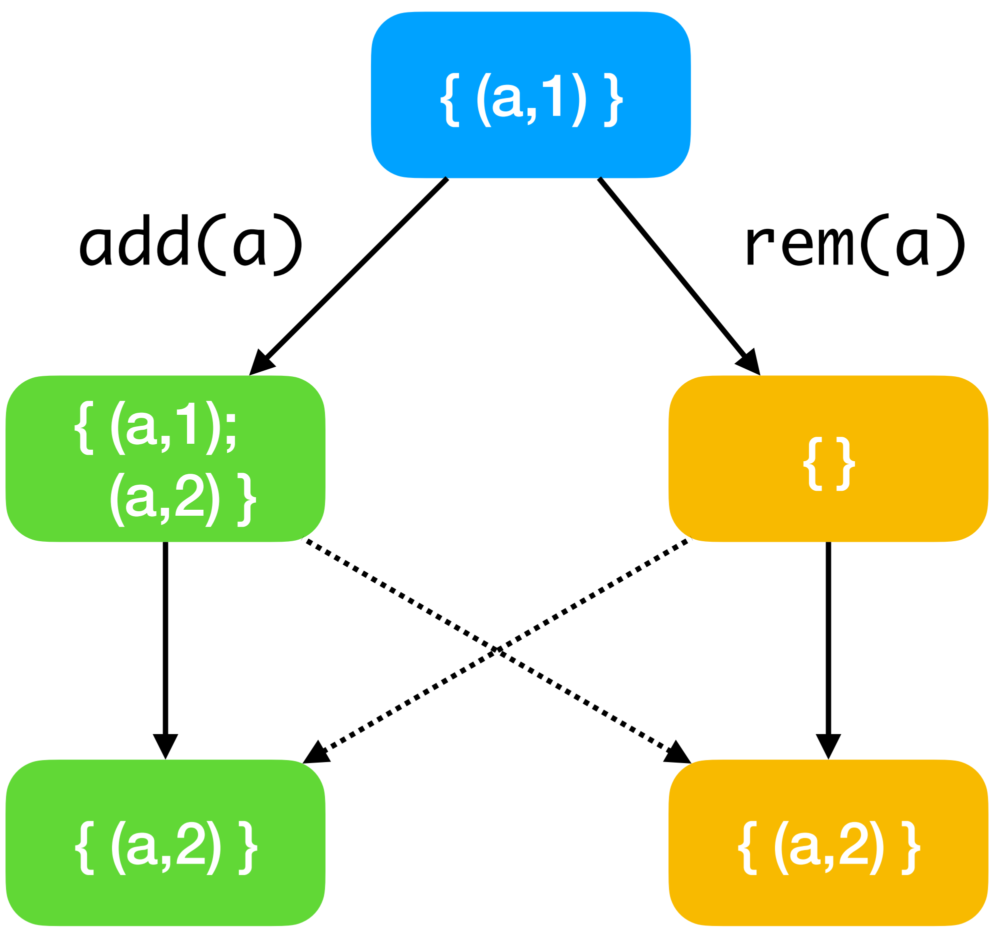

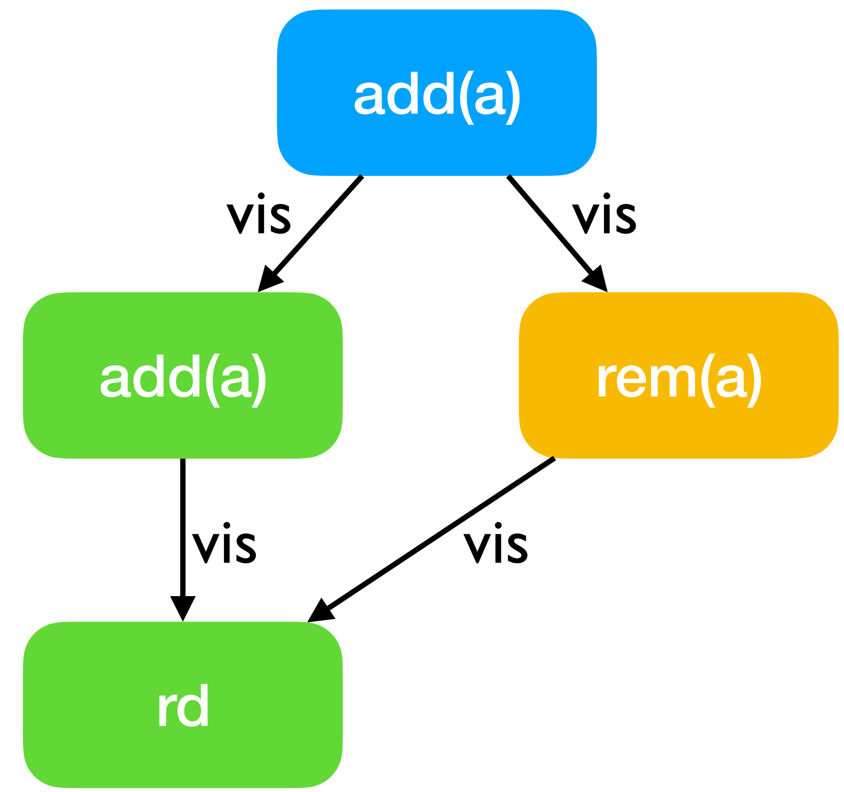

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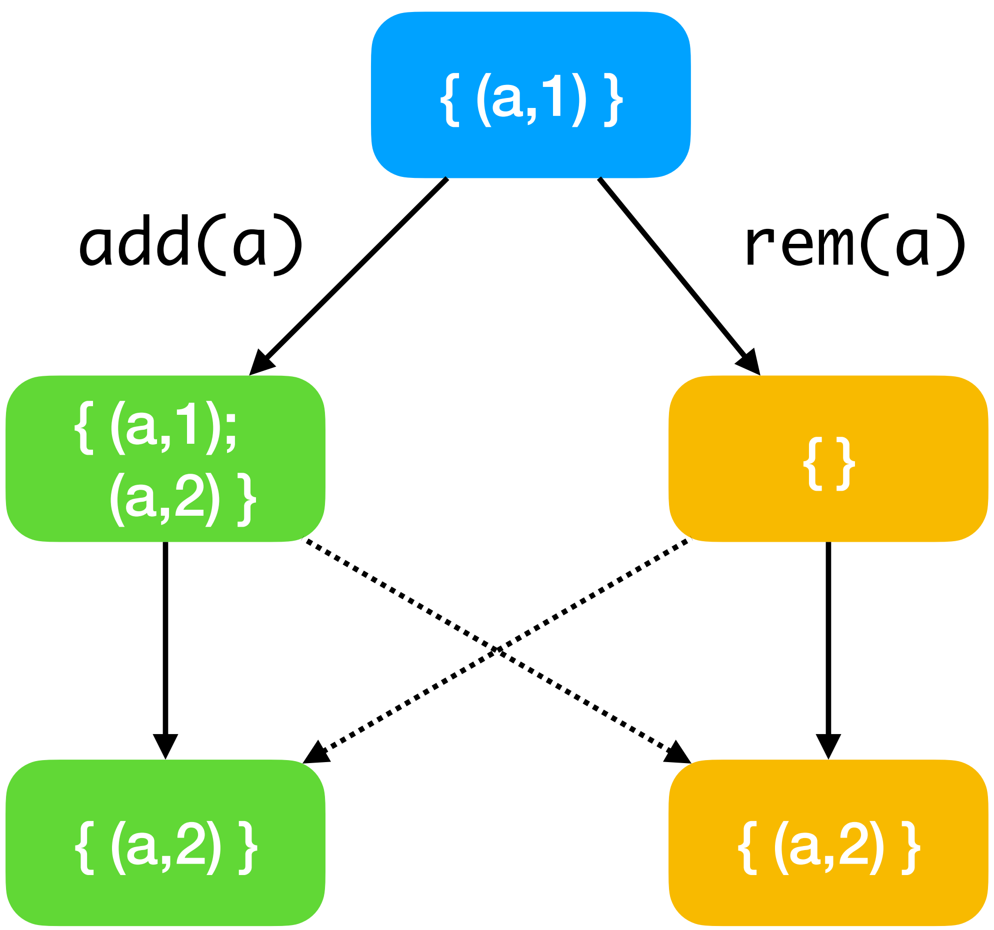

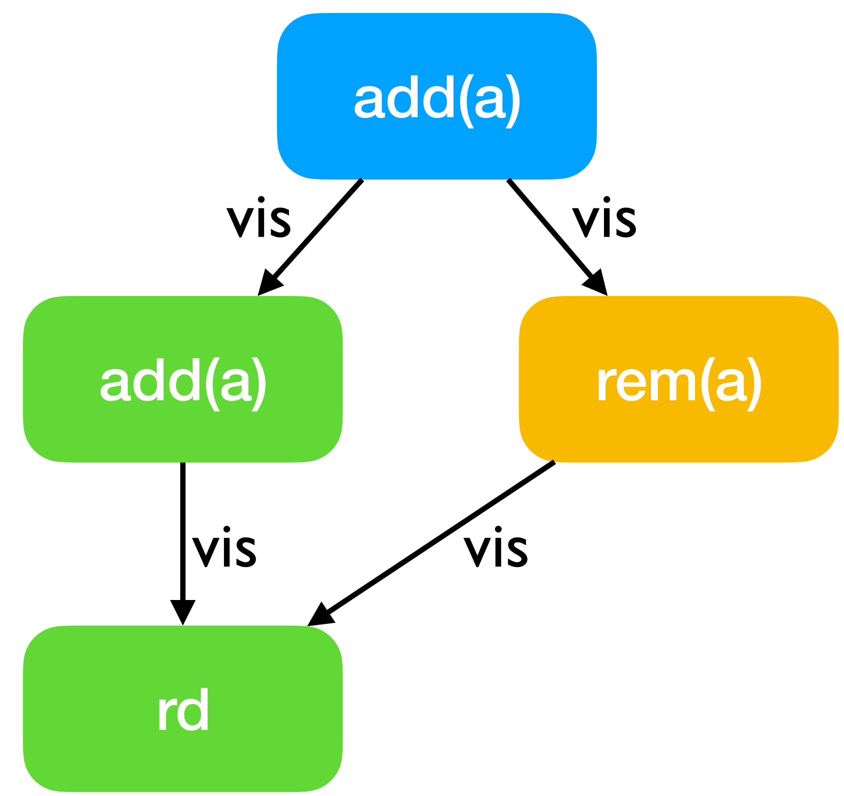

$$\mathcal{F}_{orset}(\text{rd}, \langle E, oper, rval, time, vis \rangle) = \{a \mid \exists e \in E.\ oper(e)$$
$$= \text{add}(a) \wedge \neg(\exists f \in E.\ oper(f) = \text{remove}(a) \wedge e \xrightarrow{vis} f)\}$$

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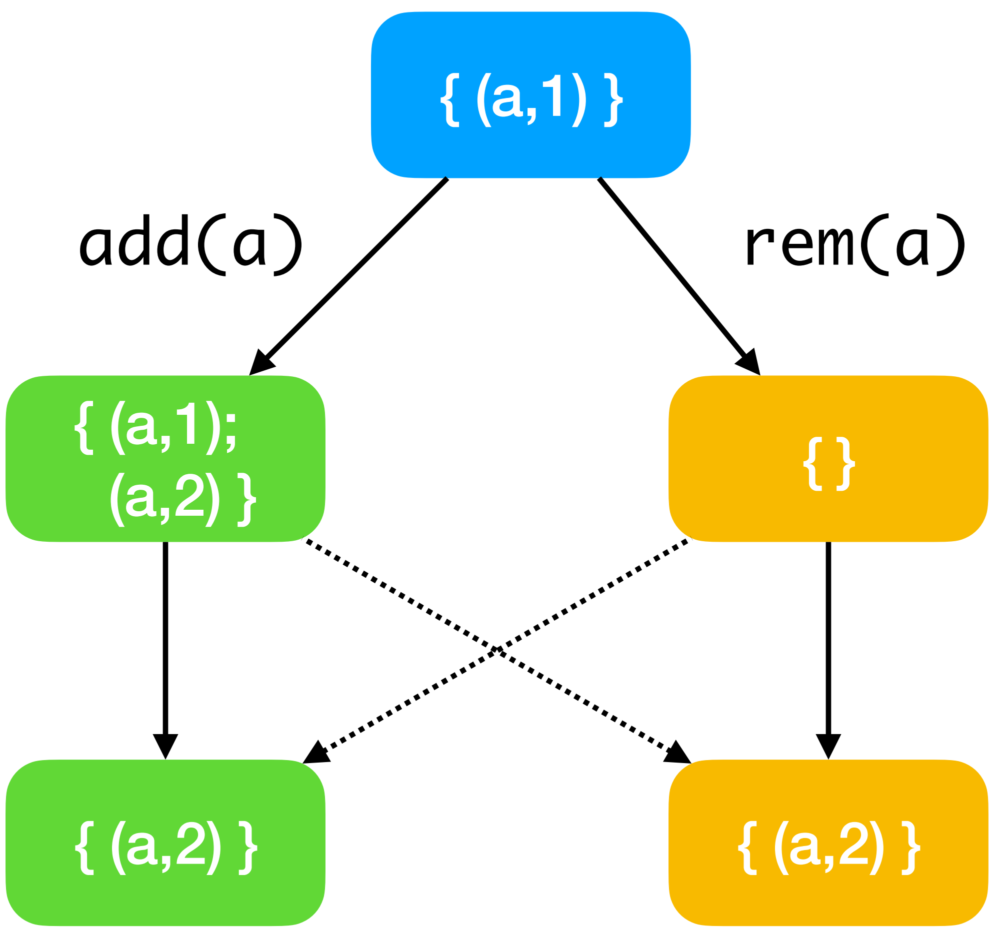

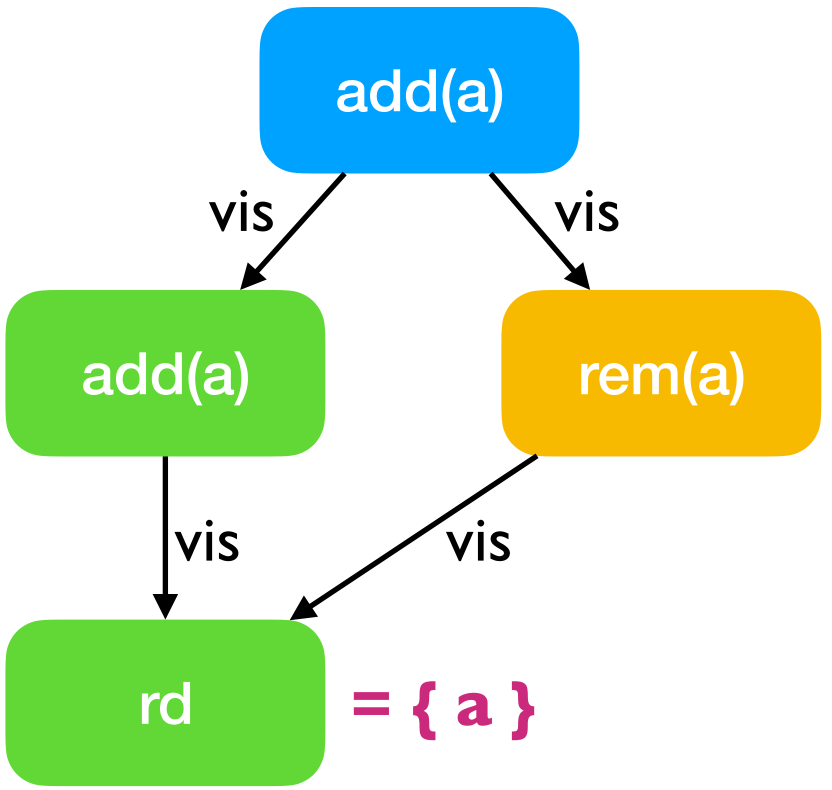

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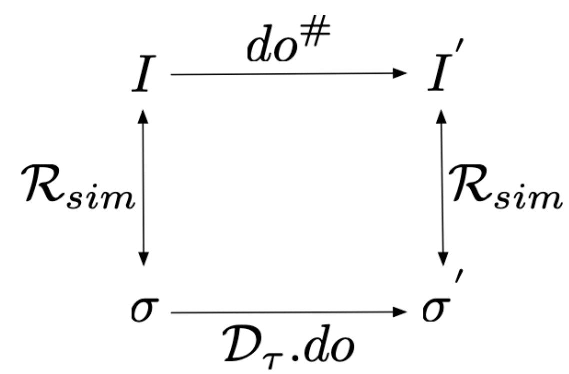
Simulation Relation

- Connects the abstract execution with the concrete state
- For the OR-set,

$$\mathcal{R}_{sim}(I, \sigma) \iff (\forall (a, t) \in \sigma \iff$$
$$(\exists e \in I.E \land I.\,oper(e) = add(a) \land I.time(e) = t \land$$
$$\neg(\exists f \in I.E \land I.\,oper(f) = remove(a) \land e \xrightarrow{vis} f)))$$

Verifying Operations

1. Show that the simulation holds for operations



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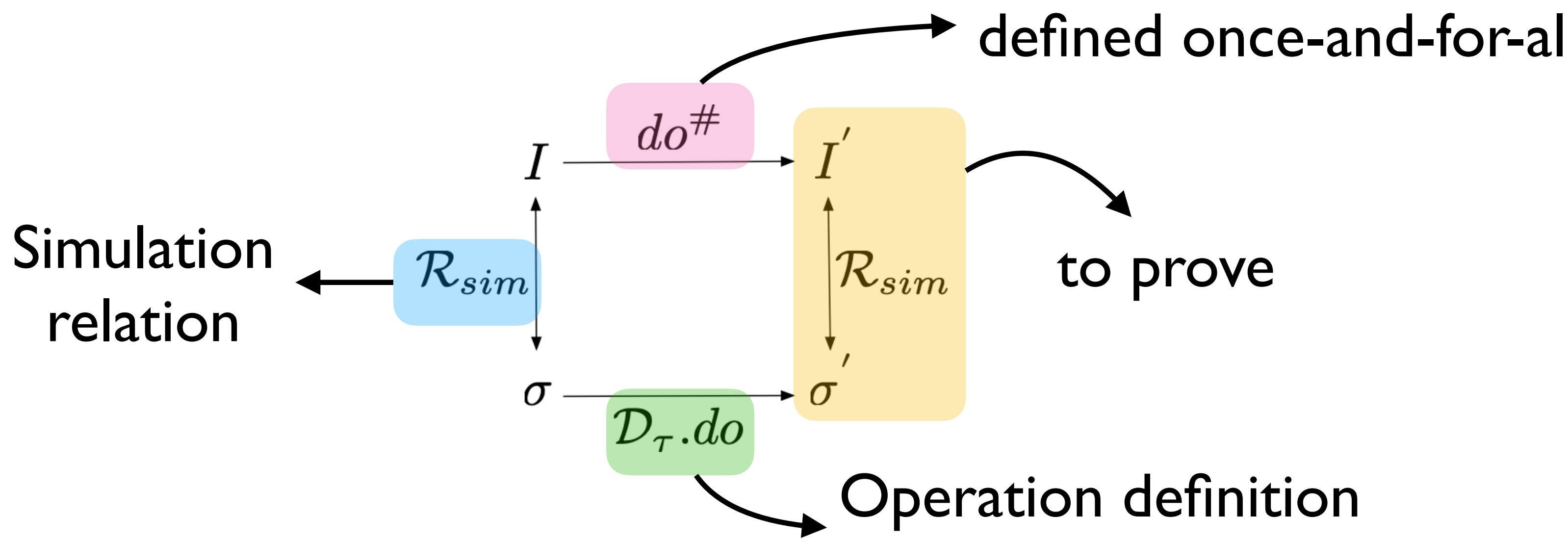

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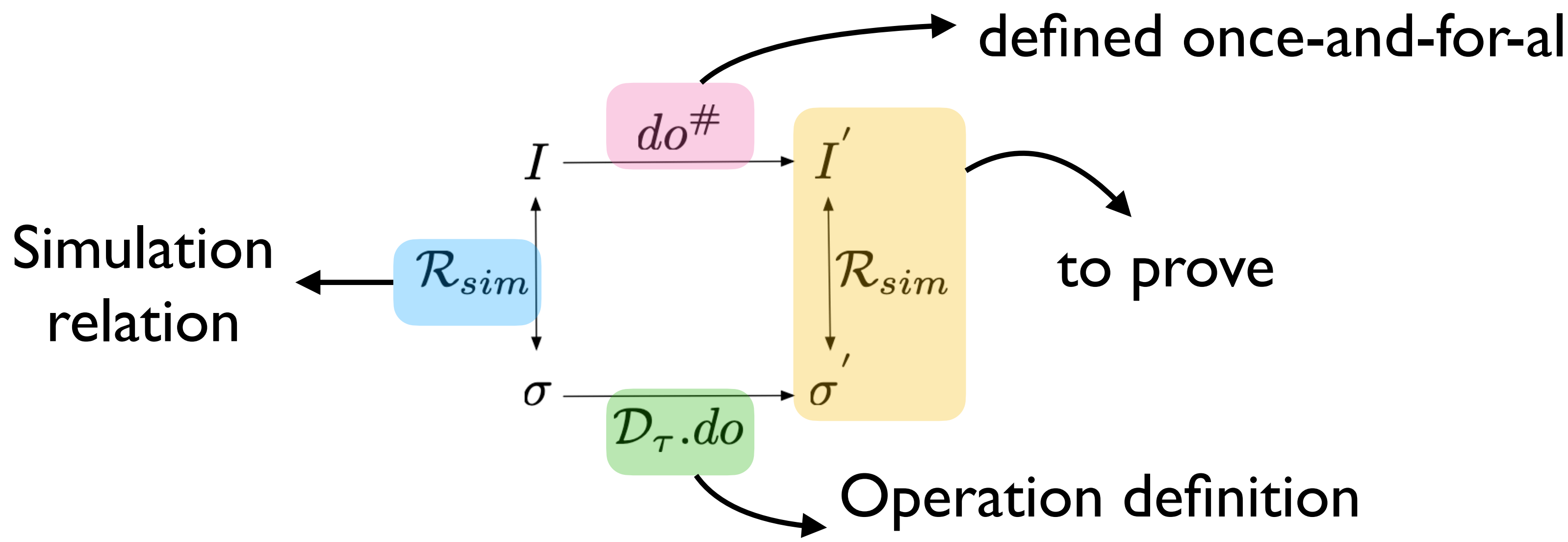

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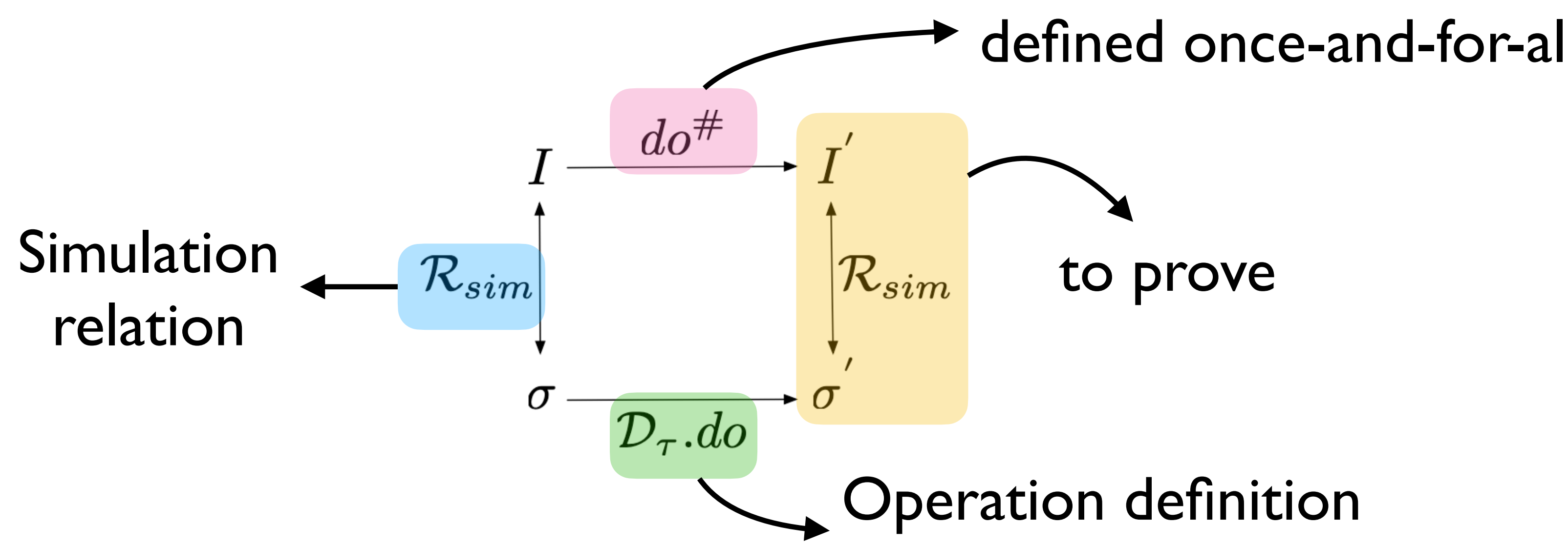

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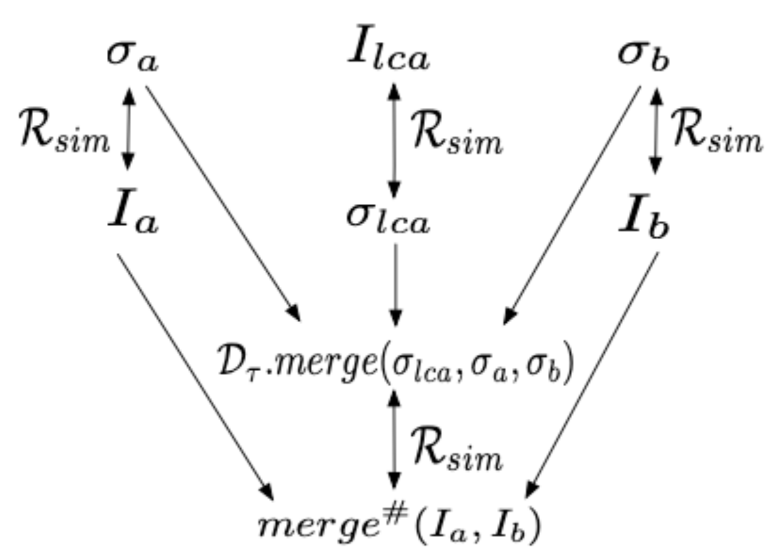

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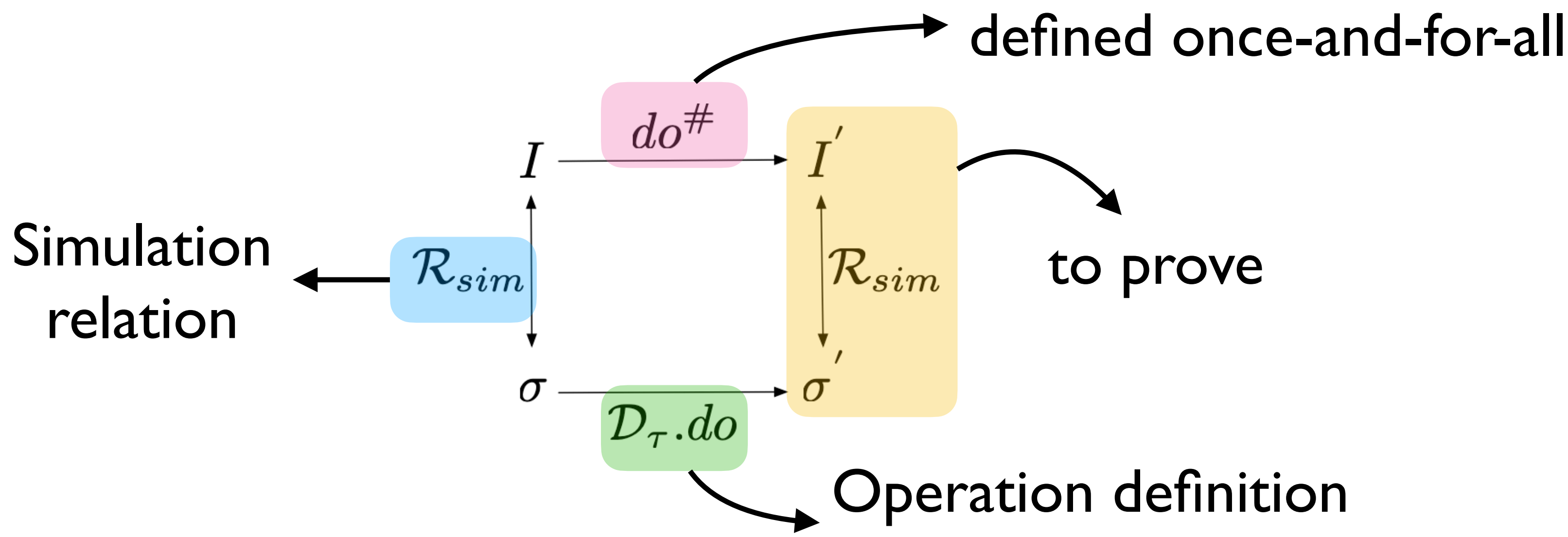

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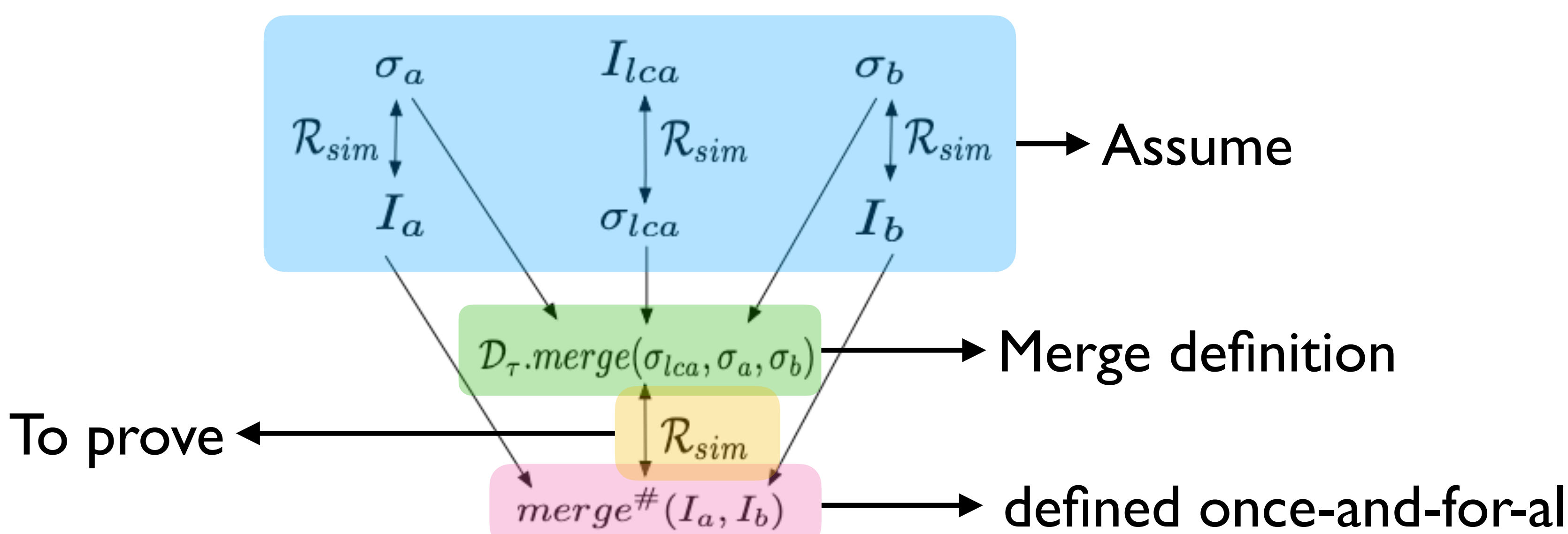

Verifying Operations

3. Show that the specification and the implementation agree on the return values of operations

$\Phi_{spec}(\mathcal{R}_{sim})$

$$\forall I, \sigma, e, op, a, t.\ \mathcal{R}_{sim}(I, \sigma) \wedge do^{\#}(I, e, op, a, t) = I^{'}$$
$$\wedge\ \mathcal{D}_{\tau}.do(op, \sigma, t) = (\sigma^{'}, a) \wedge \Psi_{ts}(I) \implies a = \mathcal{F}_{\tau}(o, I)$$

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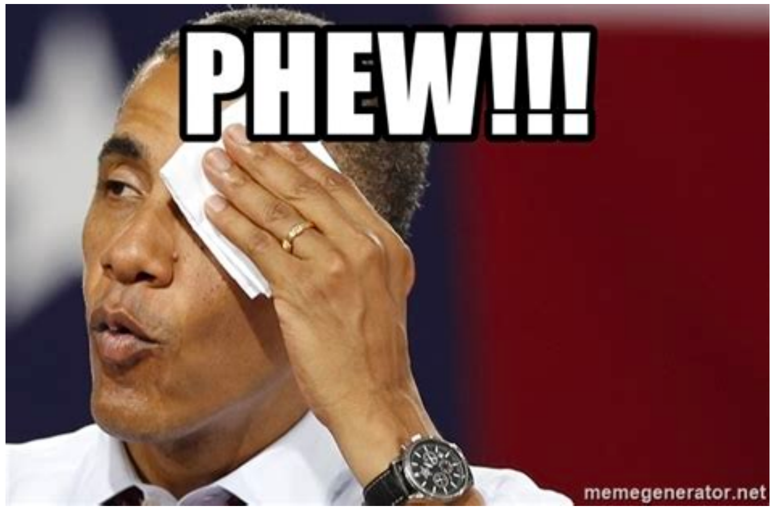
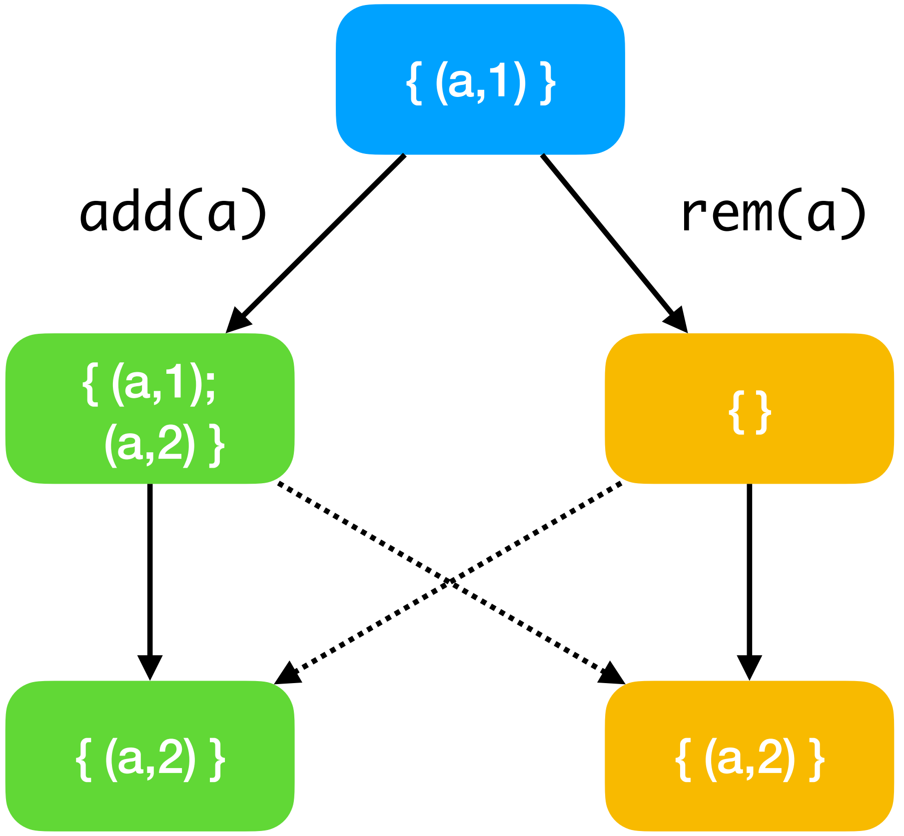


Space-efficient OR-Set

- Recall that the OR-set has duplicates



- How can we remove them?

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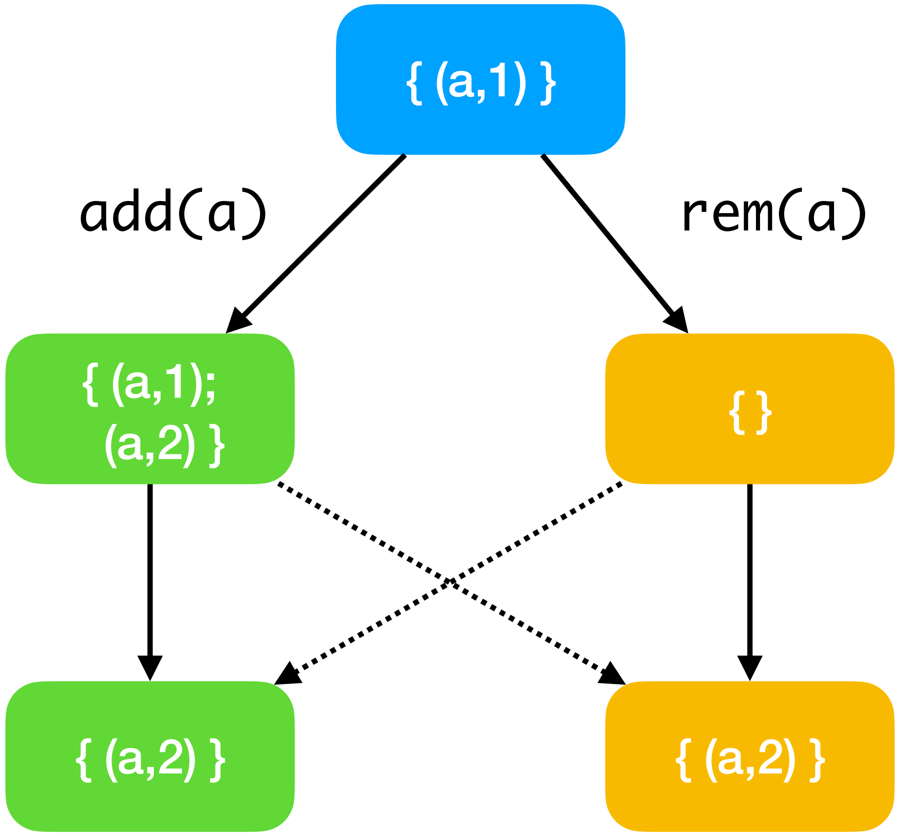

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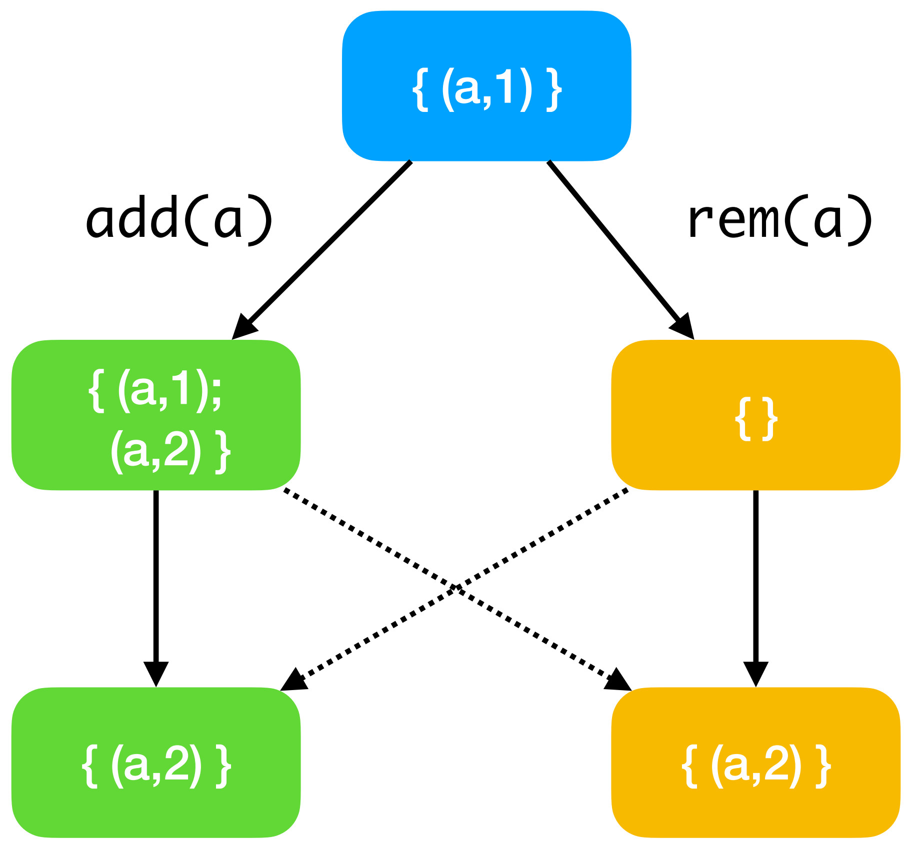

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Correctness argument is tricky

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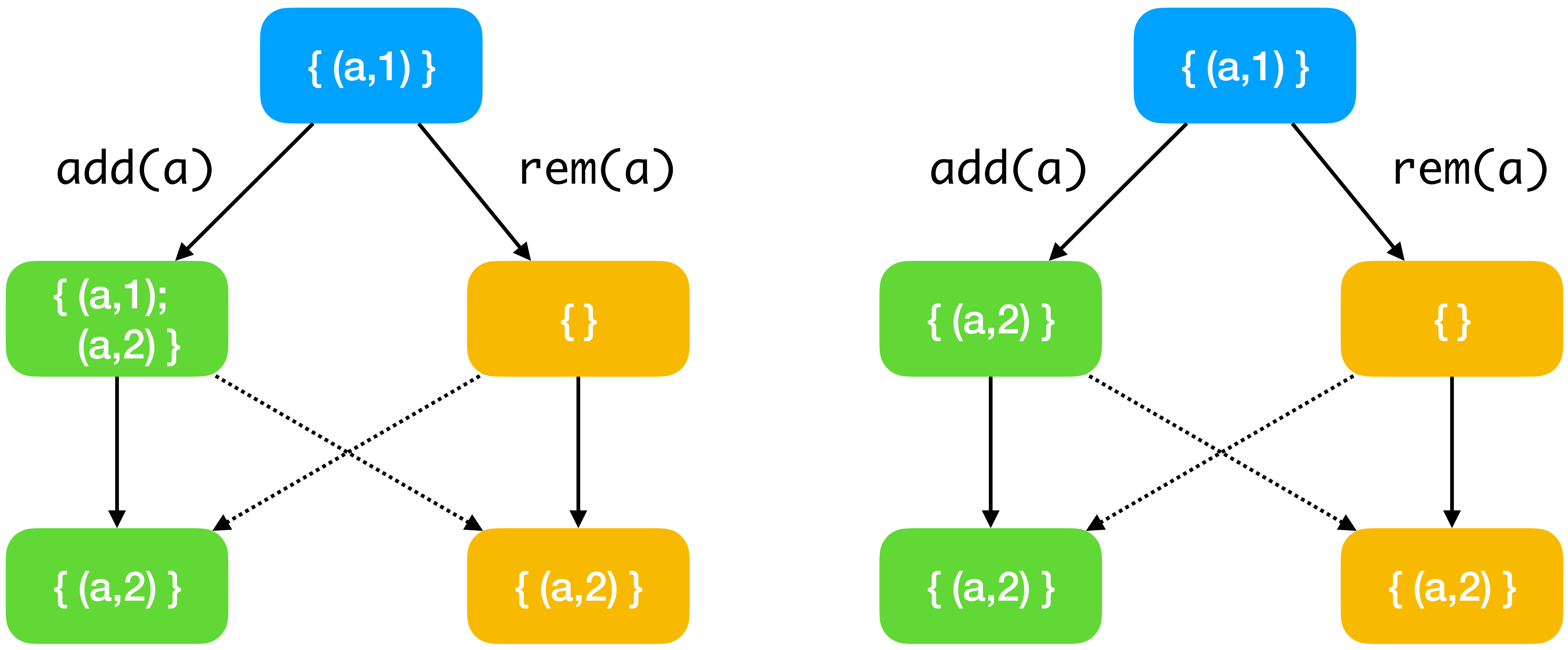
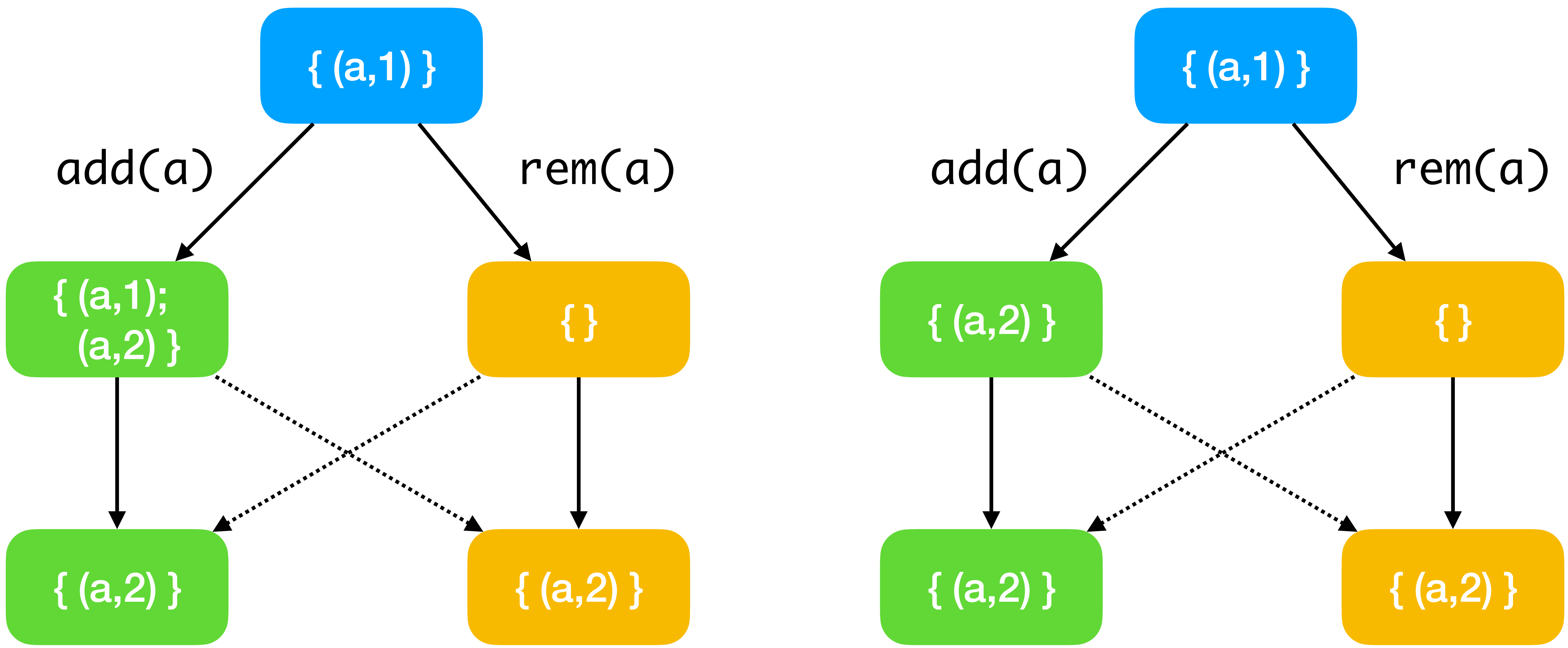


Space-efficient OR-Set



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\begin{aligned}
&\mathcal{R}_{sim}((E, oper, rval, time, vis), \sigma) \iff \\
&(\forall (a,t) \in \sigma \implies (\exists e \in E.\ oper(e) = add(a) \land time(e) = t \\
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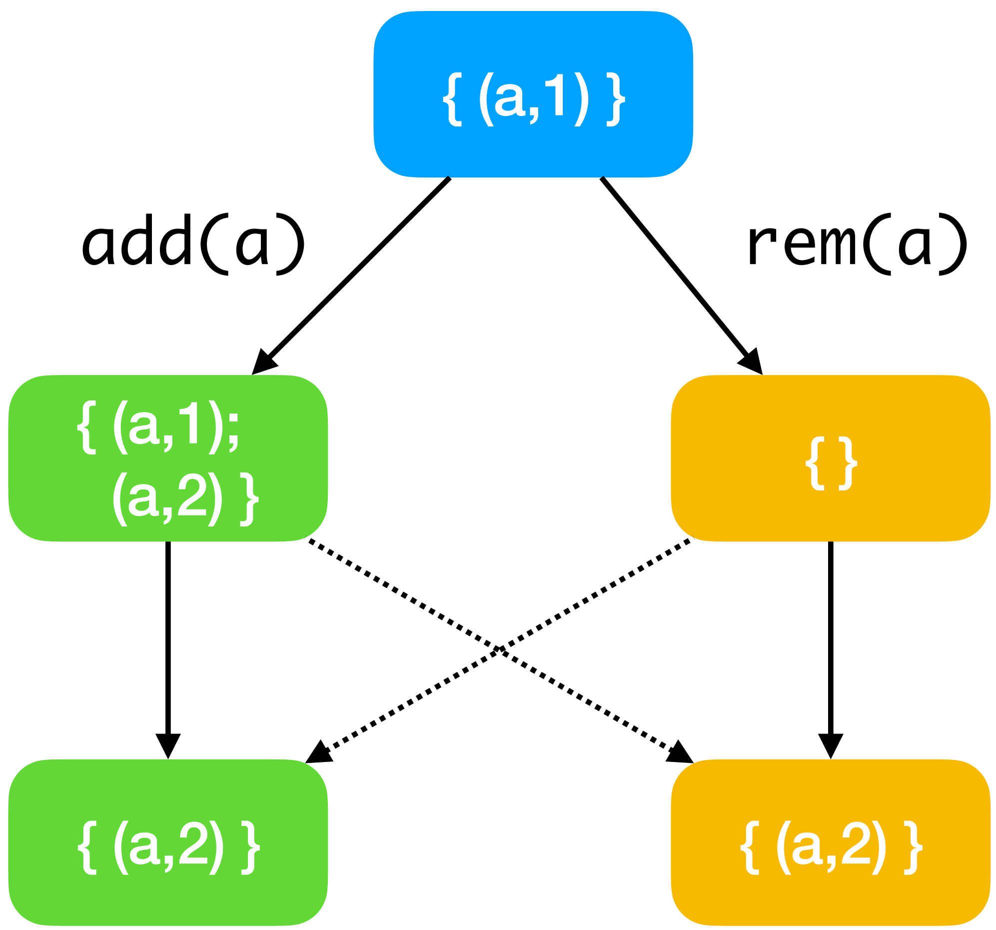

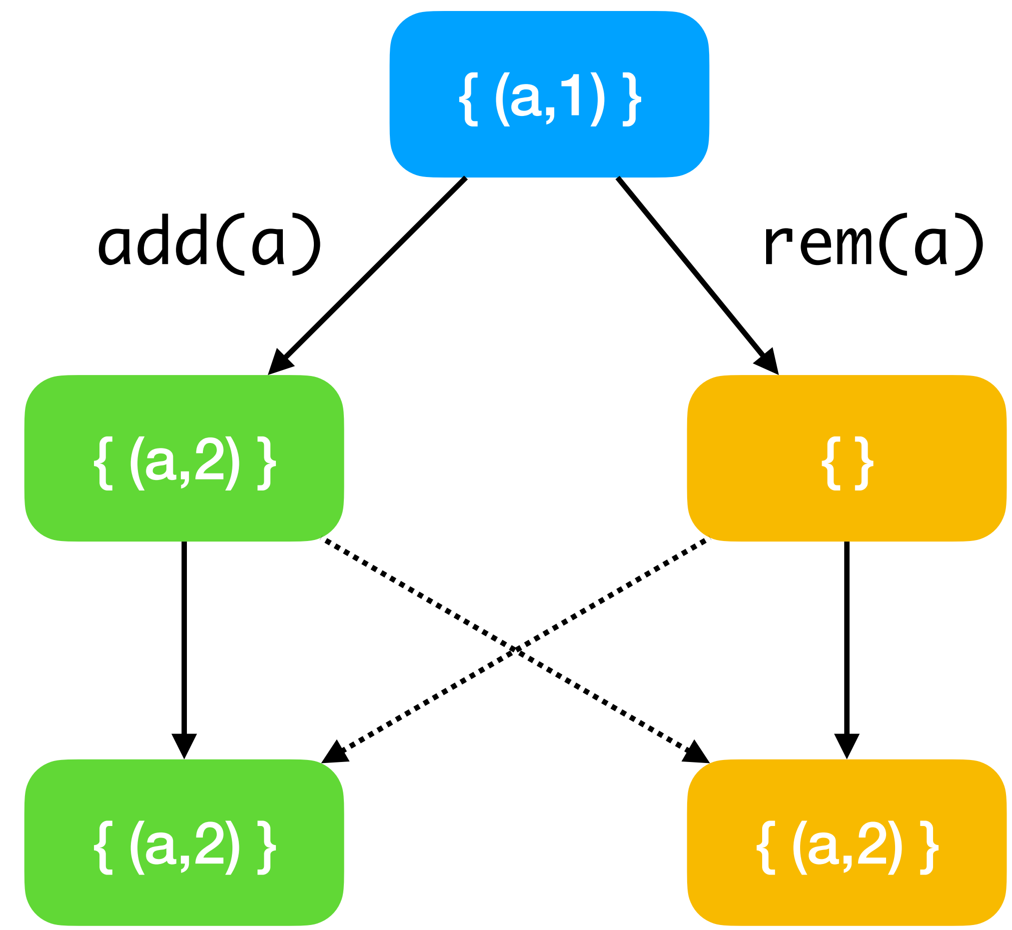

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Simulation relation is more intricate as one would expect

Verification effort

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Queue	32	1123	75	4753

Composing CRDTs is HARD!

Martin Kleppmann
@martinkl

Today in "distributed systems are hard": I wrote down a simple CRDT algorithm that I thought was "obviously correct" for a course I'm teaching. Only 10 lines or so long. Found a fatal bug only after spending hours trying to prove the algorithm correct. 😭

4:18 AM · Nov 13, 2020 · Tweetbot for iOS

41 Retweets **4** Quote Tweets **541** Likes

Martin Kleppmann @martinkl · Nov 13, 2020

The interesting thing about this bug is that it comes about only from the interaction of two features. A LWW map by itself is fine. A set in which you can insert and delete elements (but not update them) is fine. The problem arises only when delete and update interact.

1 16

Composing IRC-style chat

- Build IRC-style group chat
  - ★ Send and read messages in channels
  - ★ For simplicity, channels and messages cannot be deleted
- Represent application state as a grow-only map with string (channel name) keys and mergeable-log as values
- **Goal:**
  - ★ *map* and *log* proved correct separately
  - ★ Use the proof of underlying RDTs to prove chat application correctness

Generic Map MRDT

- Specification

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$$\mathcal{F}_{\alpha-map}(get(k, o_\alpha), I) = \text{let } I_\alpha = project(k, I) \text{ in } \mathcal{F}_\alpha(o_\alpha, I_\alpha)$$

where

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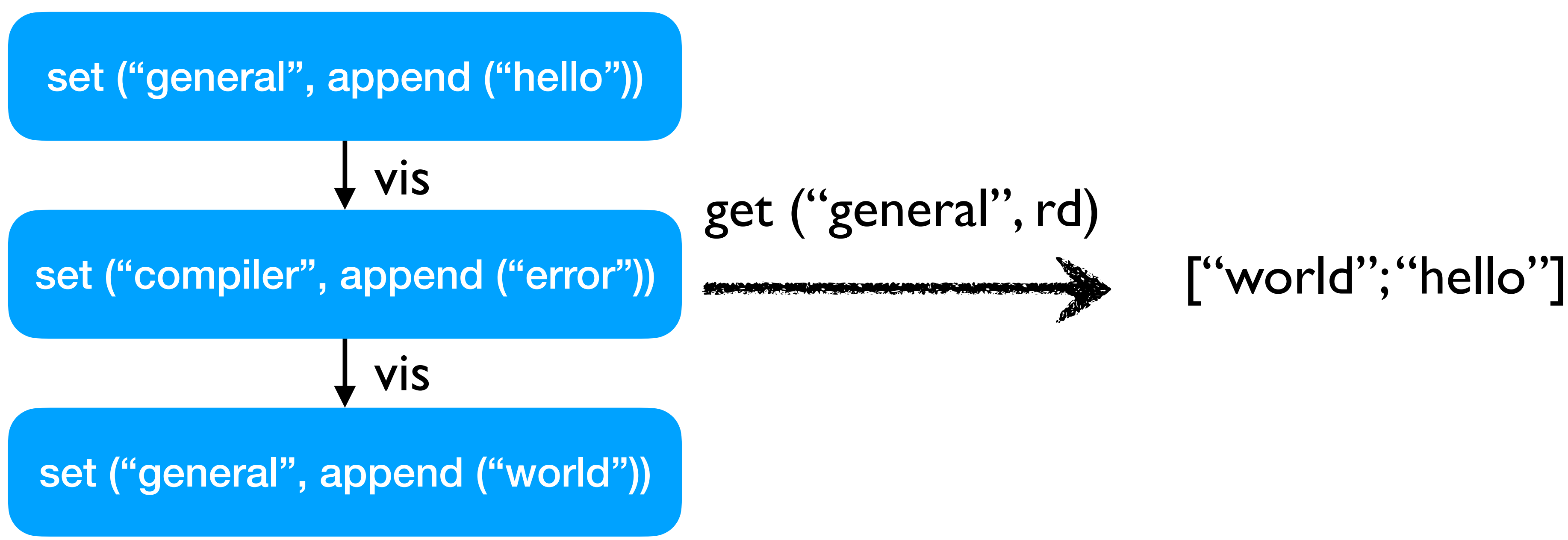

Generic Map MRDT

Implementation

$$\mathcal{D}_{\alpha-map} = (\Sigma, \sigma_0, do, merge_{\alpha-map}) \text{ where}$$

1: $\Sigma_{\alpha-map} = \mathcal{P}(string \times \Sigma_\alpha)$

2: $\sigma_0 = \{\}$

3: $\delta(\sigma, k) = \begin{cases} \sigma(k), & \text{if } k \in dom(\sigma) \\ \sigma_{0_\alpha}, & \text{otherwise} \end{cases}$

4: $do(set(k, o_\alpha), \sigma, t) =$
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$$\mathcal{R}_{sim-\alpha-map}(I, \sigma) \iff \forall k.$$

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Simulation relation appeals to the value type's simulation relation!

Composing IRC-style chat

- Program state is constructed by instantiating *generic map* with *mergeable log*
    - ★ The proof of correctness of the chat application directly follows from the composition!

Mergeable Queues

- Replicated queue with *at-least-once* dequeue semantics
  - ★ First verified queue RDT!

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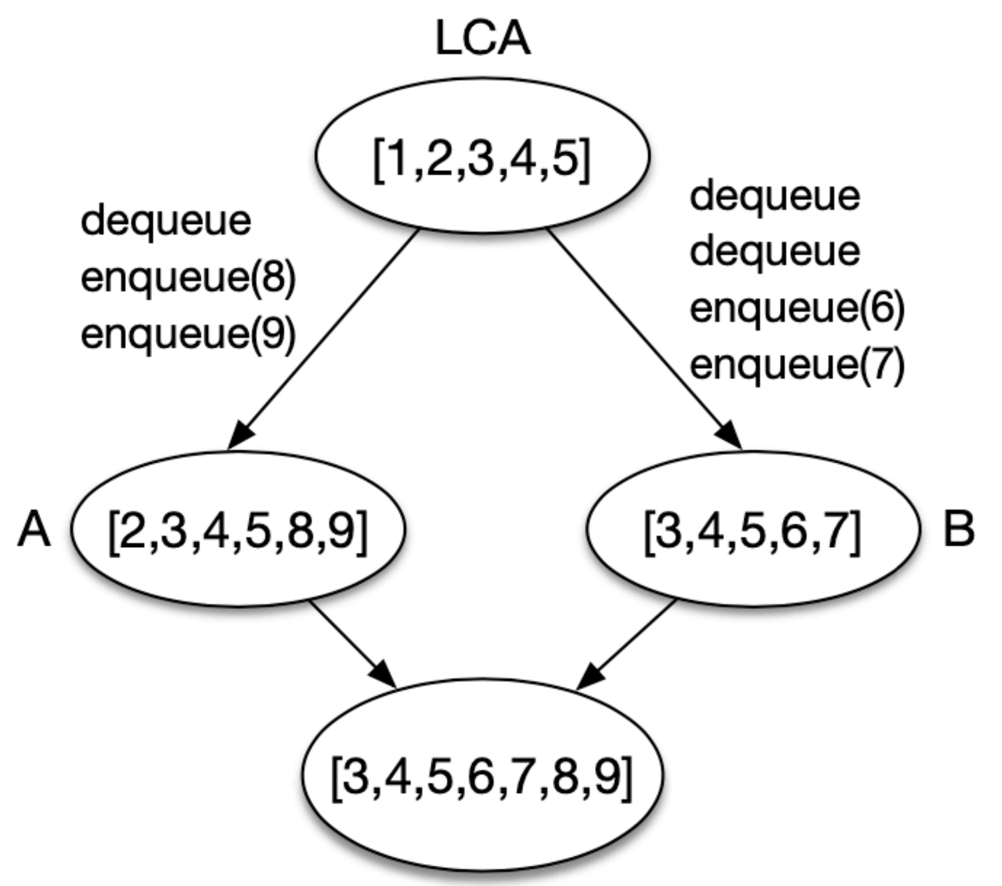

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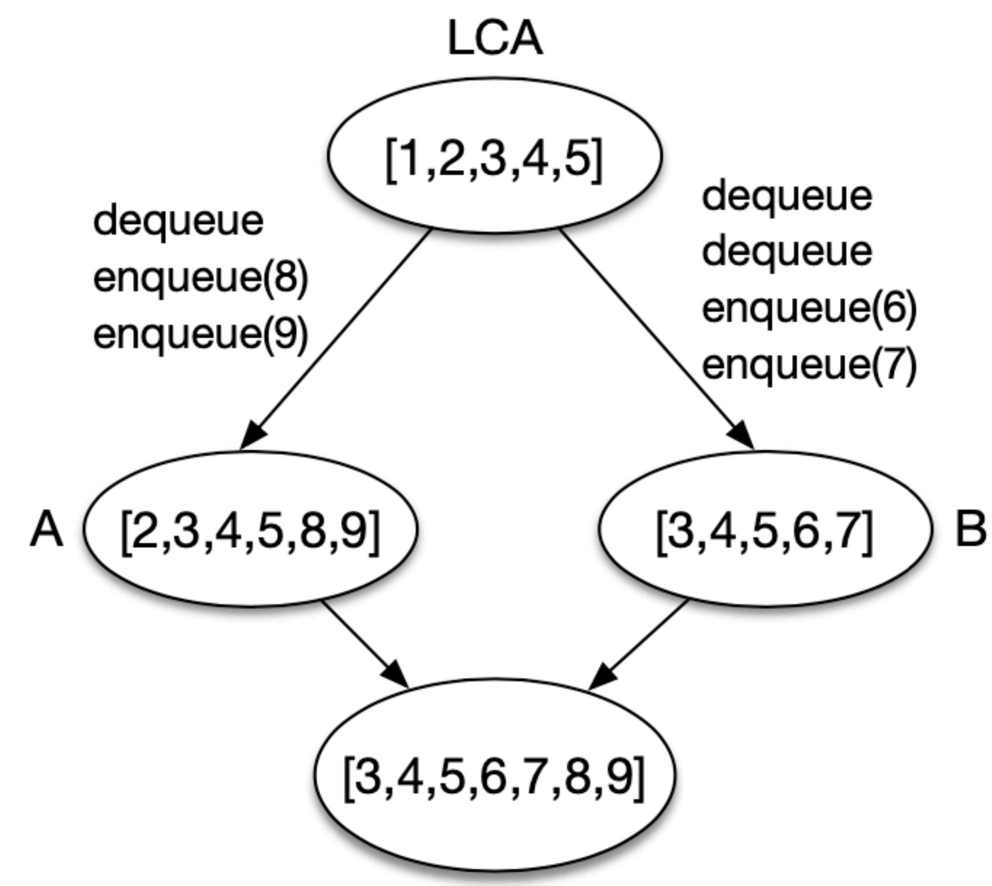

- Our aim is to have O(1) enqueue and dequeue and O(n) merge

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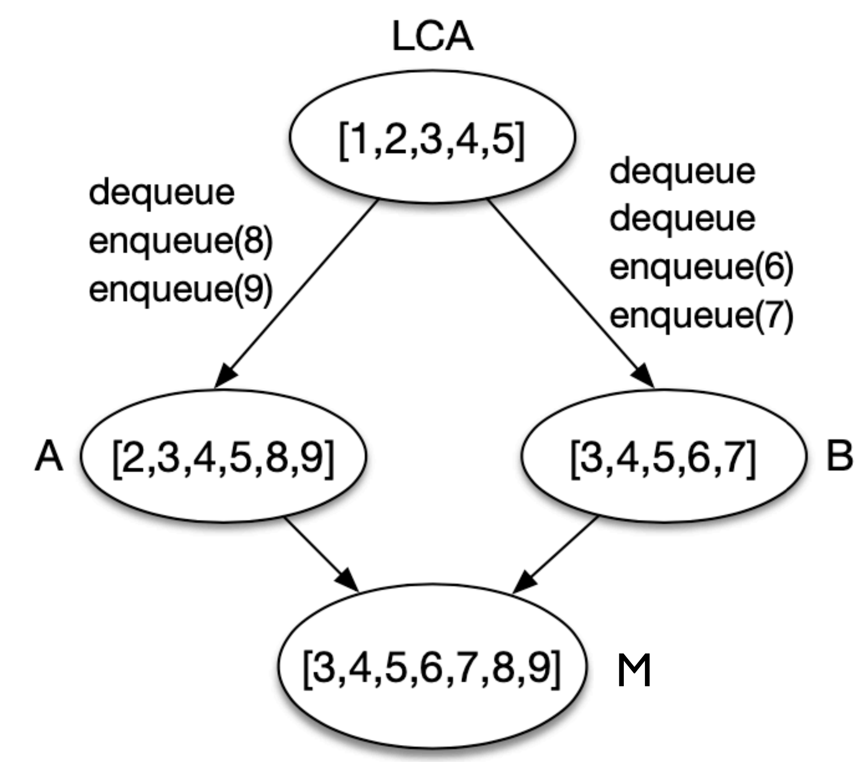

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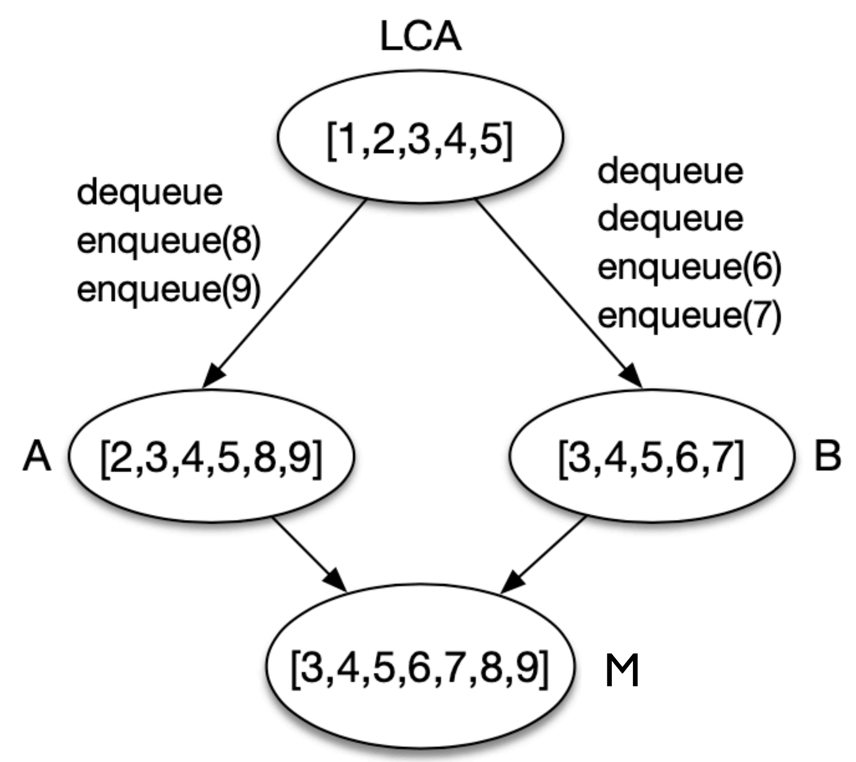

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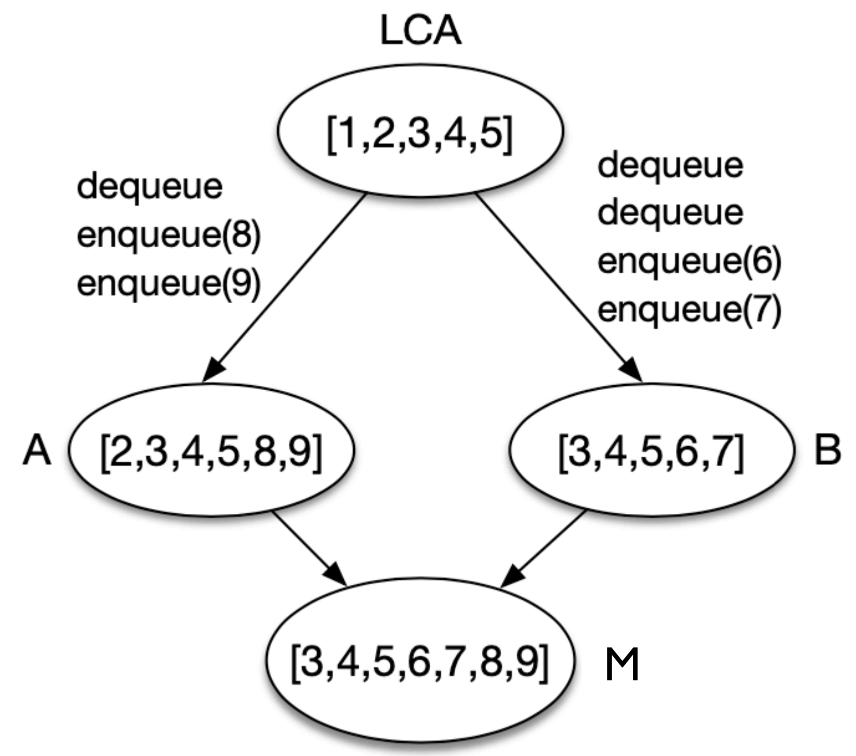

Implementation far removed from the specification!

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- MRDTs simplify RDTs by implementing them as extensions of sequential data types
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- F* allows us to strike a balance between automated and interactive proofs
  - ★ Extract to OCaml and run on Irmin!

Backup Slides

Queue Performance