Certified Mergeable Replicated Data Types

"KC" Sivaramakrishnan

joint work with

Vimala Soundarapandian, Adharsh Kamath and Kartik Nagar

Collaborative Apps

Airtable Google Docs Figma Notion Overleaf

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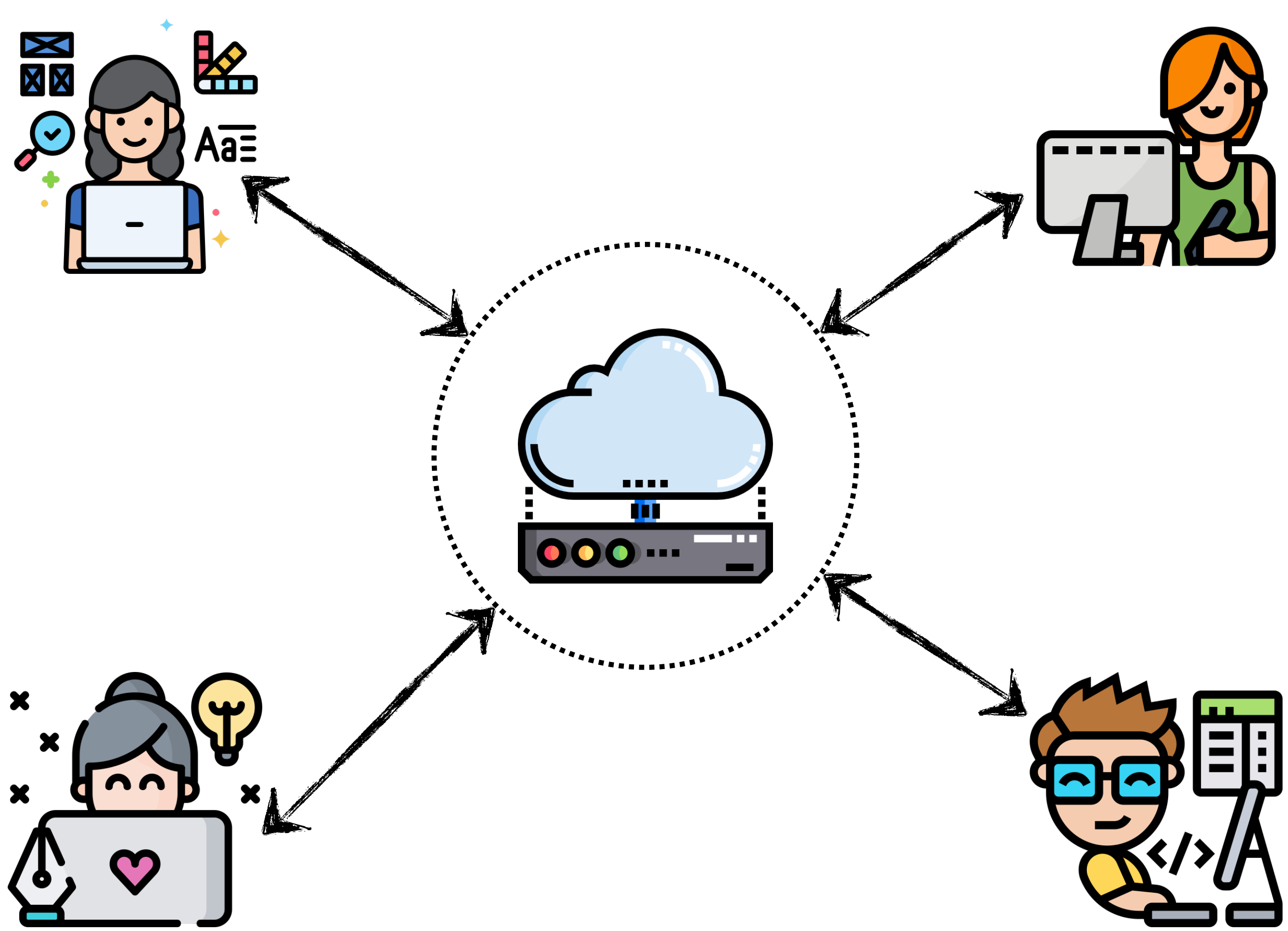

Network Partitions

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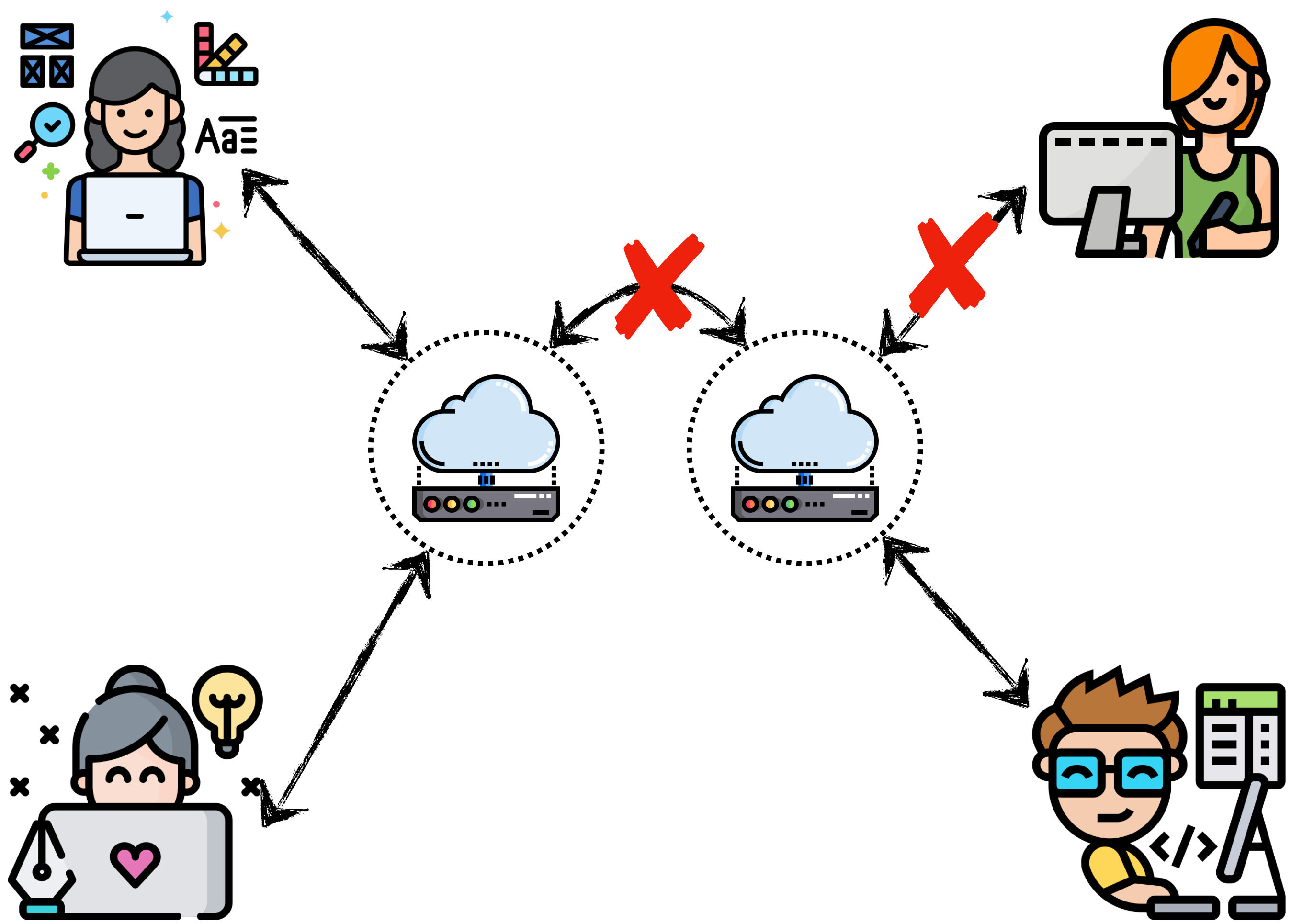

Local-first software

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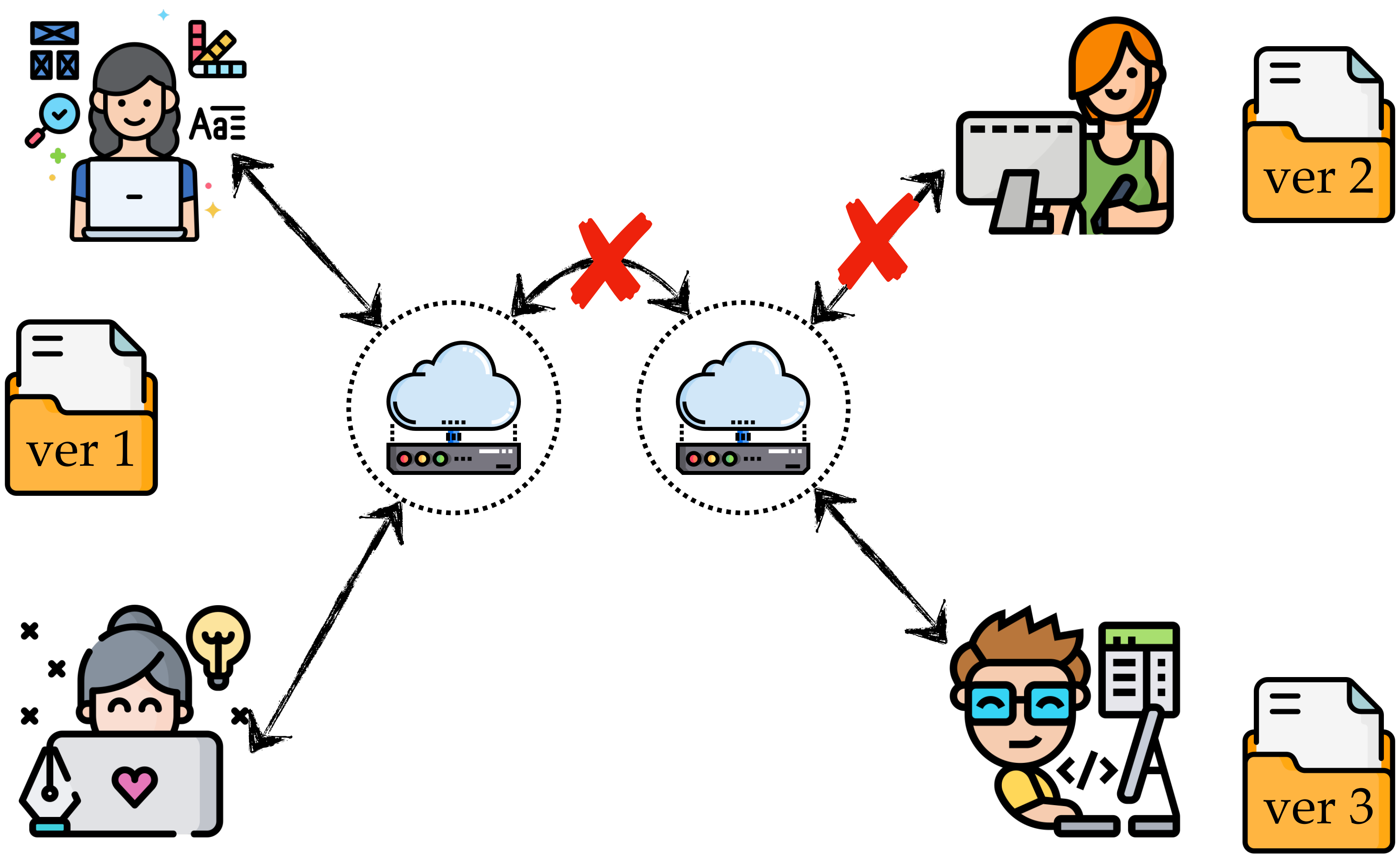

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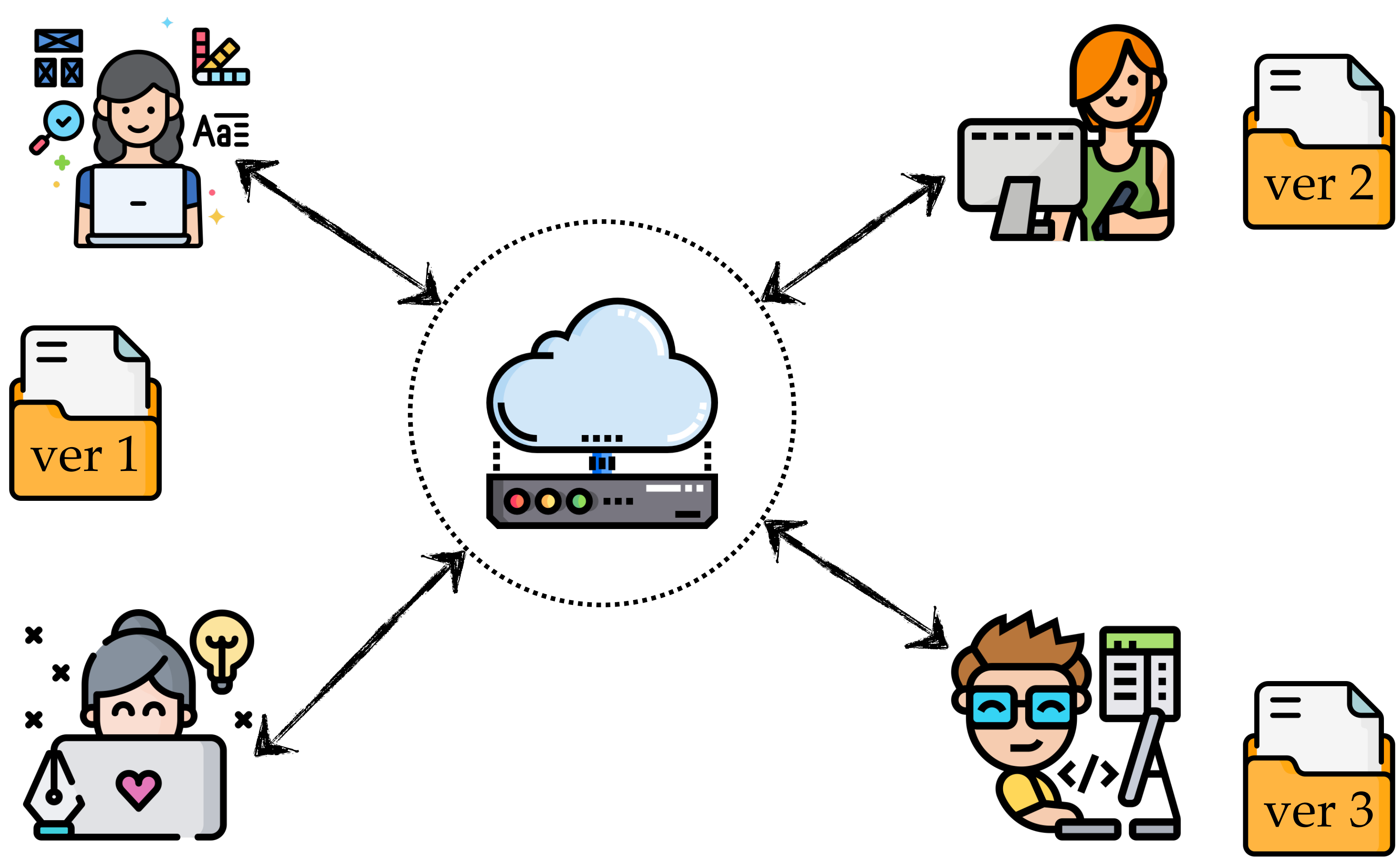

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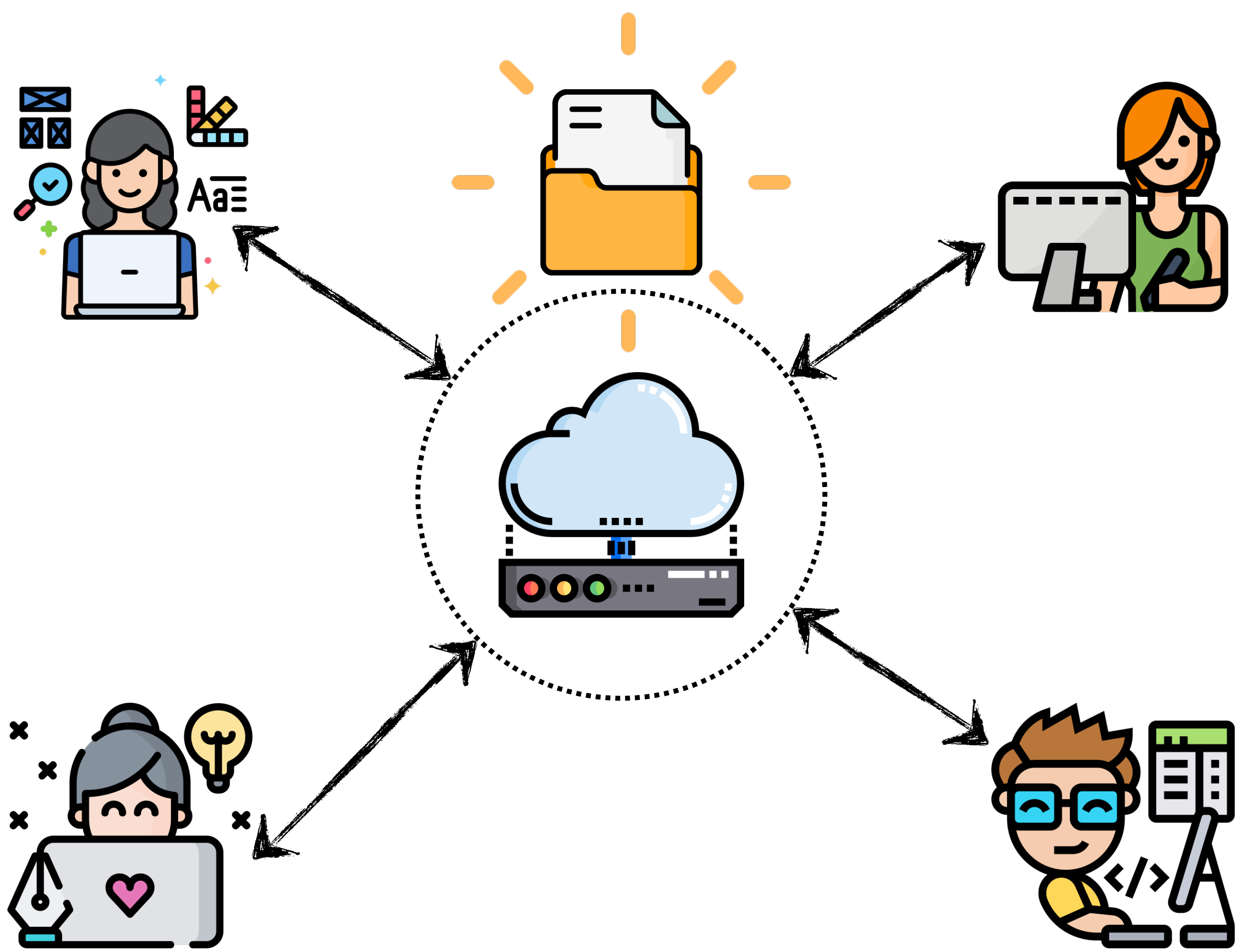
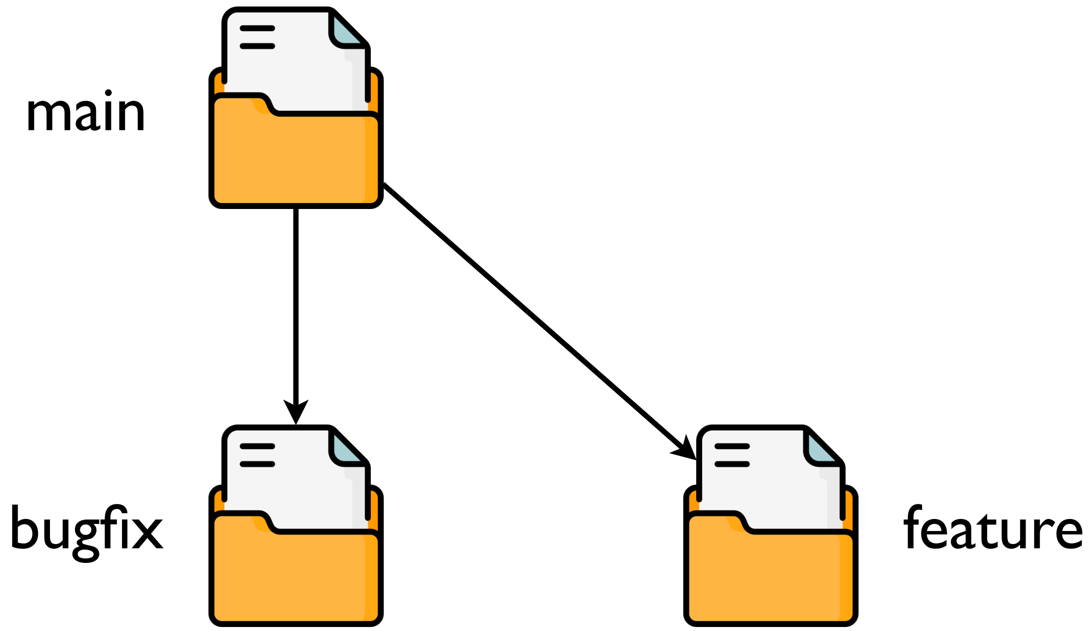


Distributed Version Control Systems



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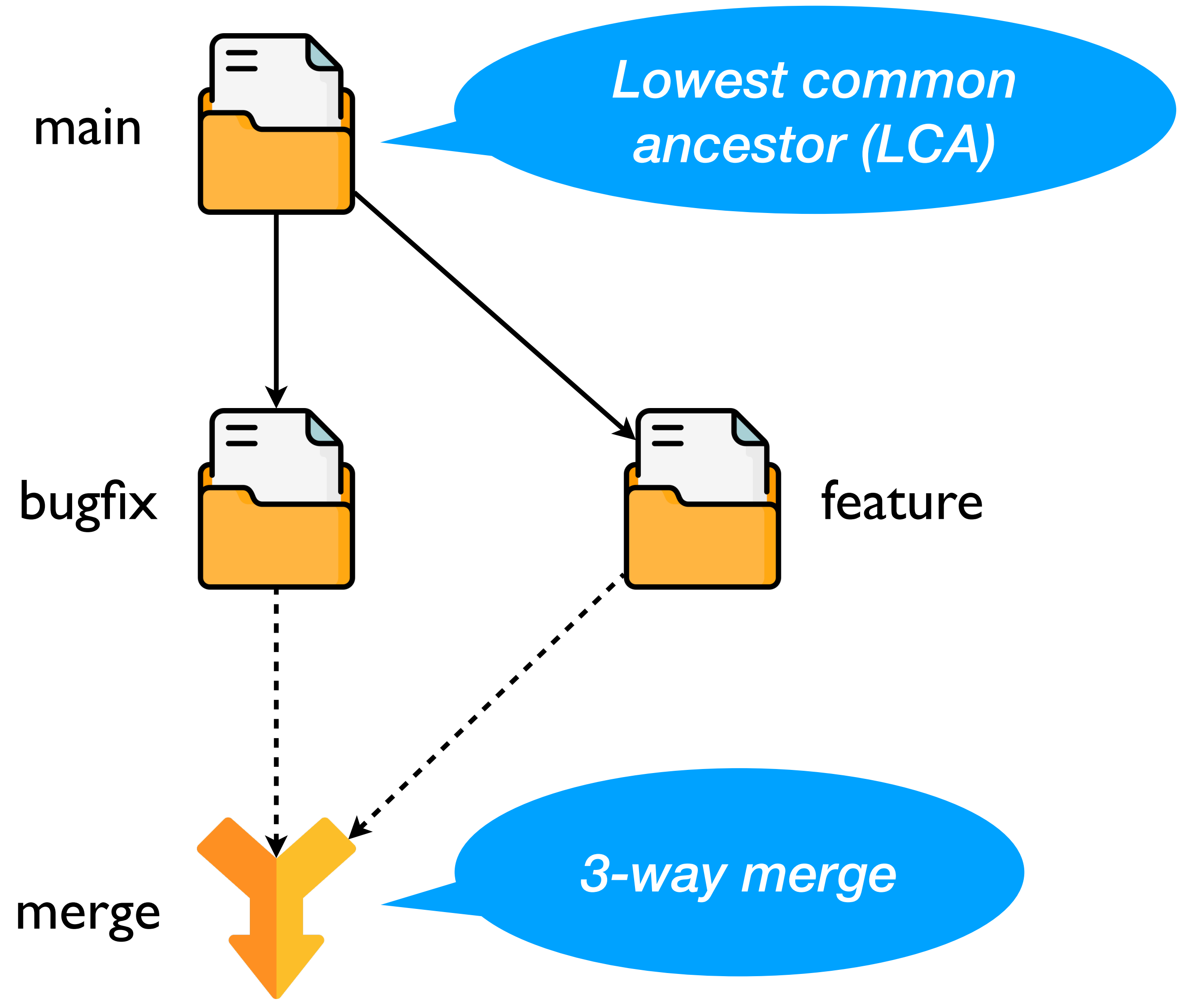

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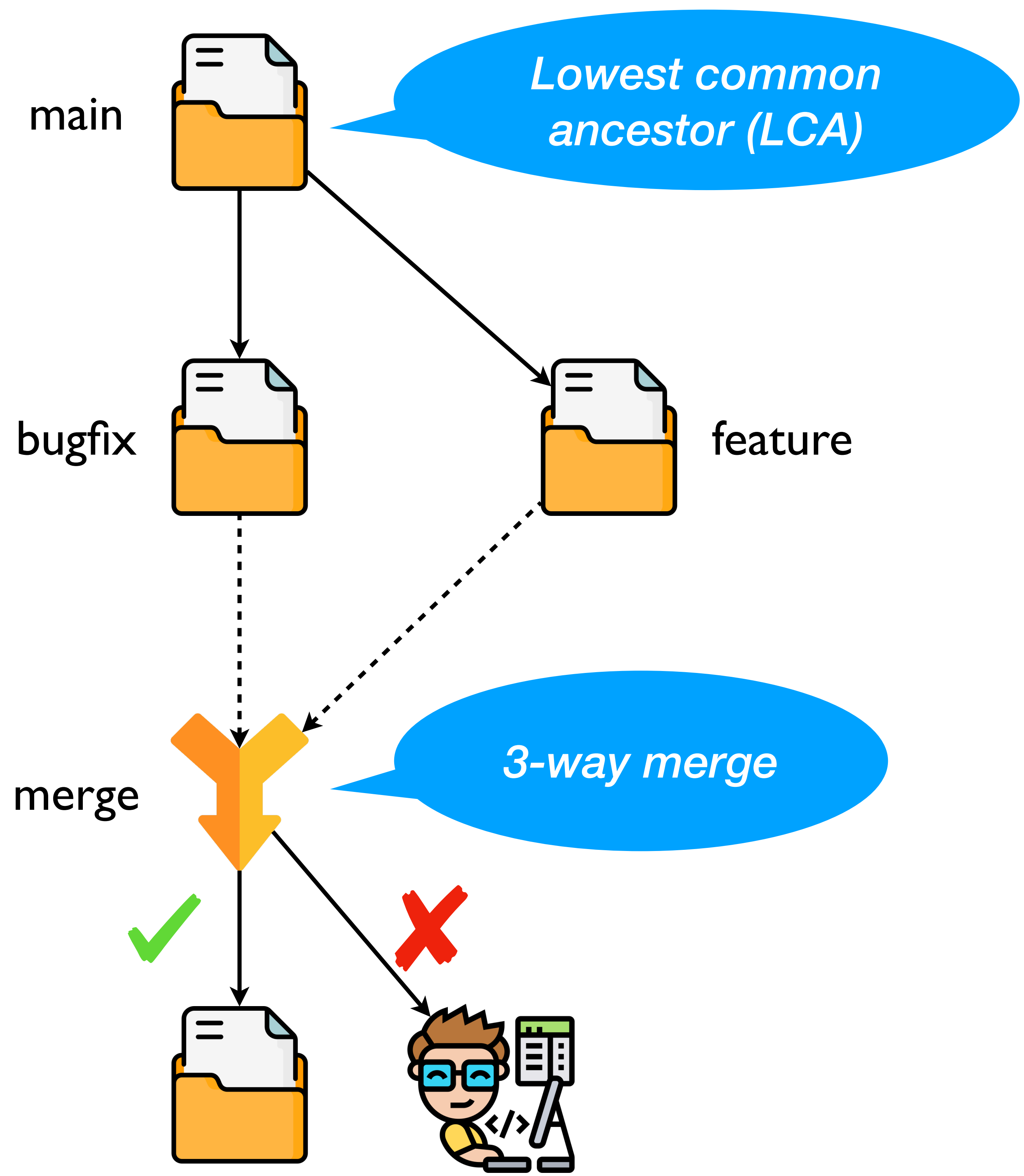

Mergeable Replicated Data Types

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  val add  : t -> int -> t
  val mult : t -> int -> t
  val merge : lca:t -> v1:t -> v2:t -> t
end = struct
  type t = int
  let read x = x
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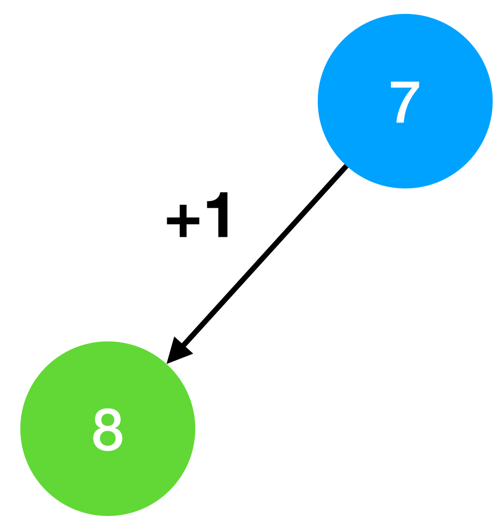

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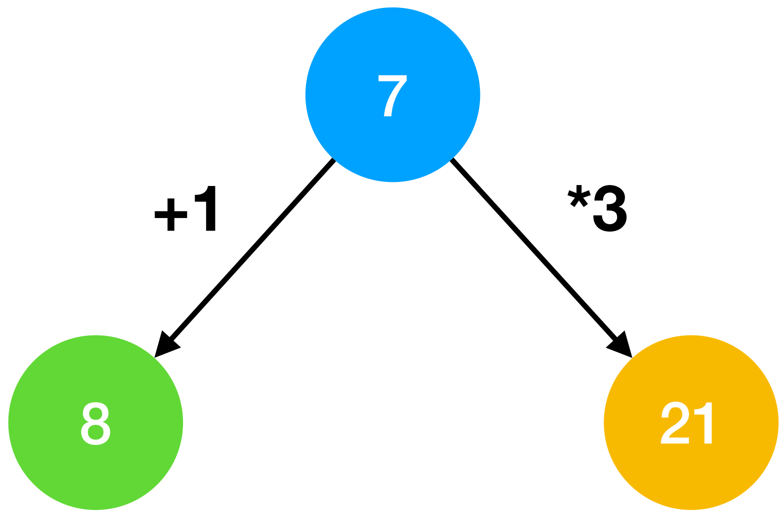

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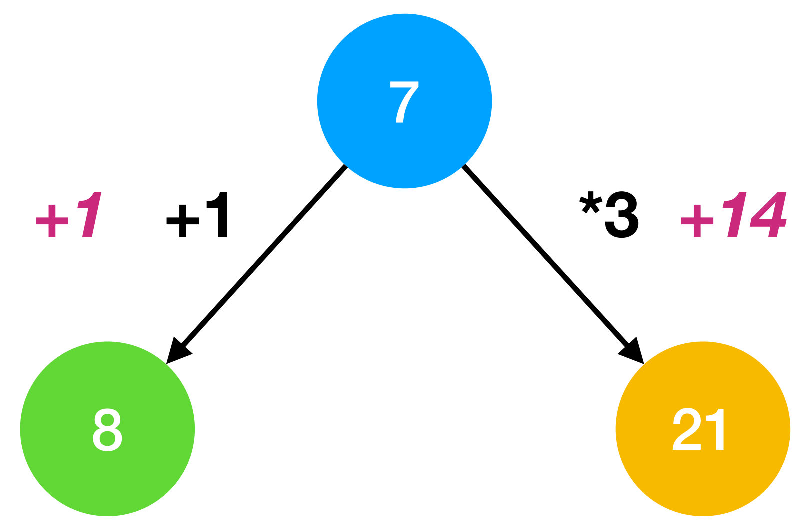

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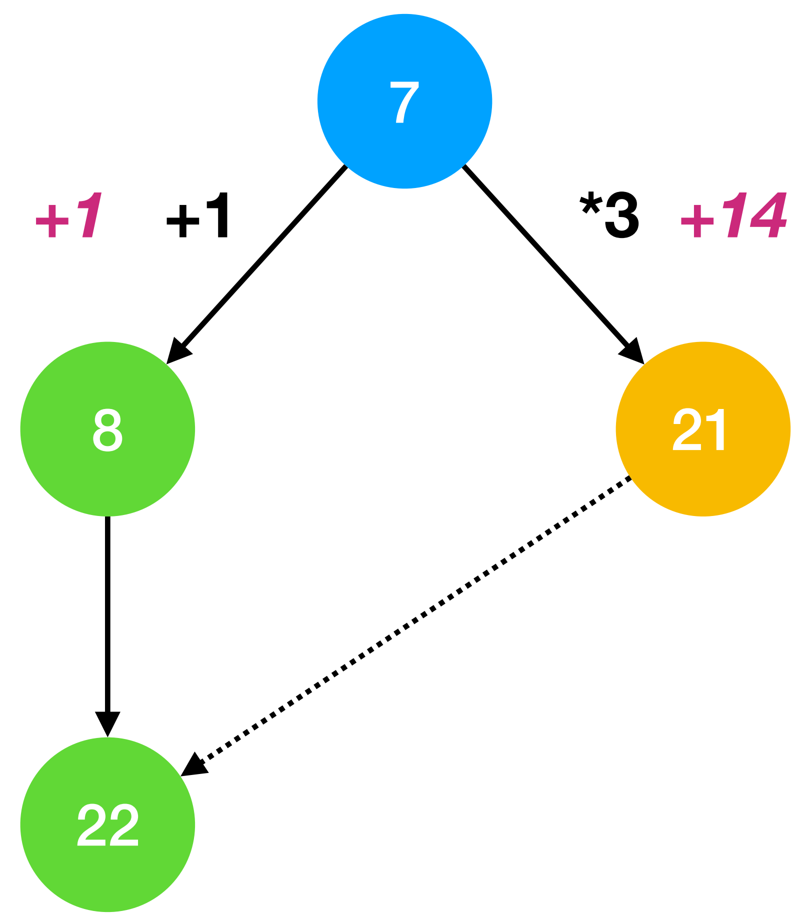

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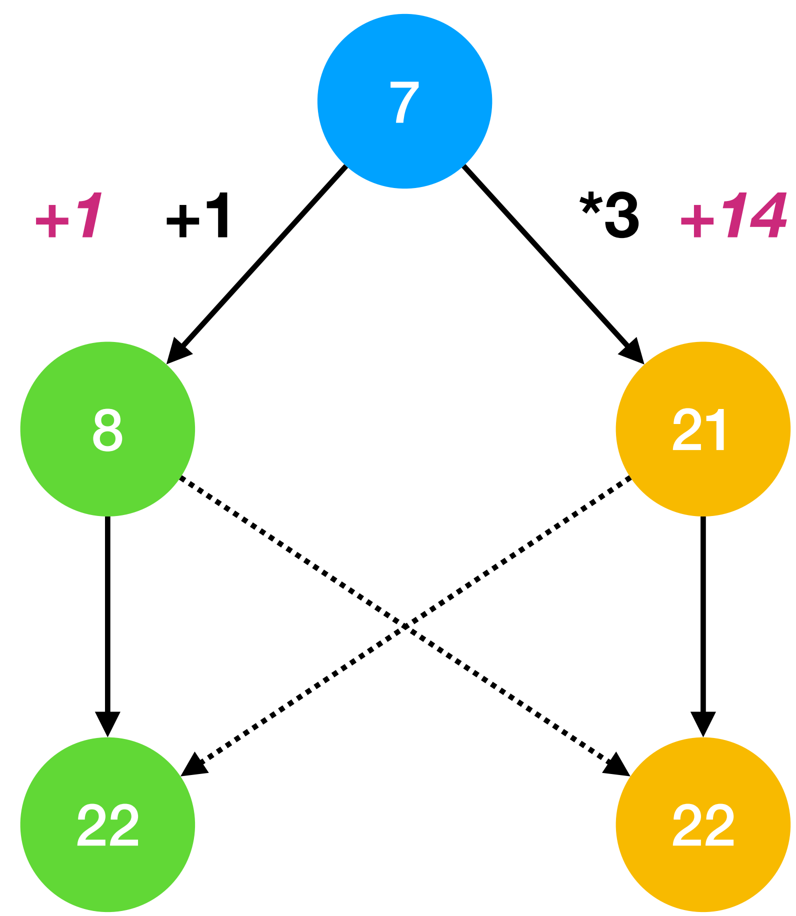

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Does the 3-way merge idea generalise?

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Sort of…

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- OR-set — *add-wins* when there is a concurrent add and remove of the same element

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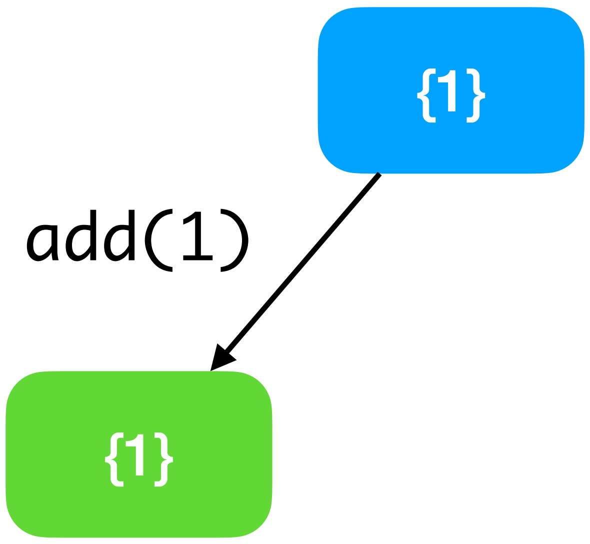

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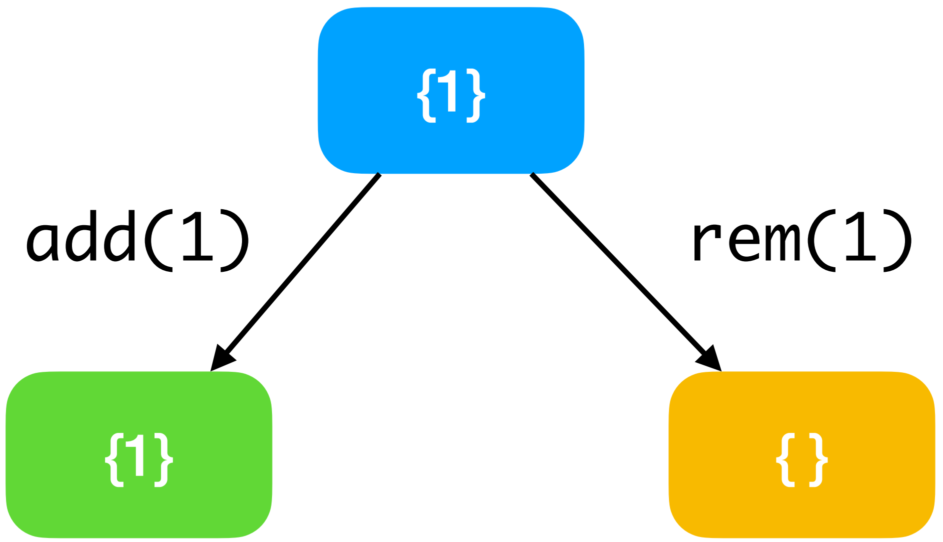

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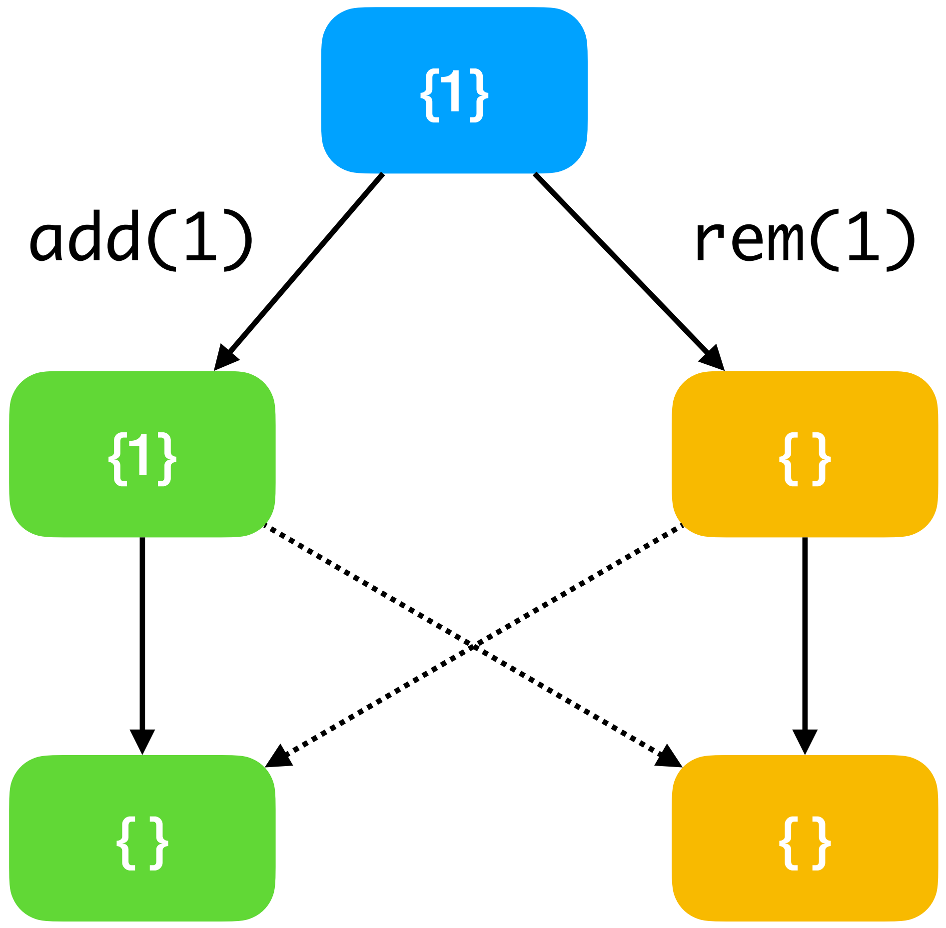
$$= \{\} \cup \{\} \cup \{\}$$
$$= \{\} \text{ (expected } \{1\})$$



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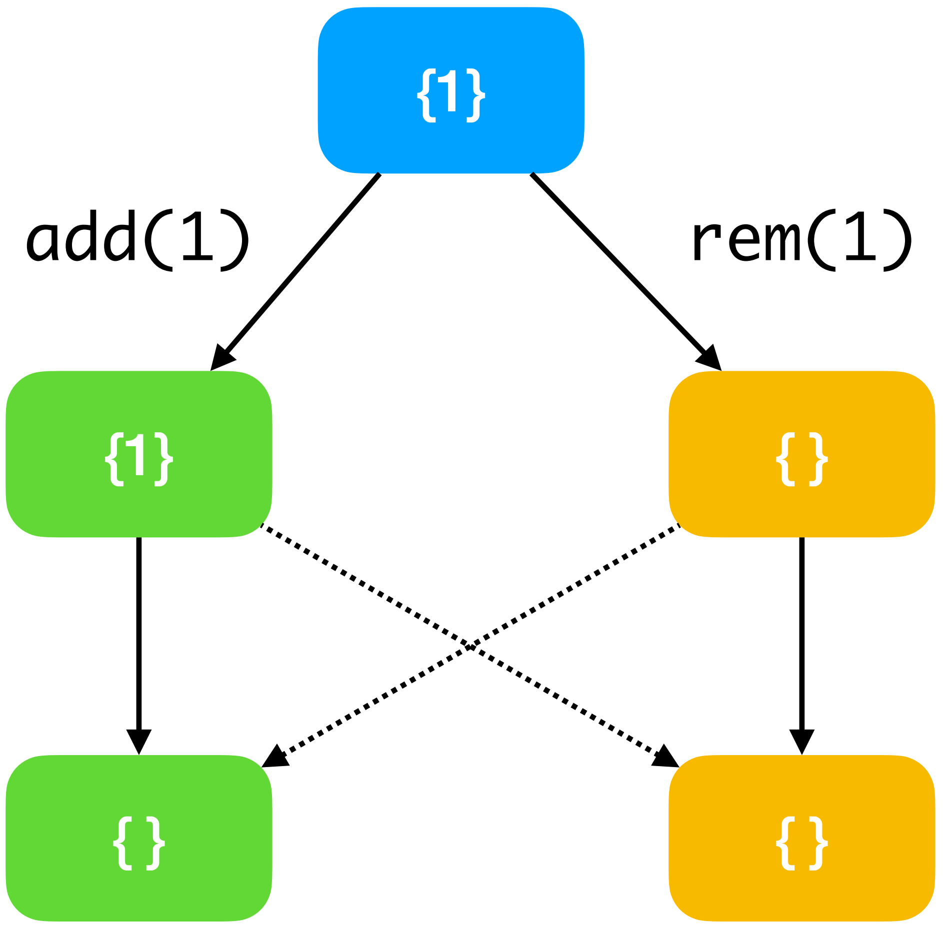

$$
\begin{aligned}
& \{\,\} \cup (\{1\} - \{1\}) \cup (\{\,\} - \{1\}) \\
&= \{\,\} \cup \{\,\} \cup \{\,\} \\
&= \{\,\} \text{ \textit{(expected \{1\})}}
\end{aligned}
$$

- Convergence is not sufficient; *Intent* is not preserved

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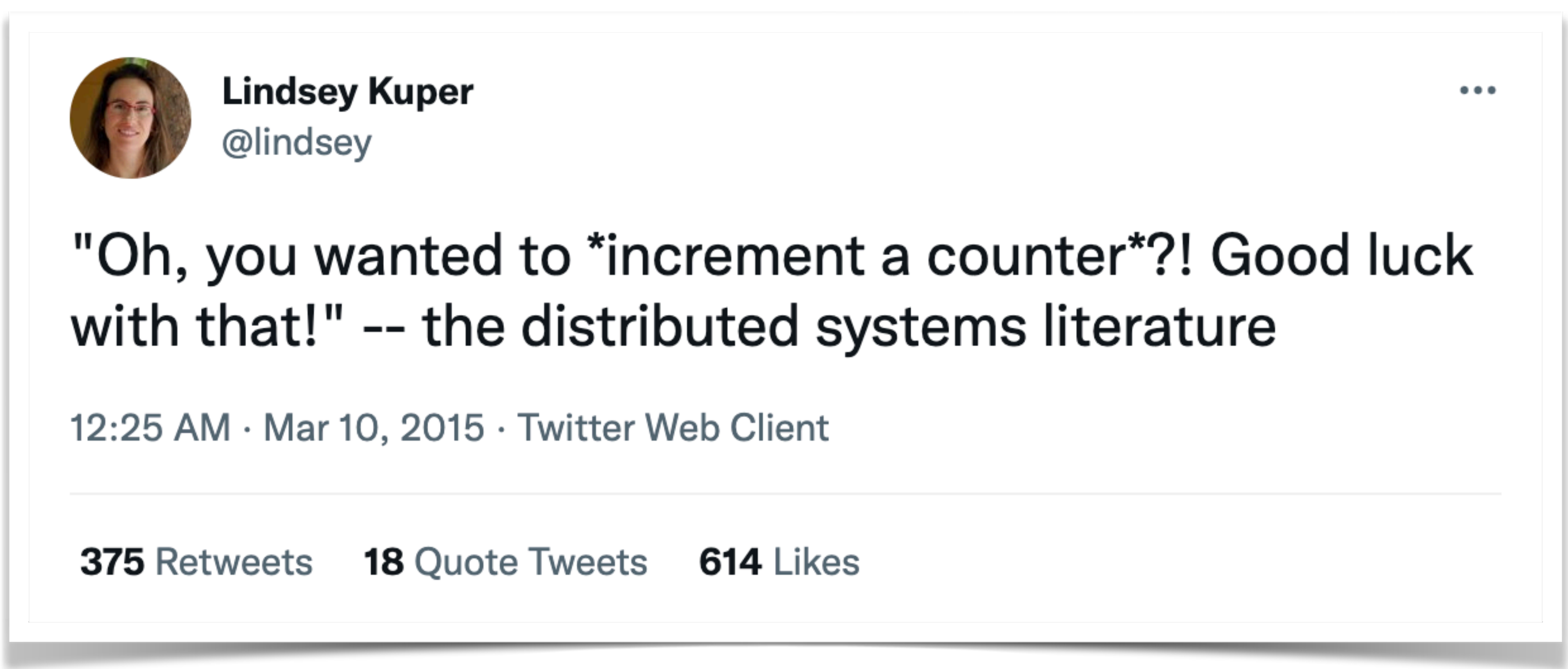

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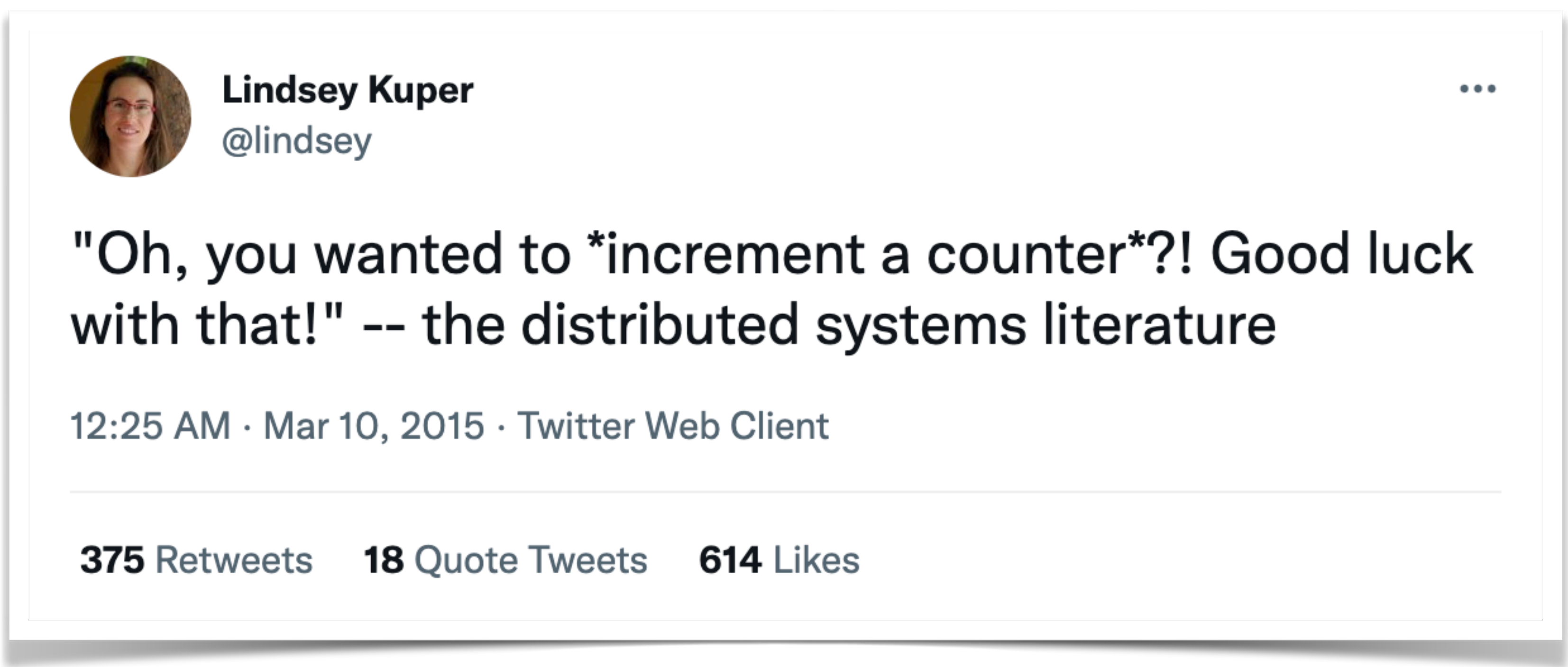

- *Mechanization* to bridge the gap between spec and impl

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- *Composition* of MRDTs and their proofs!
- Extracted RDTs are compatible with *Irmin* — a Git-like distributed database

Fixing OR-Set

- Discriminate duplicate additions by associating a unique id

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{ (a,1) }

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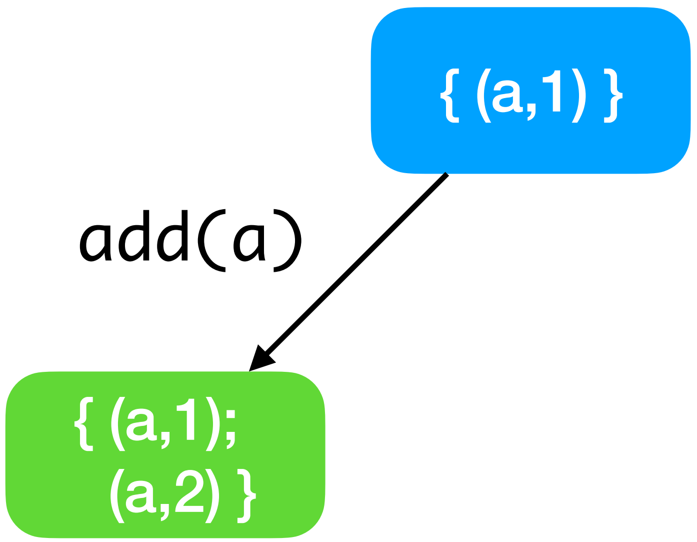

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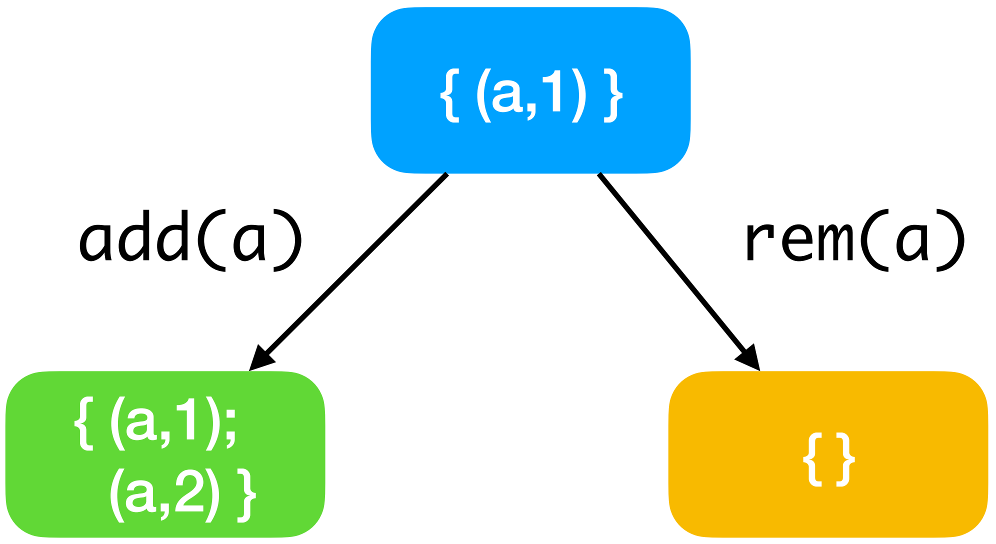

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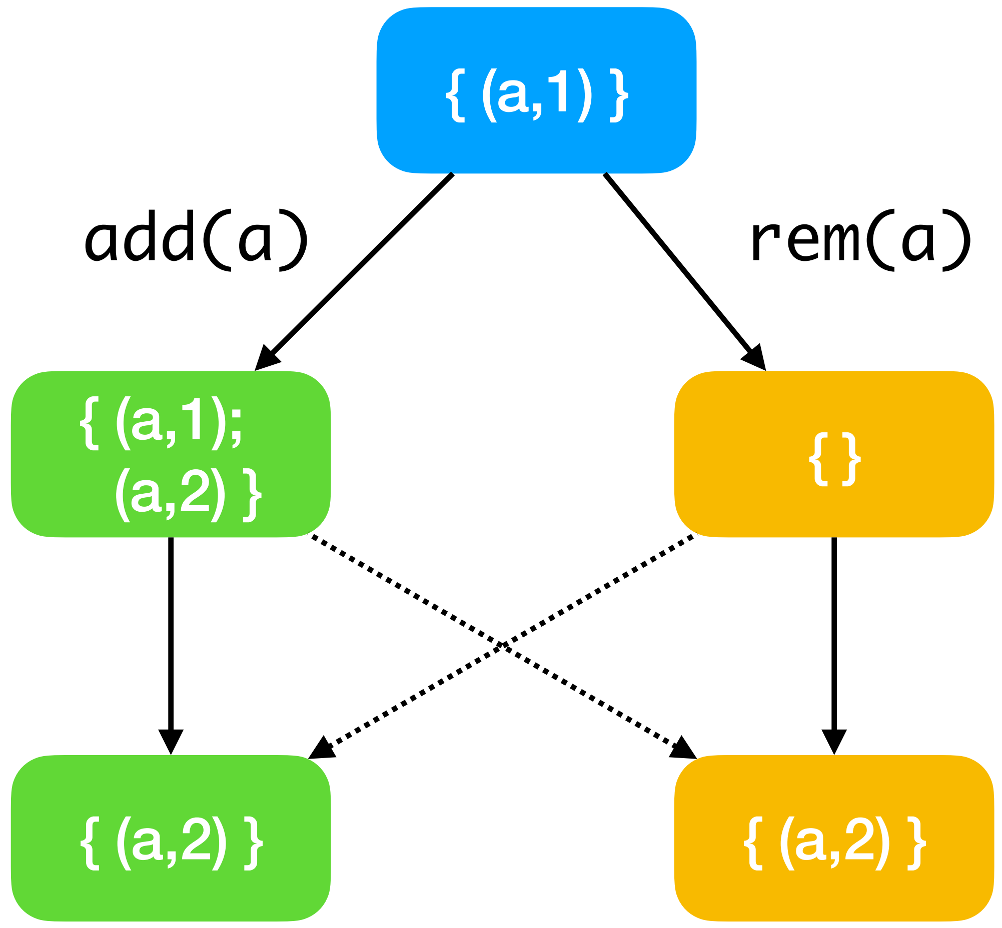
$$
\begin{aligned}
& \{\,\} \\
& \cup\ (\{\,(a,1);\ (a,2)\,\} - \{\,(a,1)\,\}) \\
& \cup\ (\{\,\} - \{\,(a,1)\,\}) \\
=\ & \{\,\} \cup \{\,(a,2)\,\} \cup \{\,\} \\
=\ & \{\,(a,2)\,\}
\end{aligned}
$$



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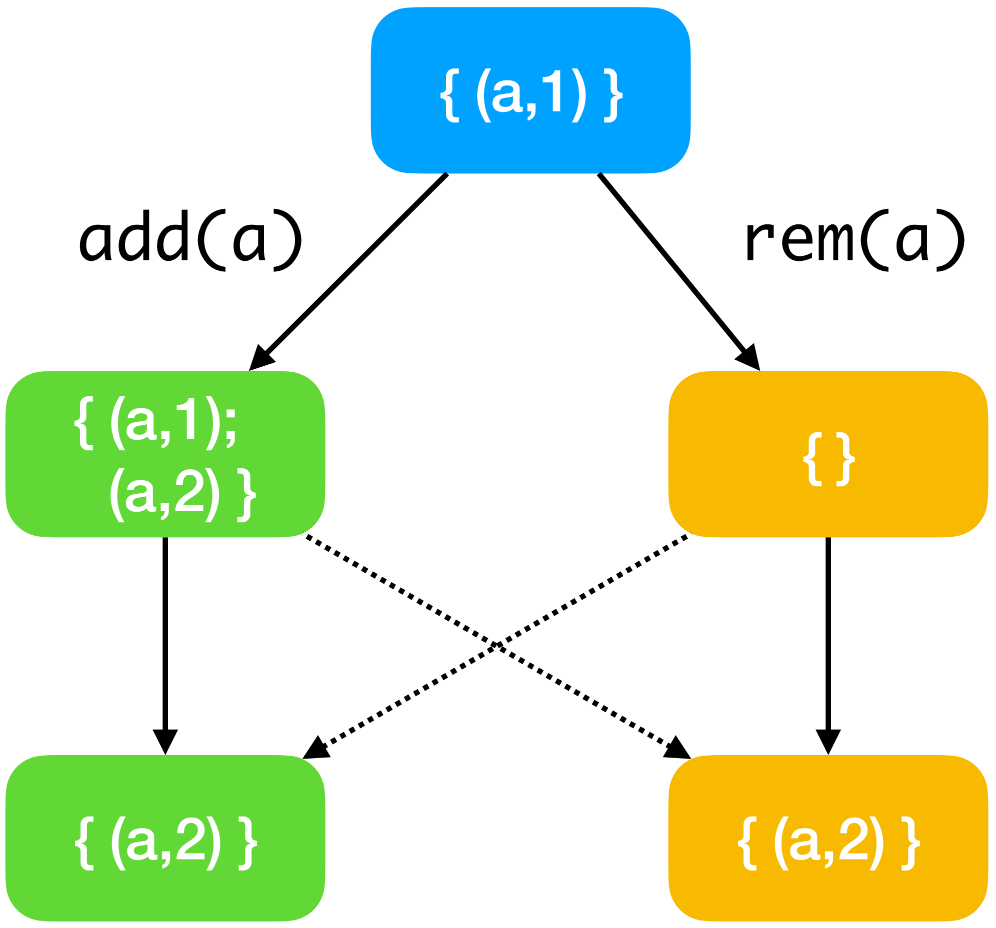
  { }
  ∪ ({ (a,1); (a,2) } - { (a,1) })
  ∪ ({ } - { (a,1) })
  = { } ∪ { (a,2) } ∪ { }
  = { (a,2) }

- MRDT implementation

$$D_\tau = (\Sigma, \sigma_0, do, merge)$$



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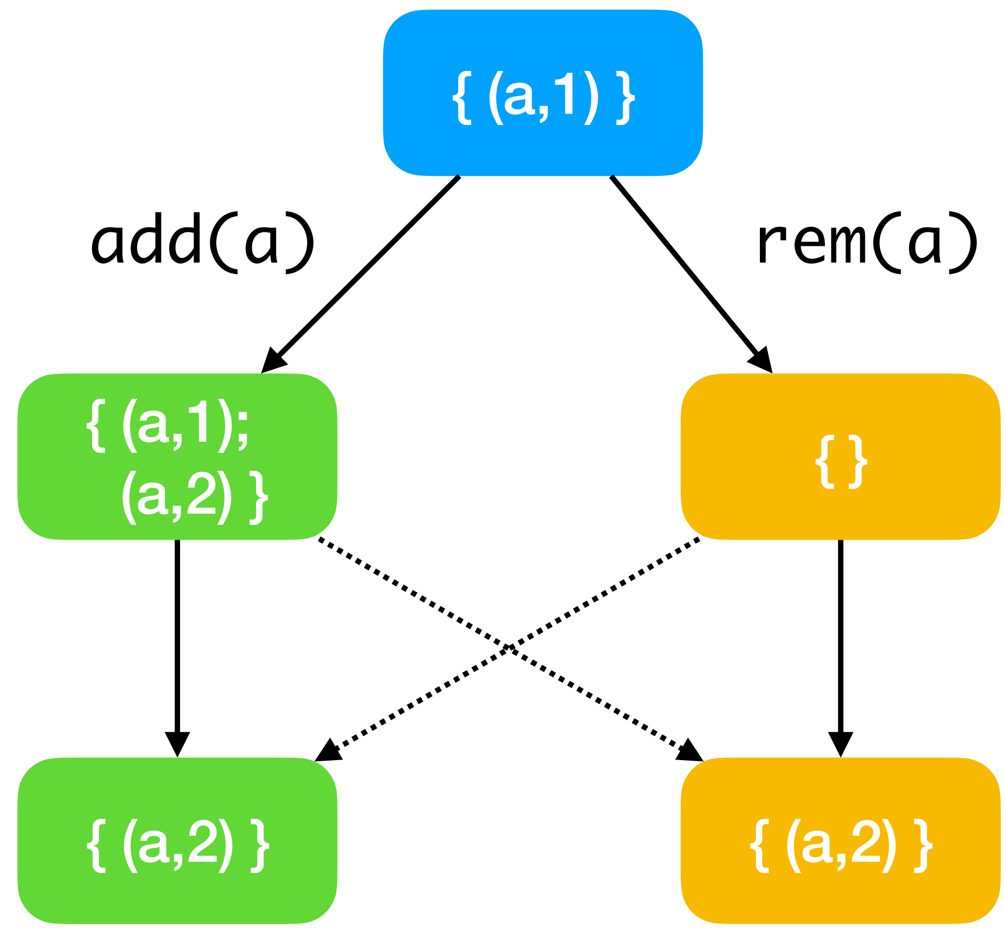

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1: $\Sigma = \mathcal{P}(\mathbb{N} \times \mathbb{N})$
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3: $do(rd, \sigma, t) = (\sigma, \{a \mid (a, t) \in \sigma\})$
4: $do(add(a), \sigma, t) = (\sigma \cup \{(a, t)\}, \bot)$
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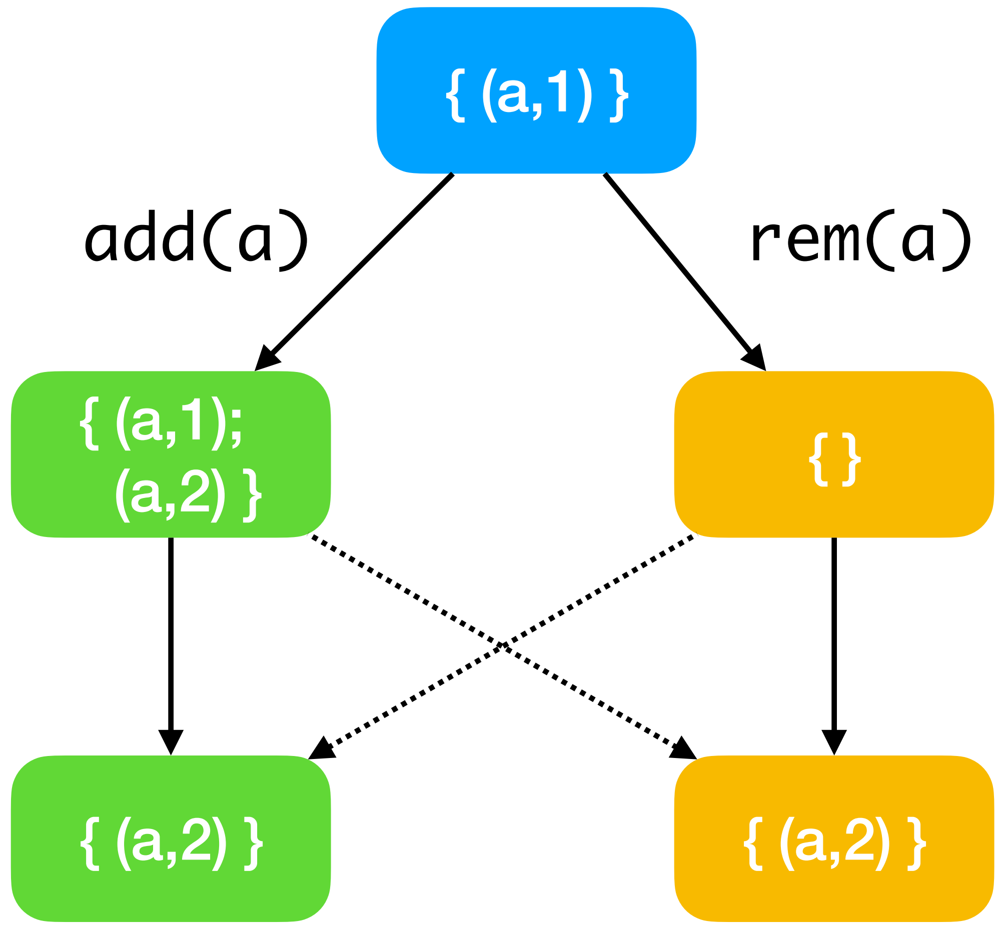

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Unique Lamport Timestamps

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Abstract state $I = \langle E, oper, rval, time, vis \rangle$

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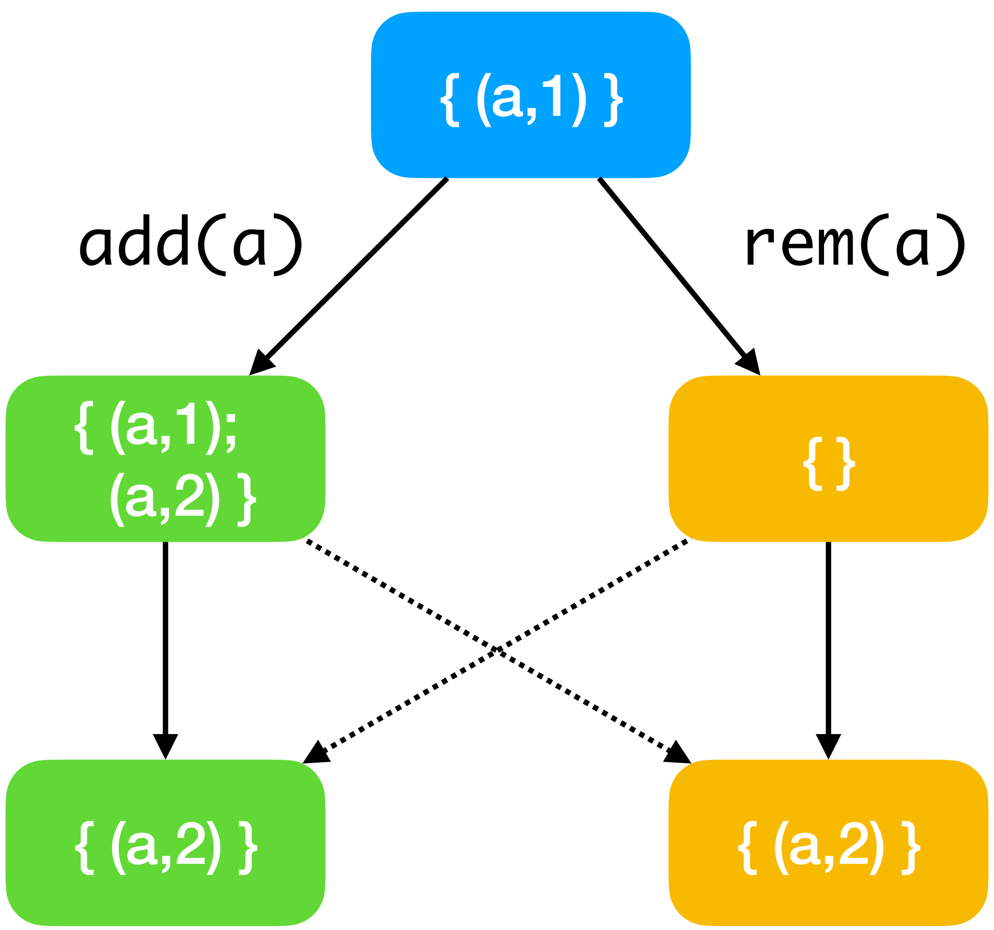

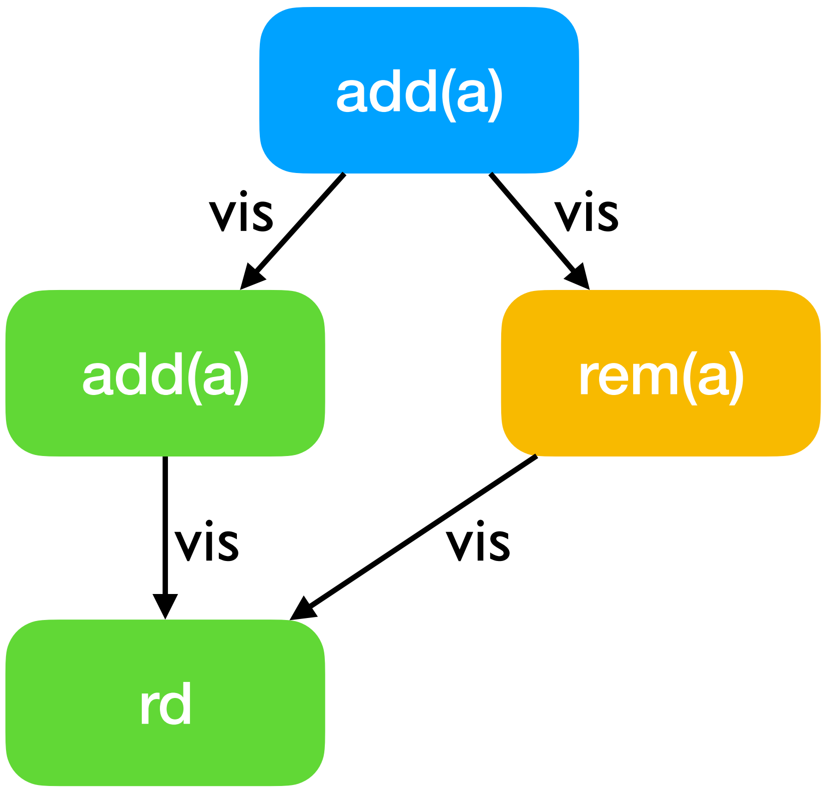

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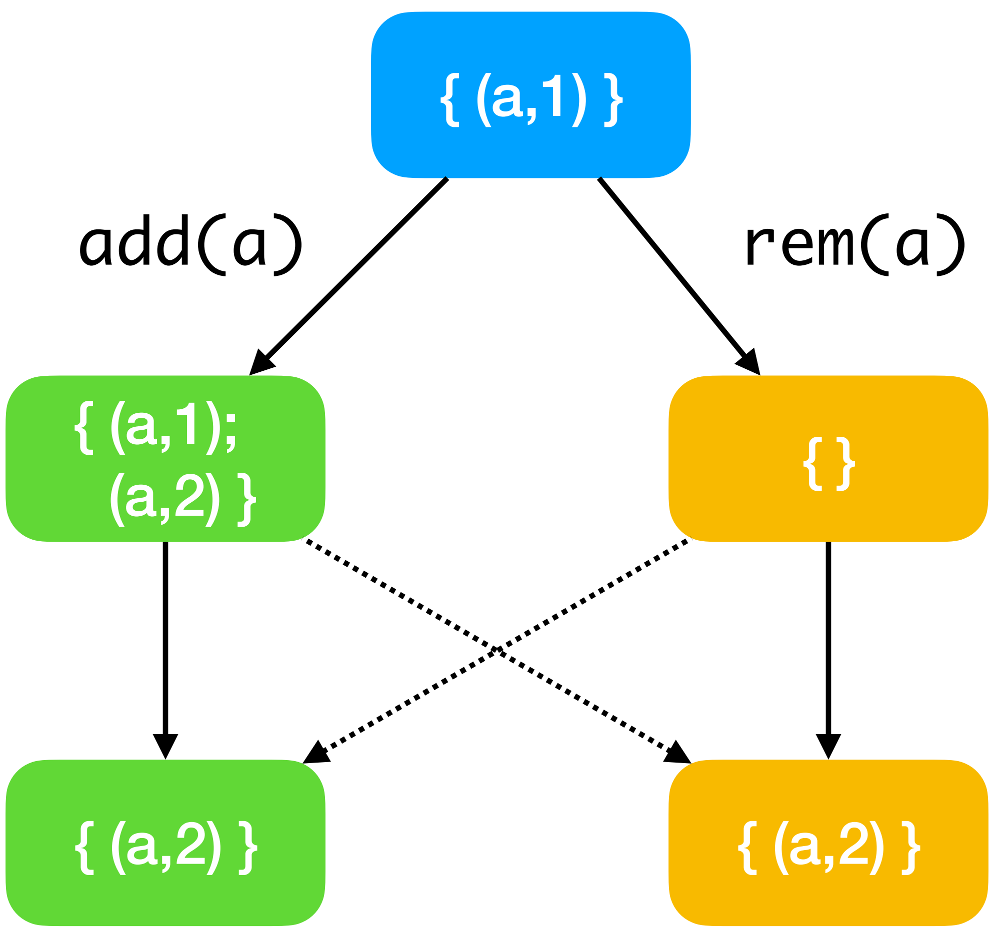

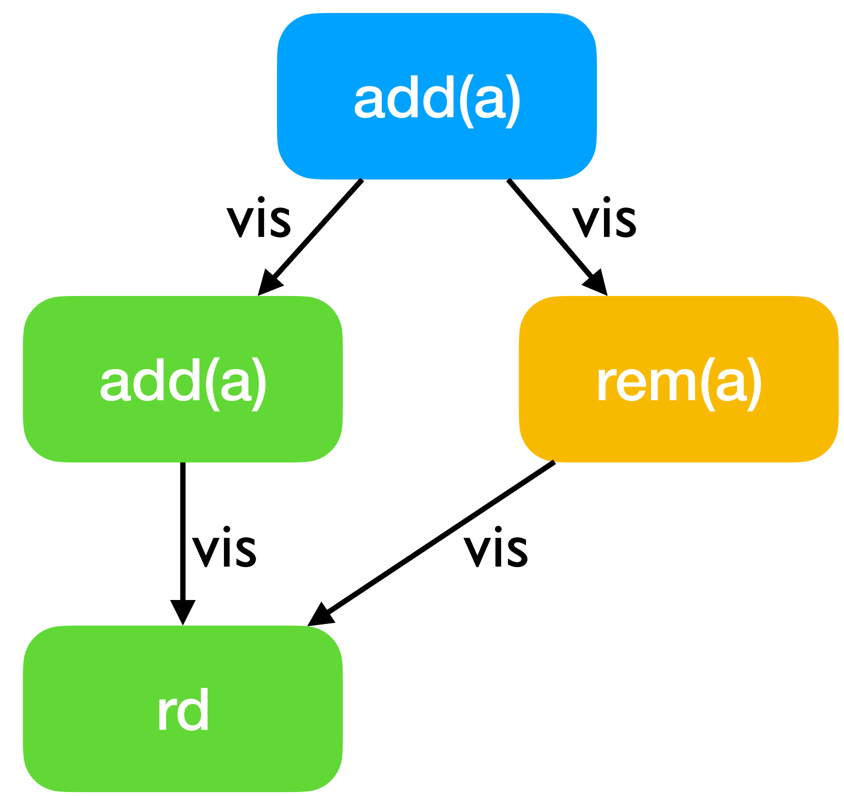

$$\mathcal{F}_{orset}(\mathrm{rd}, \langle E, oper, rval, time, vis \rangle) = \{a \mid \exists e \in E.\ oper(e)$$

$$= \mathrm{add}(a) \wedge \neg(\exists f \in E.\ oper(f) = \mathrm{remove}(a) \wedge e \xrightarrow{vis} f)\}$$

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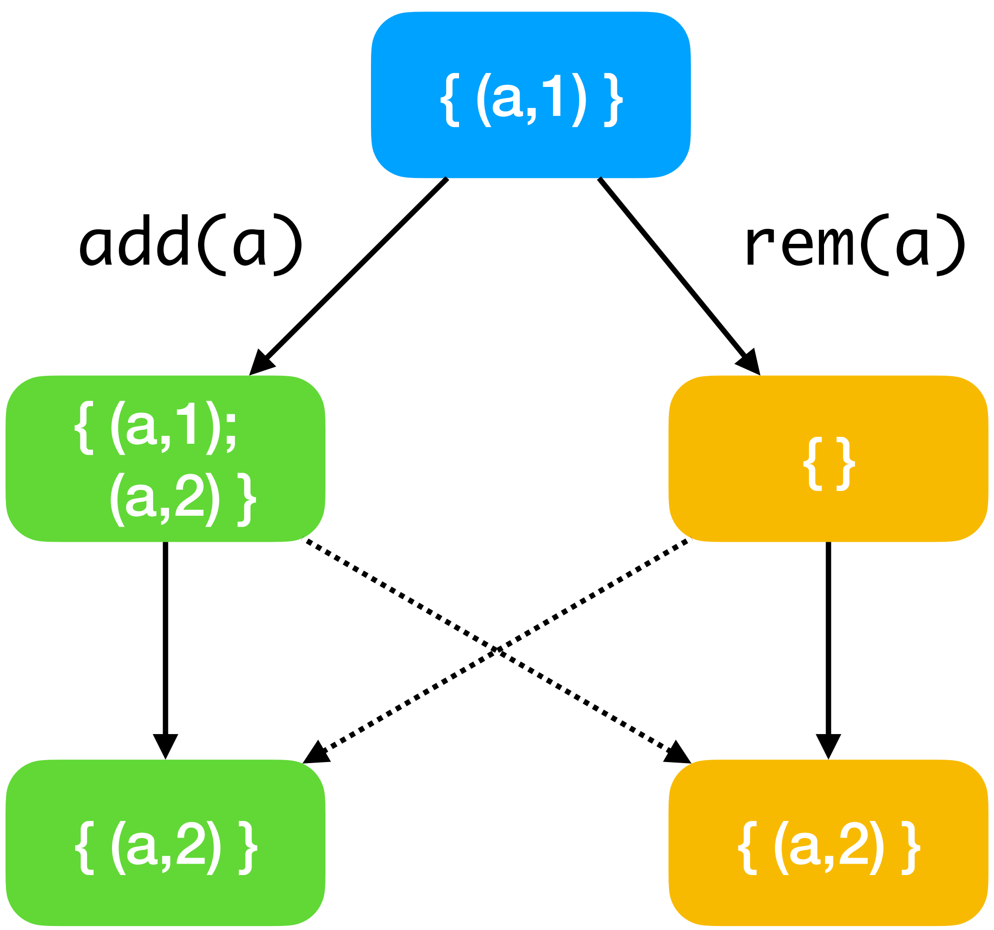

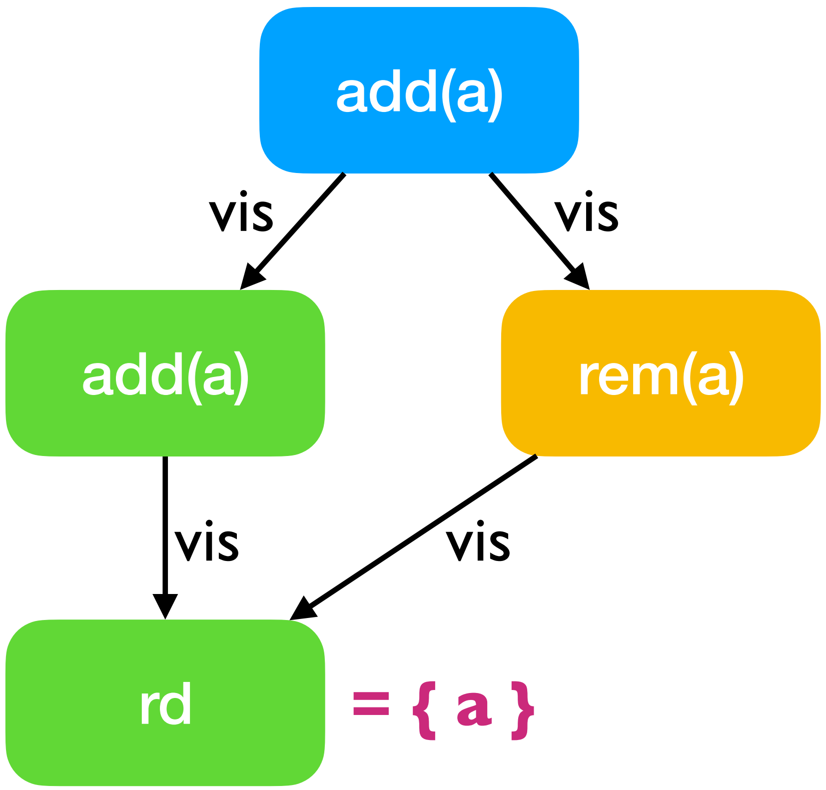

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$$\mathcal{R}_{sim}(I, \sigma) \iff (\forall (a, t) \in \sigma \iff$$
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- The main verification effort is to show that the relation above is indeed a *simulation relation*
  - ★ Shown separately for *operations* and *merge function*
  - ★ Proof by induction on the execution trace

Verification effort

MRDTs verified	#Lines code	#Lines proof	#Lemmas	Verif. time (s)
Increment-only counter	6	43	2	3.494
PN counter	8	43	2	23.211
Enable-wins flag	20	58	3	1074
		81	6	171
		89	7	104
LWW register	5	44	1	4.21
G-set	10	23	0	4.71
		28	1	2.462
		33	2	1.993
G-map	48	26	0	26.089
Mergeable log	39	95	2	36.562
OR-set (§2.1.1)	30	36	0	43.85
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Composing RDTs is HARD!

Martin Kleppmann
@martinkl

Today in "distributed systems are hard": I wrote down a simple CRDT algorithm that I thought was "obviously correct" for a course I'm teaching. Only 10 lines or so long. Found a fatal bug only after spending hours trying to prove the algorithm correct. 😭

4:18 AM · Nov 13, 2020 · Tweetbot for iOS

41 Retweets **4** Quote Tweets **541** Likes

Martin Kleppmann @martinkl · Nov 13, 2020

The interesting thing about this bug is that it comes about only from the interaction of two features. A LWW map by itself is fine. A set in which you can insert and delete elements (but not update them) is fine. The problem arises only when delete and update interact.

1 16

Composing IRC-style chat

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- Represent application state as a map MRDT
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- **Goal:**
  - ★ *map* and *log* proved correct separately
  - ★ Use the proof of underlying RDTs to prove chat application correctness

Generic Map MRDT

Implementation

$$\mathcal{D}_{\alpha-map} = (\Sigma, \sigma_0, do, merge_{\alpha-map}) \text{ where}$$

1: $\Sigma_{\alpha-map} = \mathcal{P}(string \times \Sigma_\alpha)$

2: $\sigma_0 = \{\}$

3: $\delta(\sigma, k) = \begin{cases} \sigma(k), & \text{if } k \in dom(\sigma) \\ \sigma_{0_\alpha}, & \text{otherwise} \end{cases}$

4: $do(set(k, o_\alpha), \sigma, t) =$
$\quad \text{let } (v, r) = do_\alpha(o_\alpha, \delta(\sigma, k), t) \text{ in } (\sigma[k \mapsto v], r)$

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$\quad \text{let } (\_, r) = do_\alpha(o_\alpha, \delta(\sigma, k), t) \text{ in } (\sigma, r)$

6: $merge_{\alpha-map}(\sigma_{lca}, \sigma_a, \sigma_b) =$
$\quad \{(k, v) \mid (k \in dom(\sigma_{lca}) \cup dom(\sigma_a) \cup dom(\sigma_b)) \wedge$
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Simulation Relation

$$\mathcal{R}_{sim-\alpha-map}(I, \sigma) \iff \forall k.$$

1: $(k \in dom(\sigma) \iff \exists e \in I.E.\ oper(e) = set(k, \_)) \wedge$

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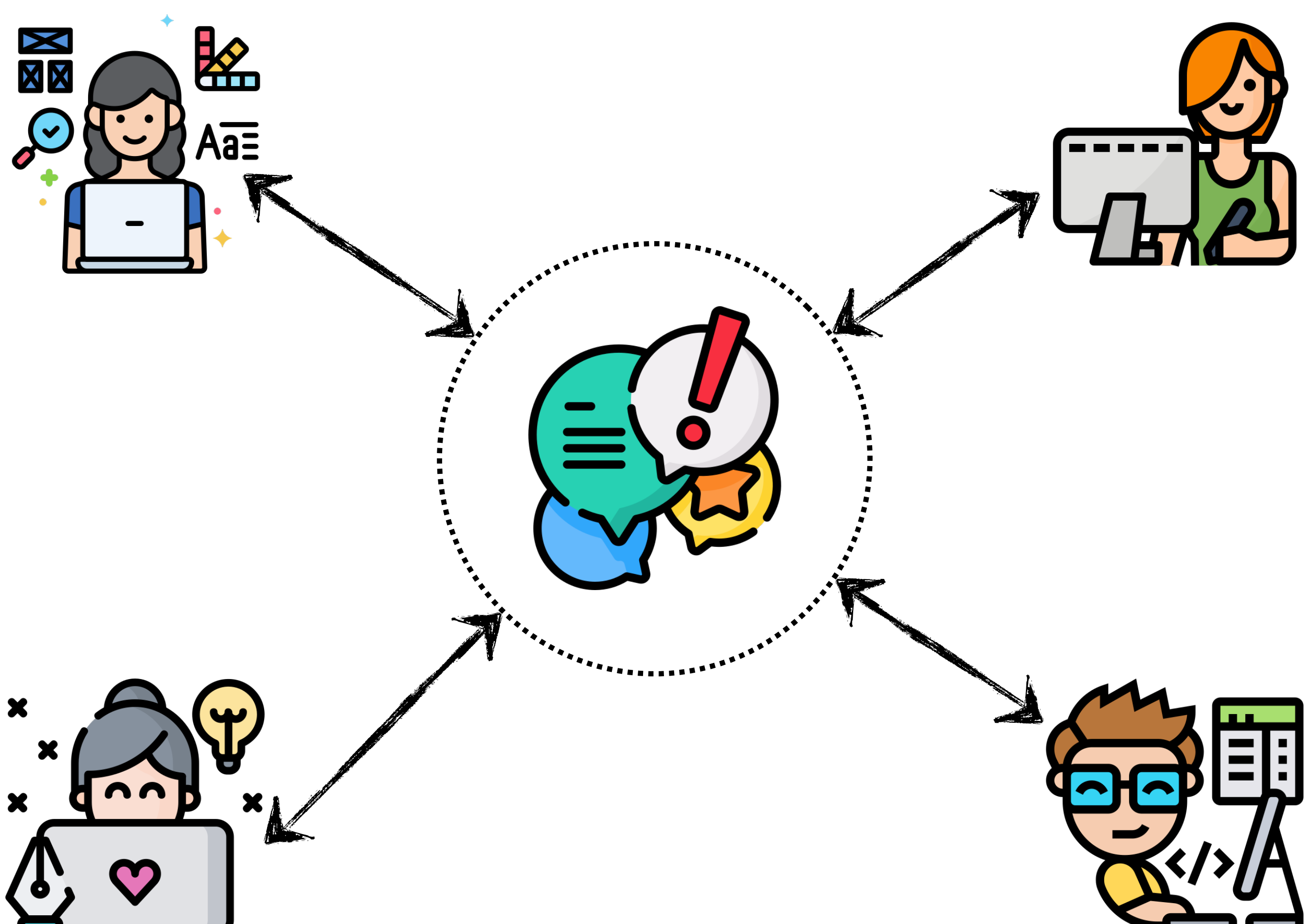
Simulation relation appeals to the value type's simulation relation!

Composing IRC-style chat

- IRC app state is constructed by instantiating *generic map* with *mergeable log*
- The proof of correctness of the chat application directly follows from the composition.
  - ★ See paper for details!



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  - ✦ An F* library implementing and proving MRDTs
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- See paper for
  - ✦ Formal description of the system + soundness proof
  - ✦ Case study on replicated queues
  - ✦ Performance results