Relational Reasoning for Mergeable Replicated Data Types

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joint work with
Gowtham Kaki, Swarn Priya, Suresh Jagannathan

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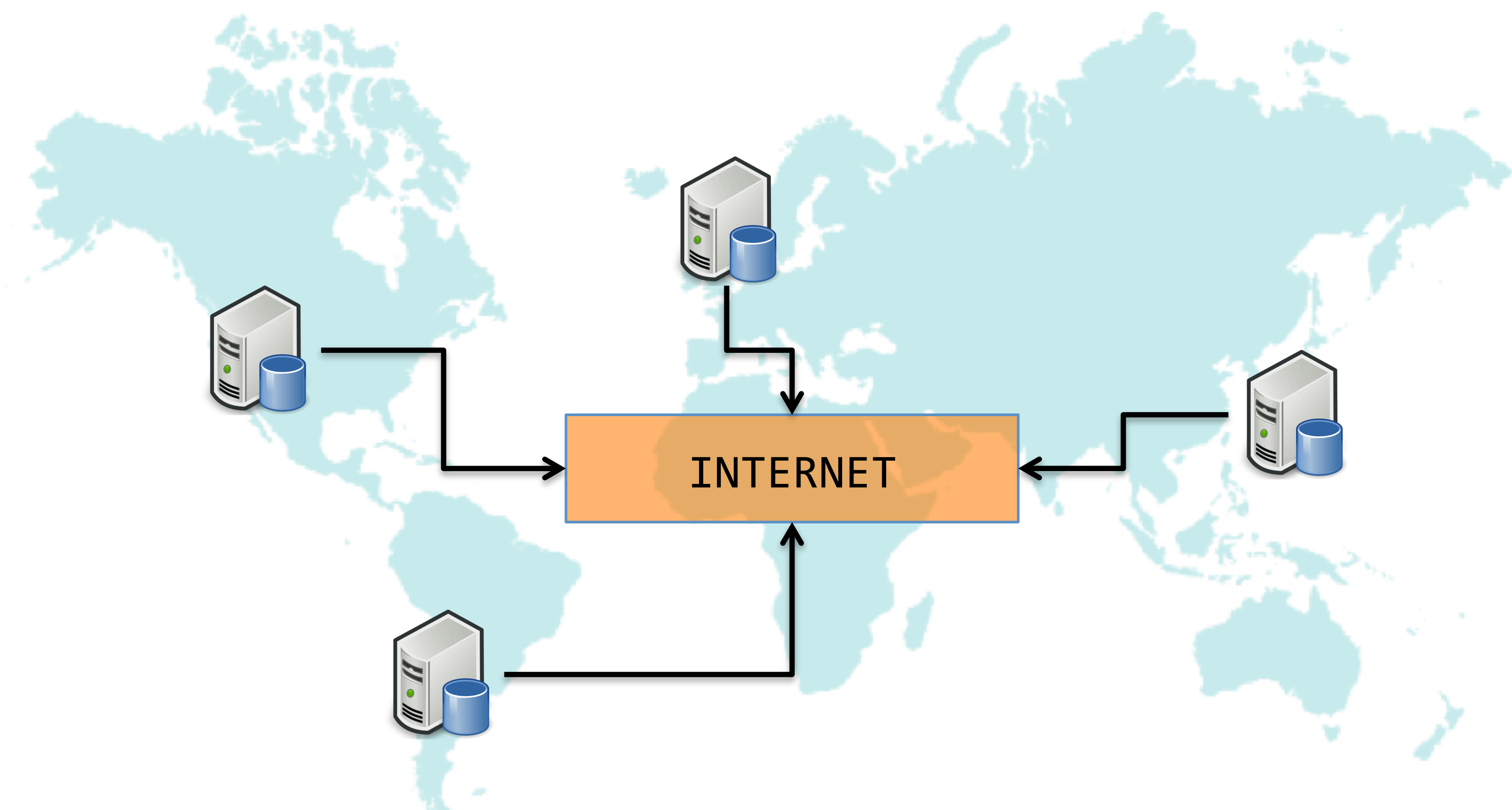
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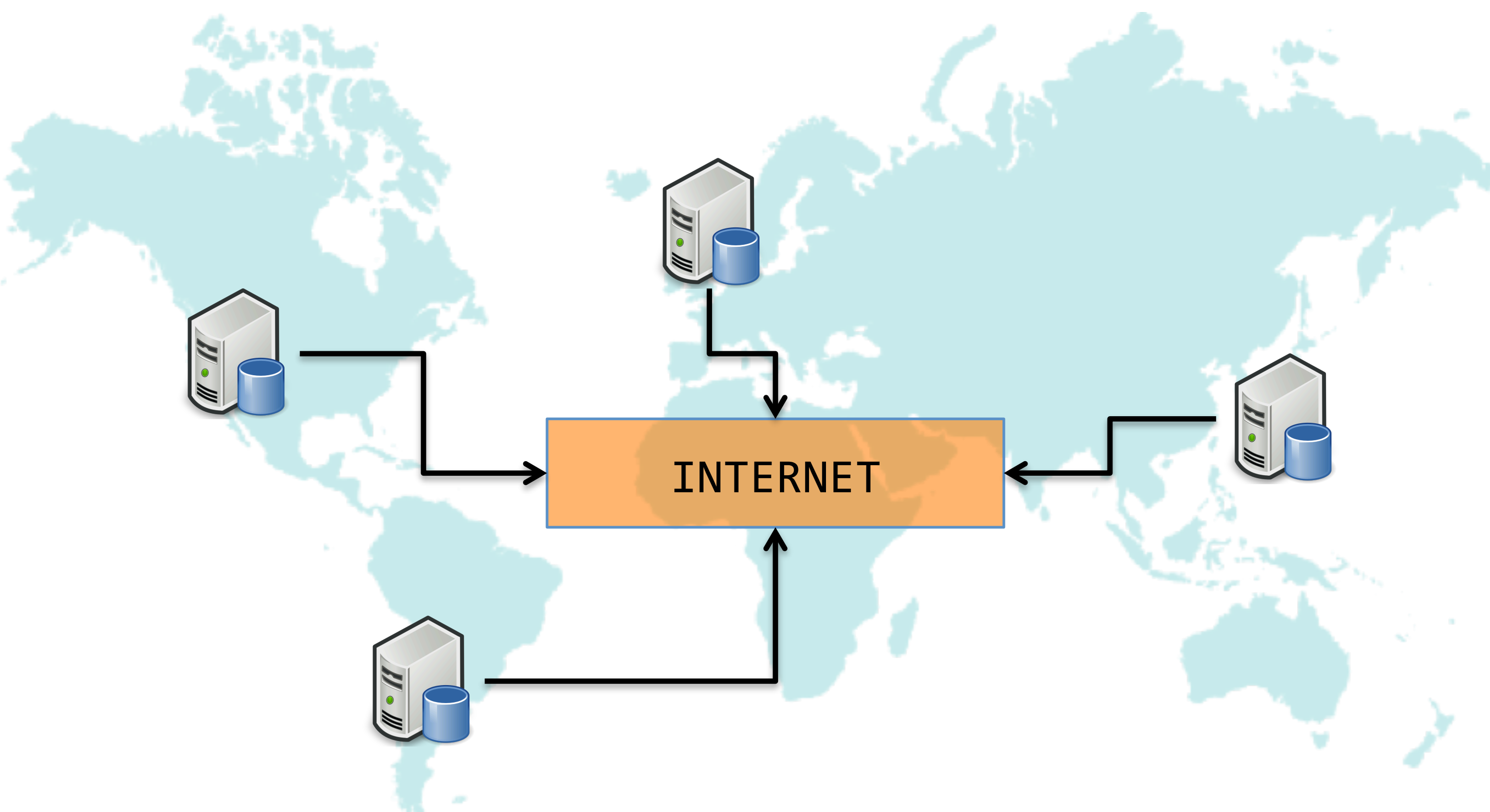

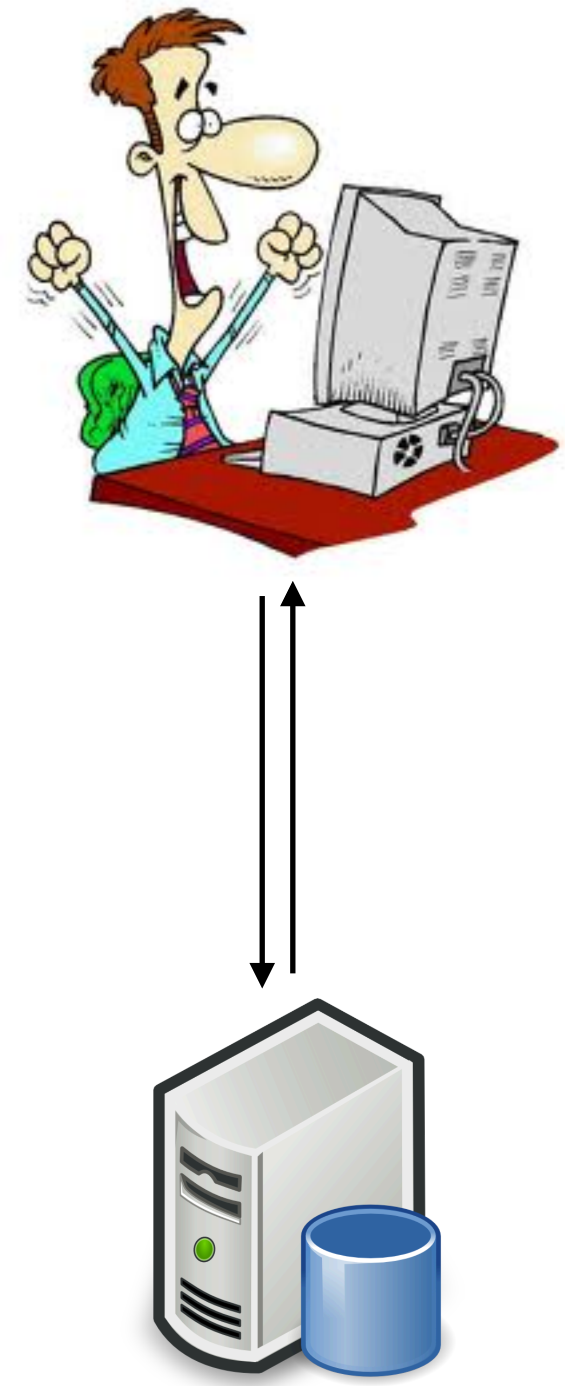

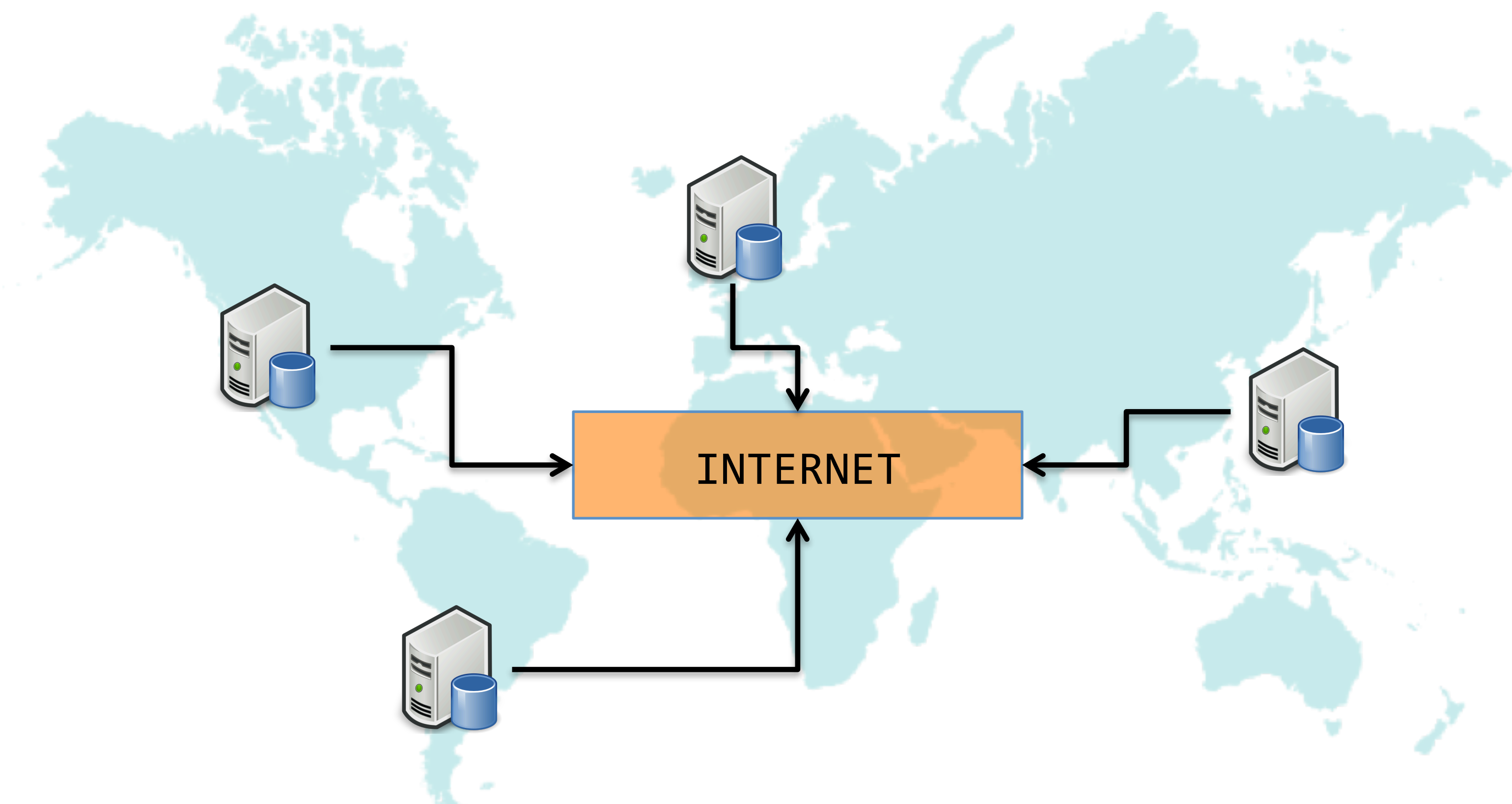

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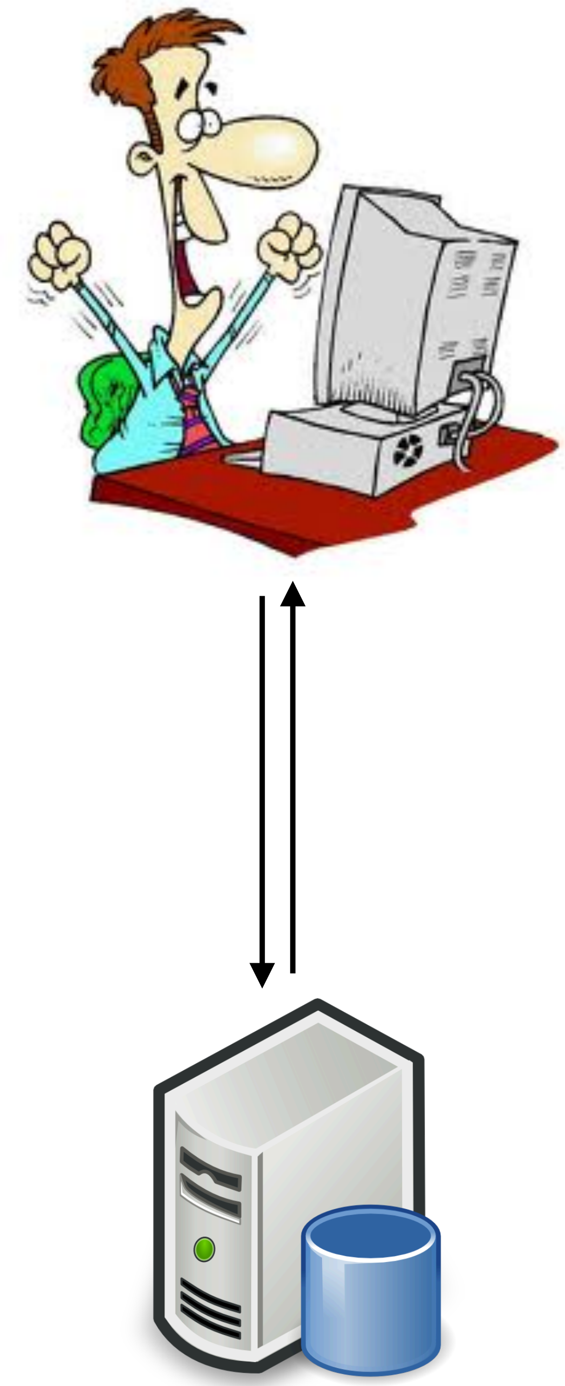

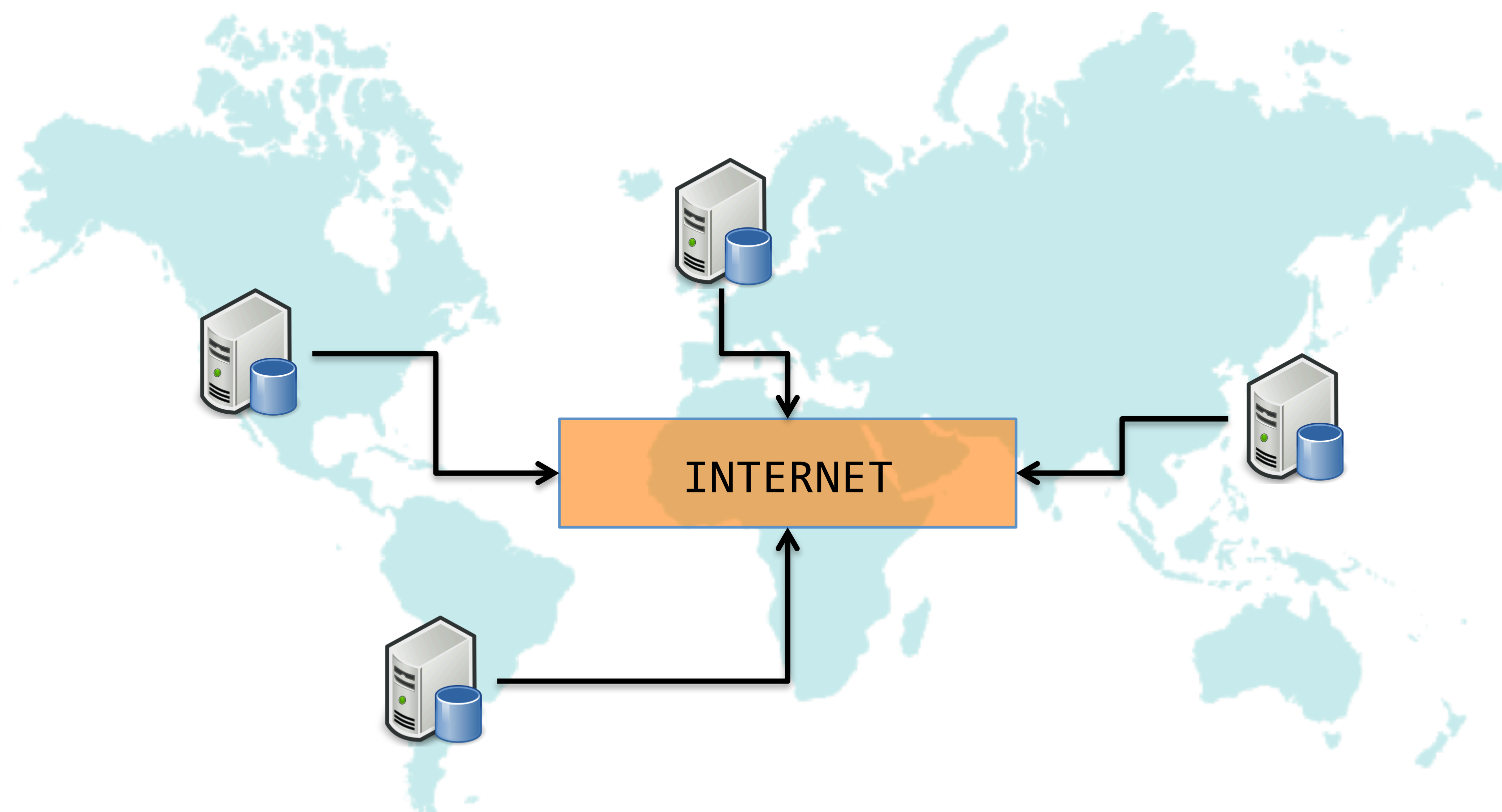

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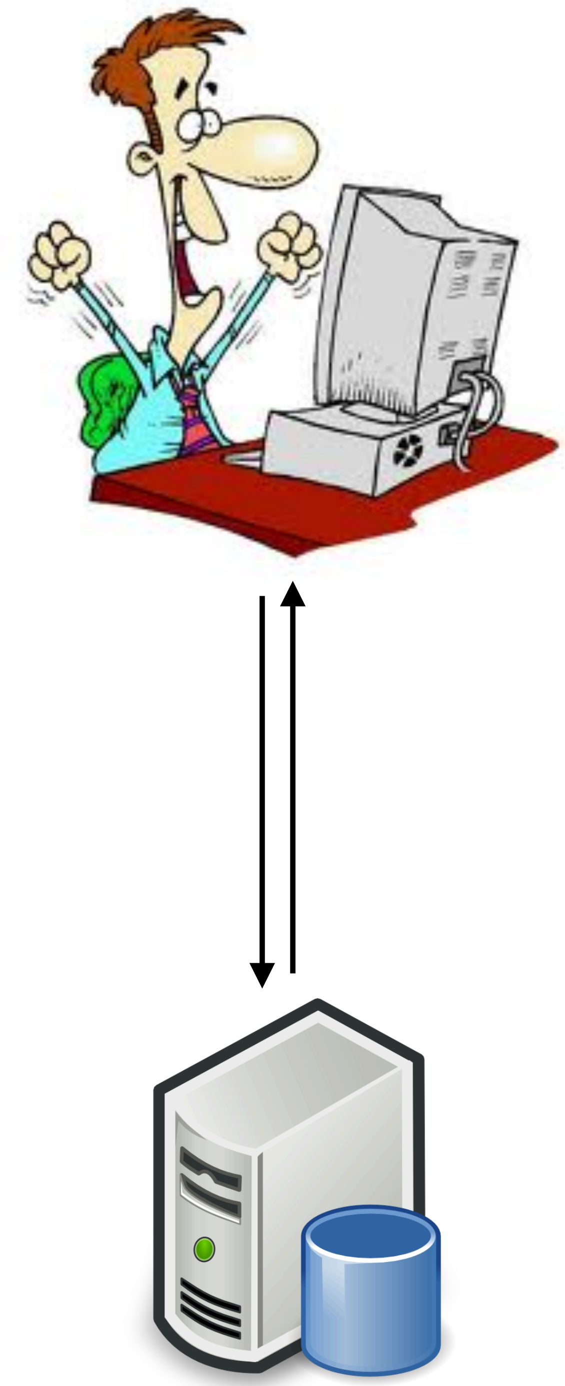

- Serializability
- Linearizability

- Weak Consistency & Isolation

When *system-level concerns* like replication & availability affect *application-level design* decisions, programming becomes *complicated*.

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@natefoster

Seems like Twitter wants me to follow this guy.

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Sequential Counter

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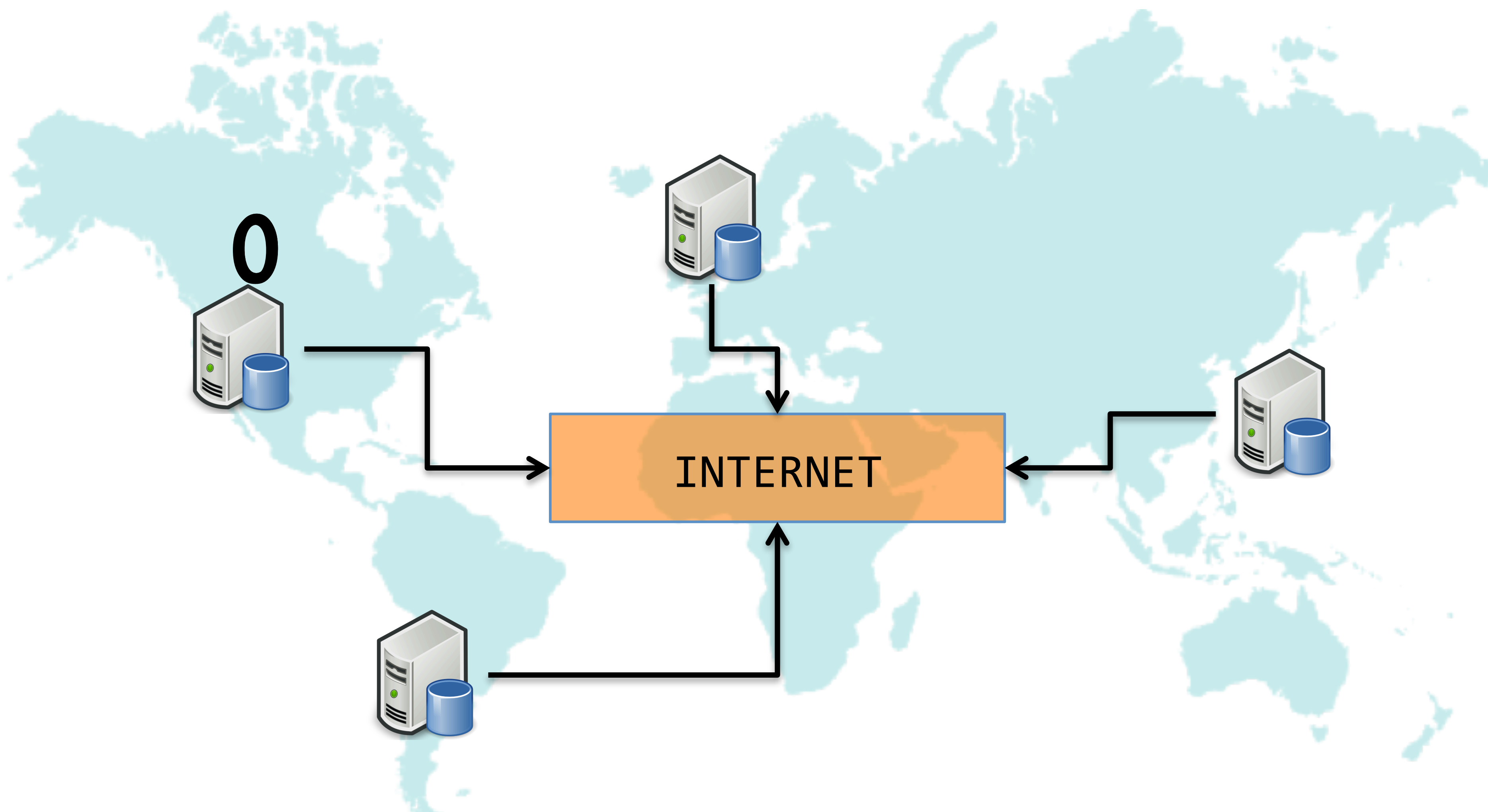
- Written in idiomatic style
- Composable

```
type counter_list = Counter.t list
```

Replicated Counter



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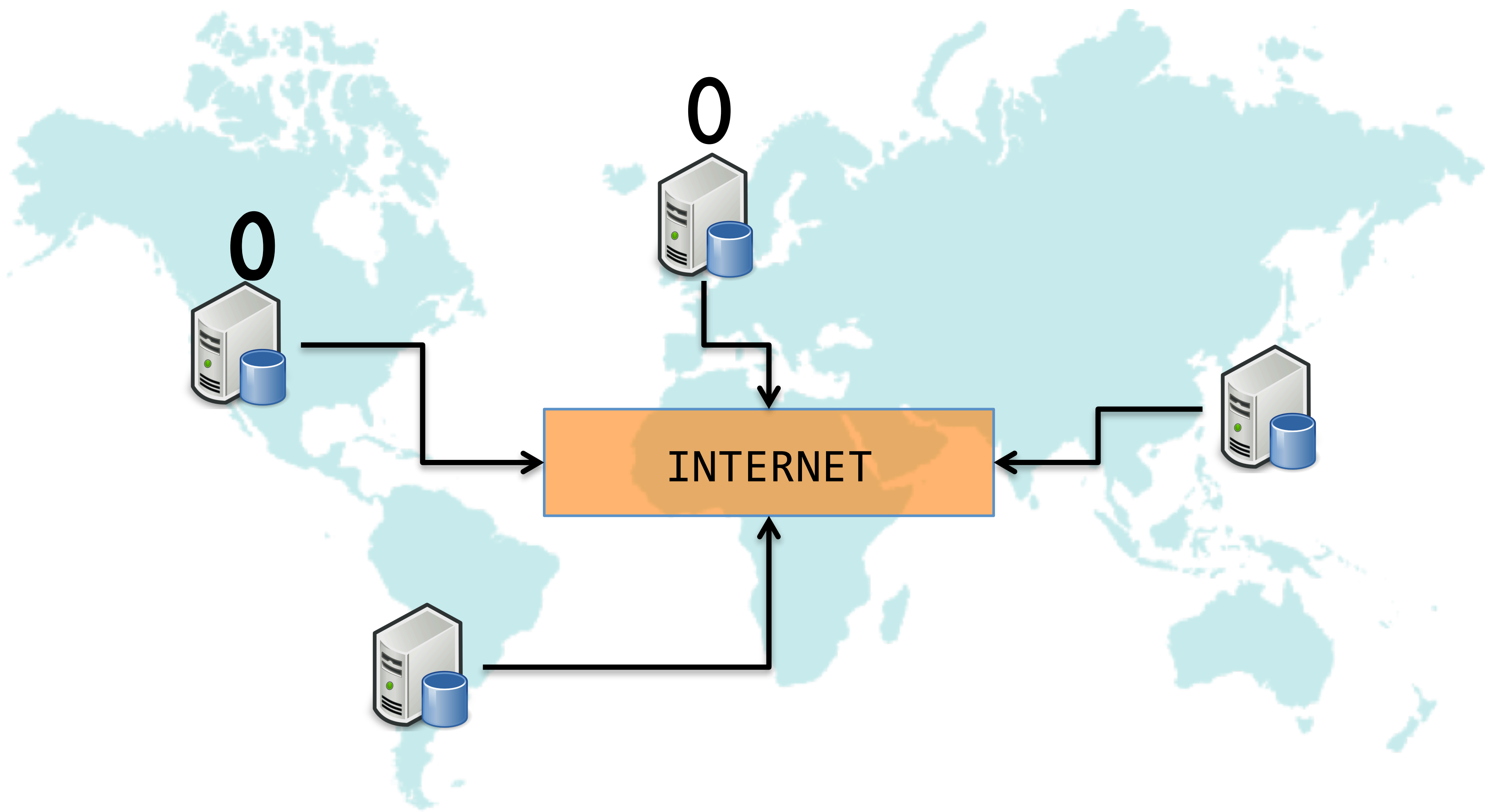

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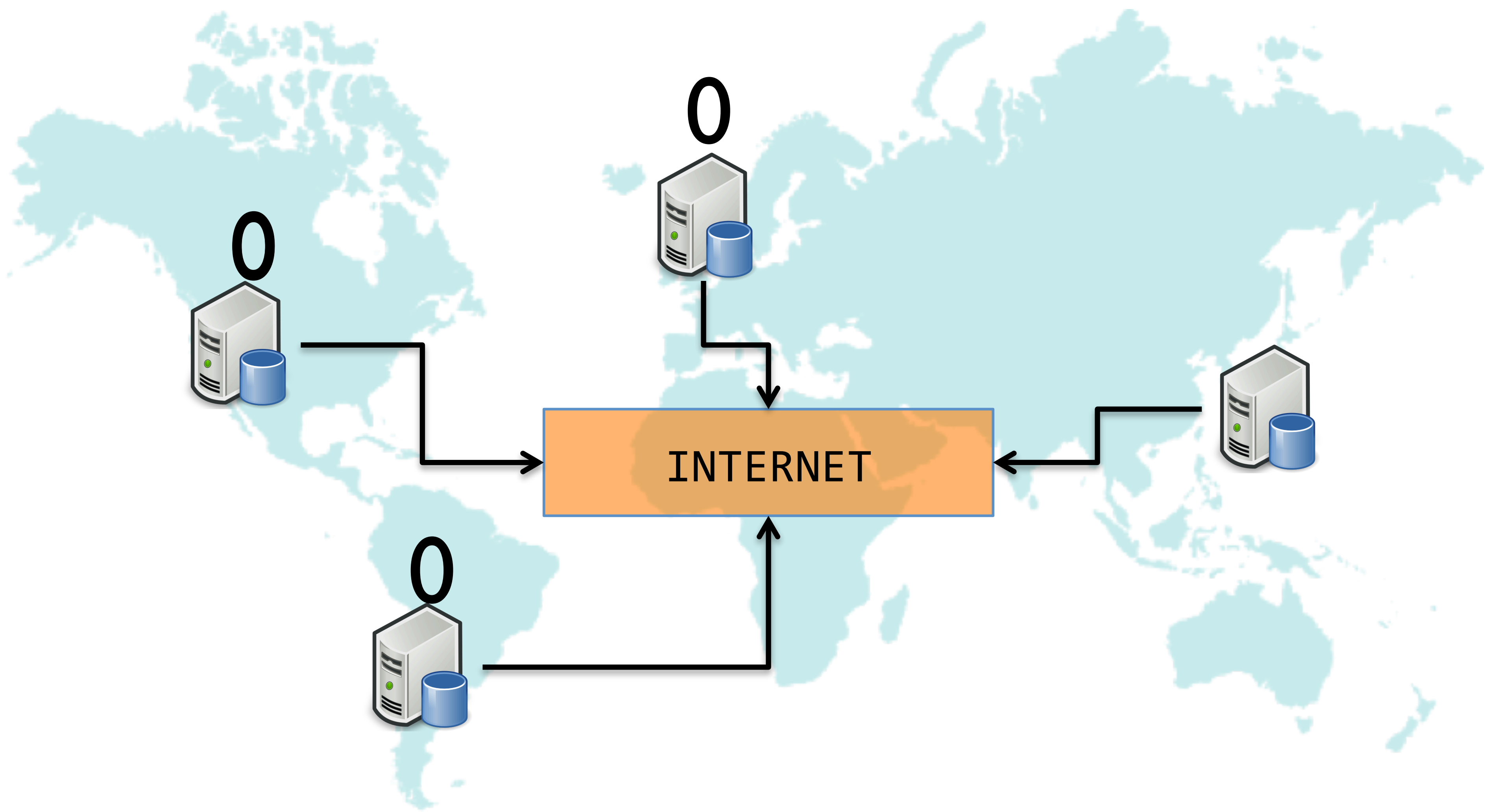

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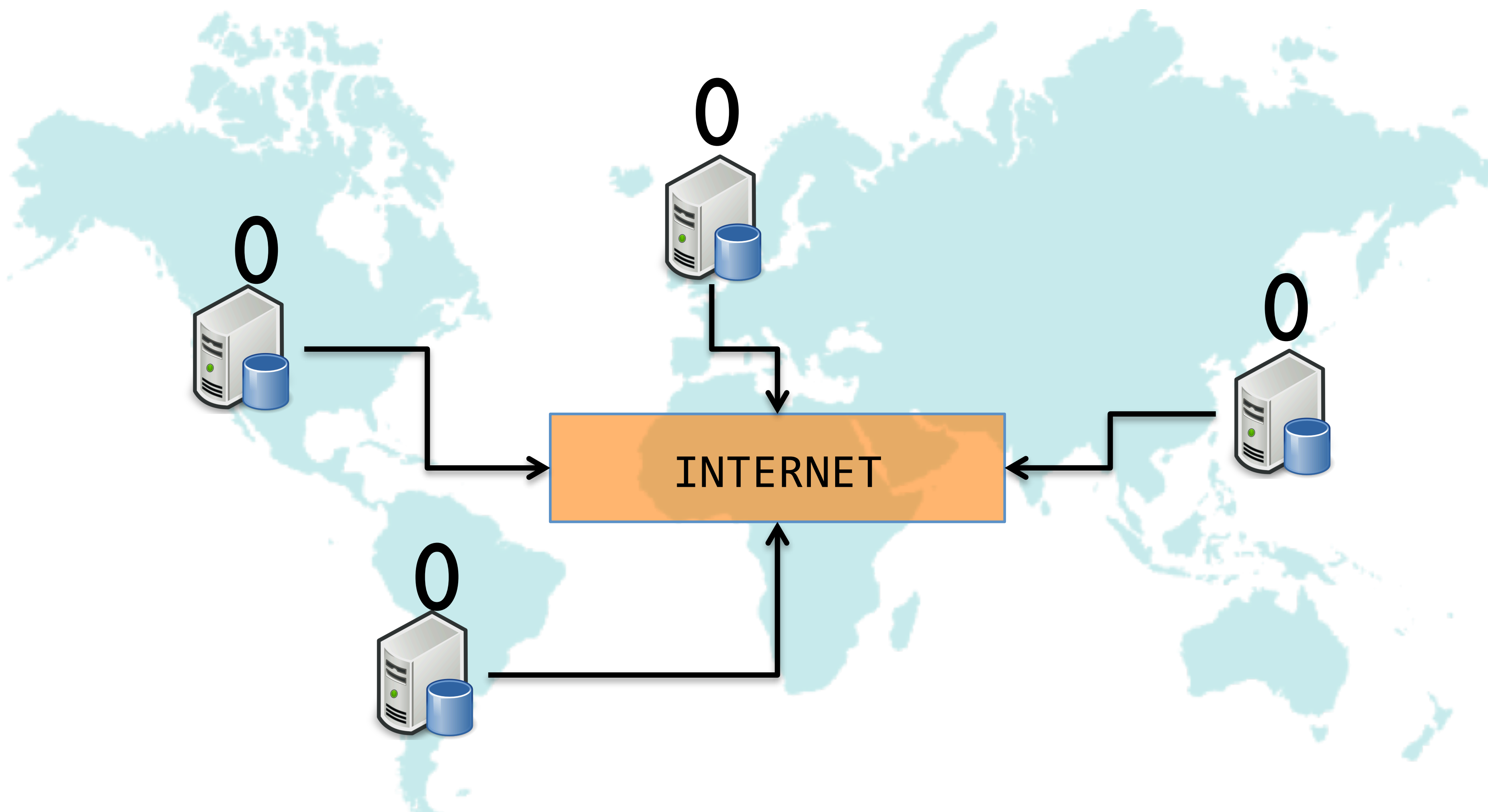

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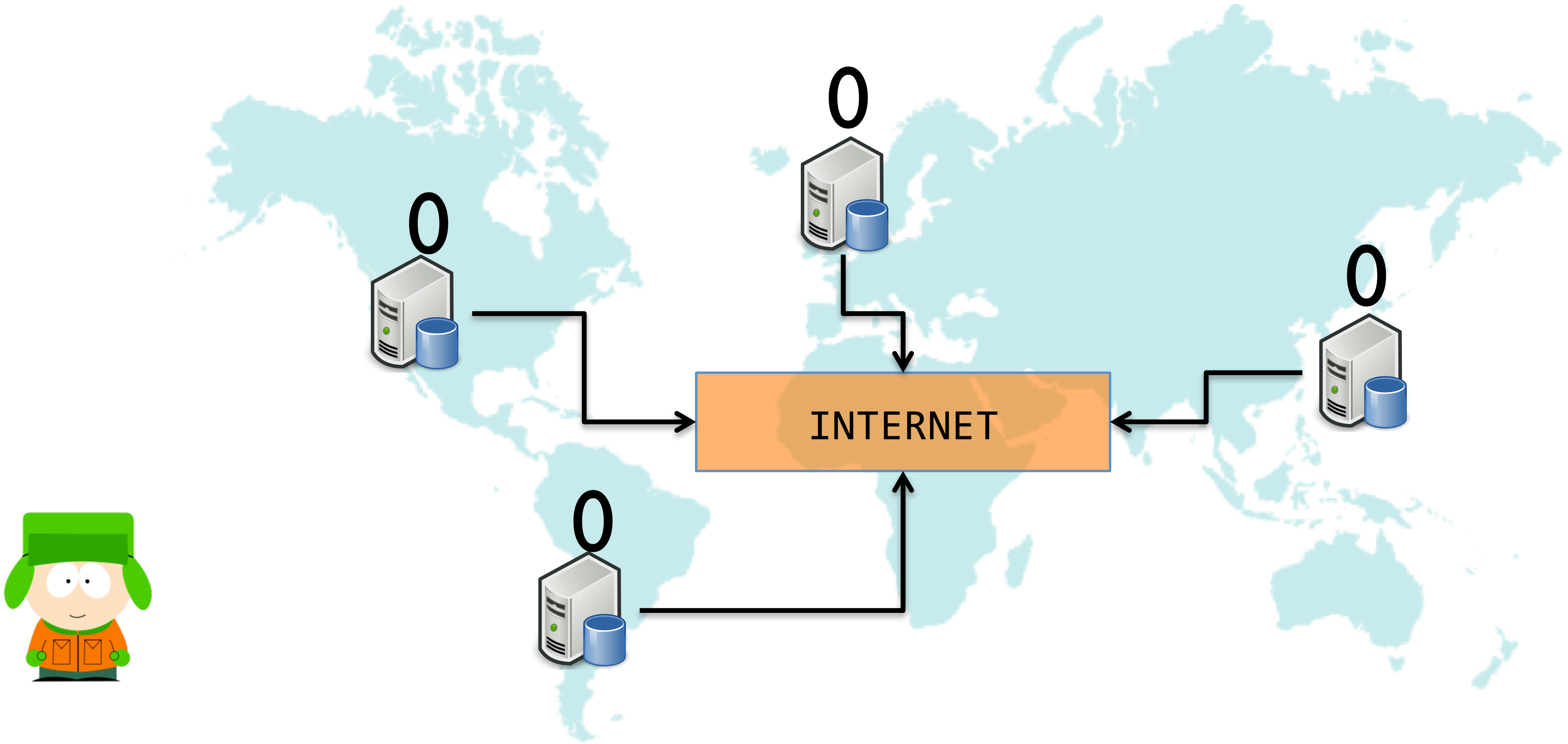

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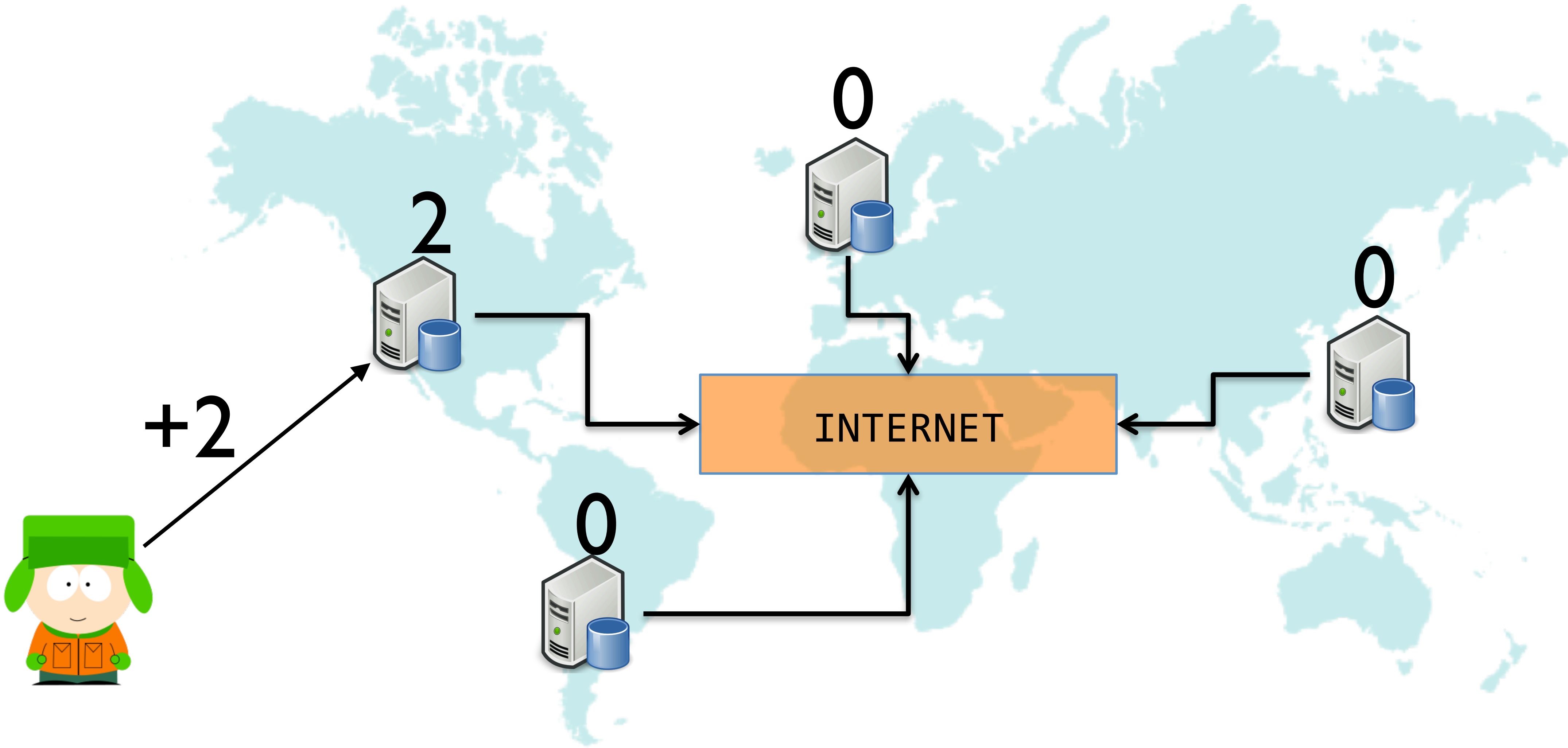

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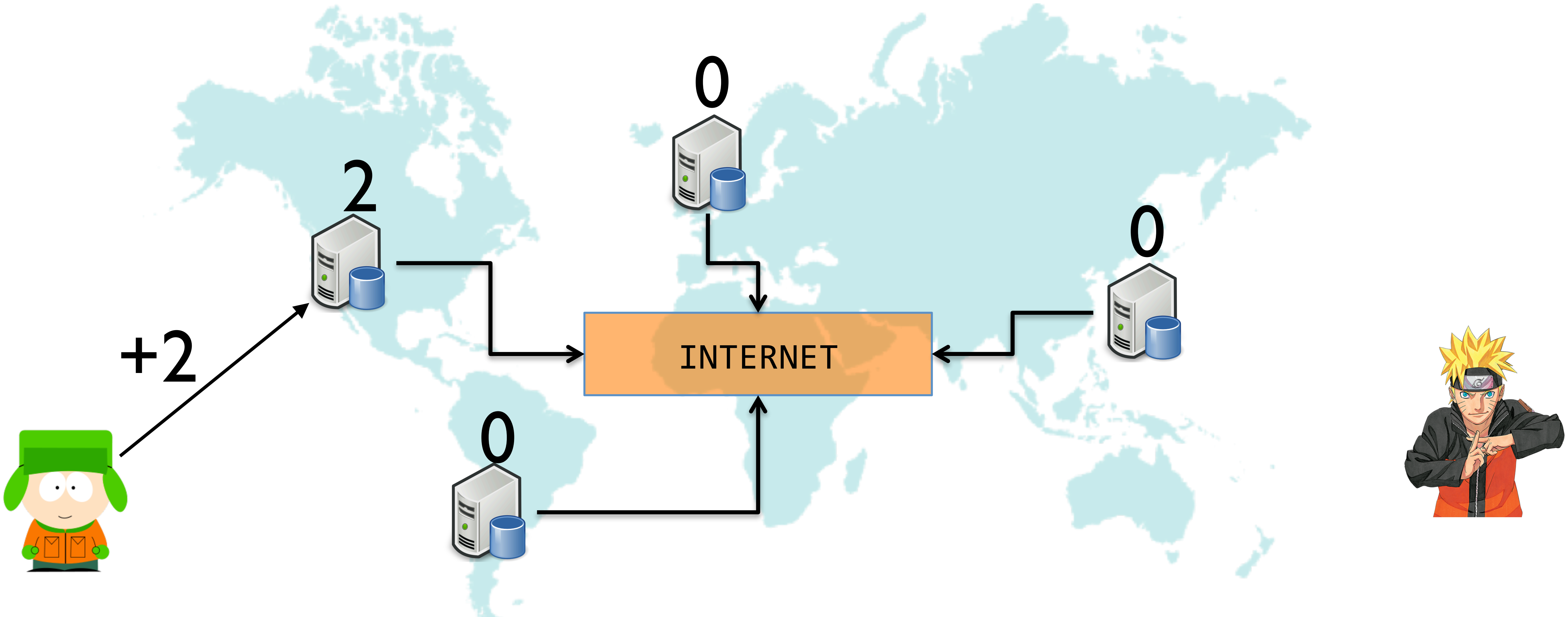

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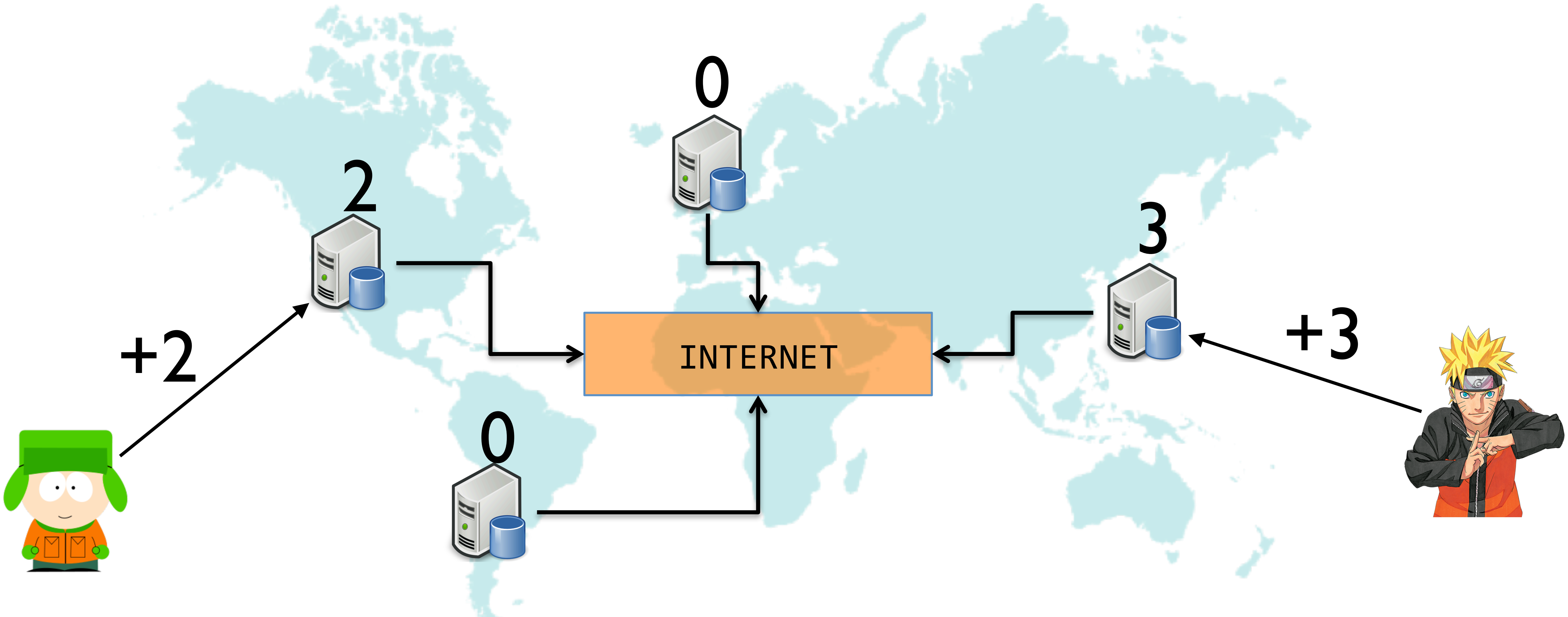

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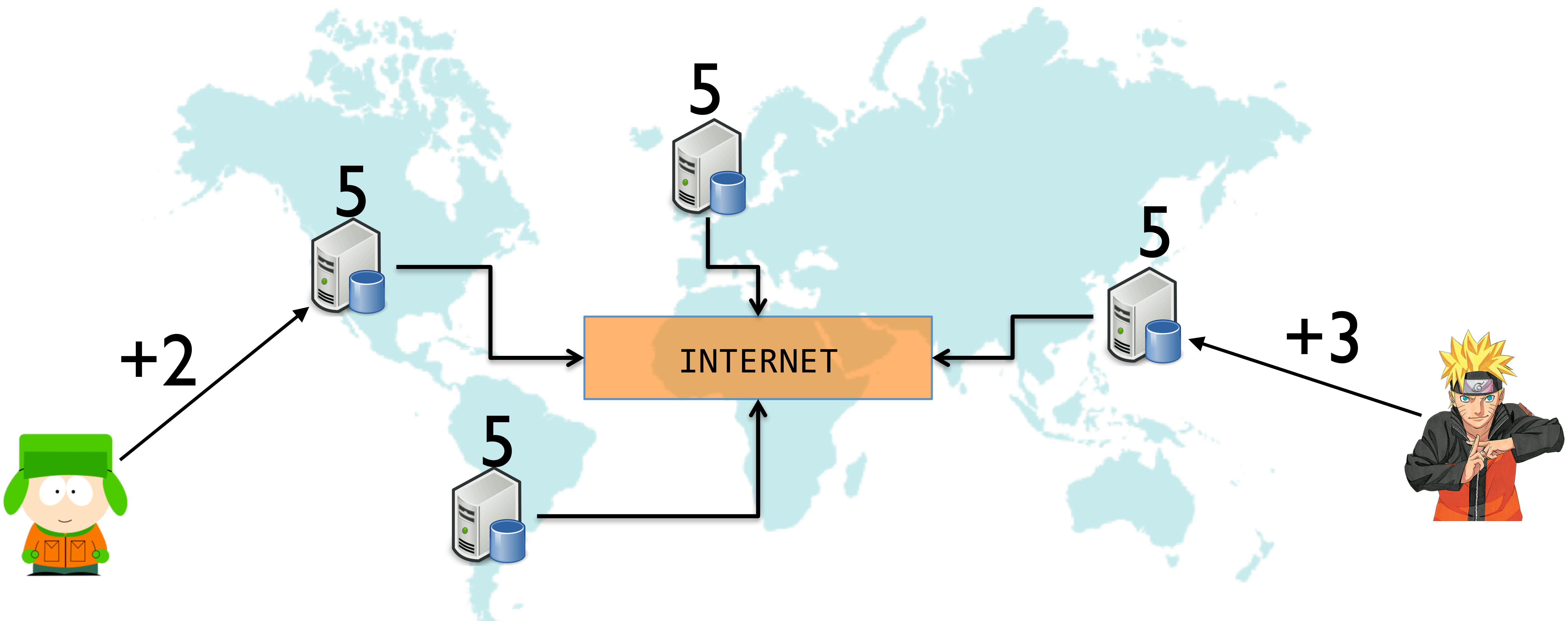

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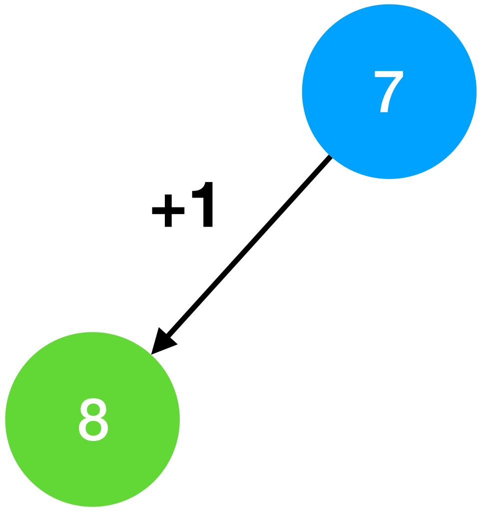

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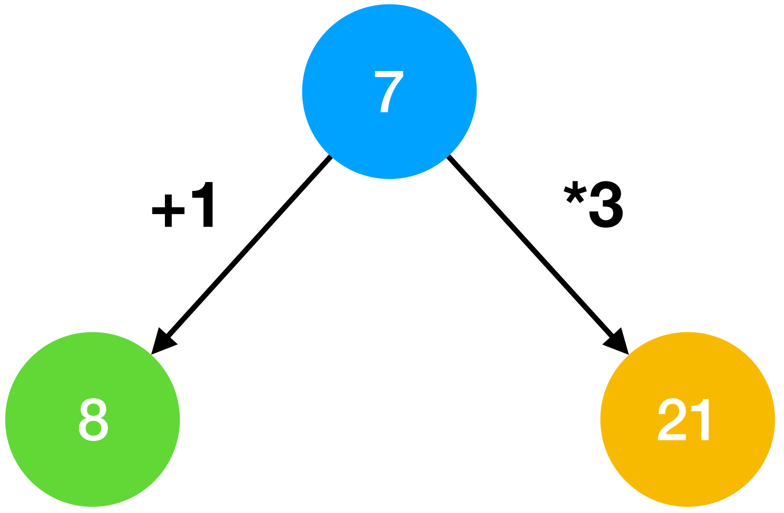

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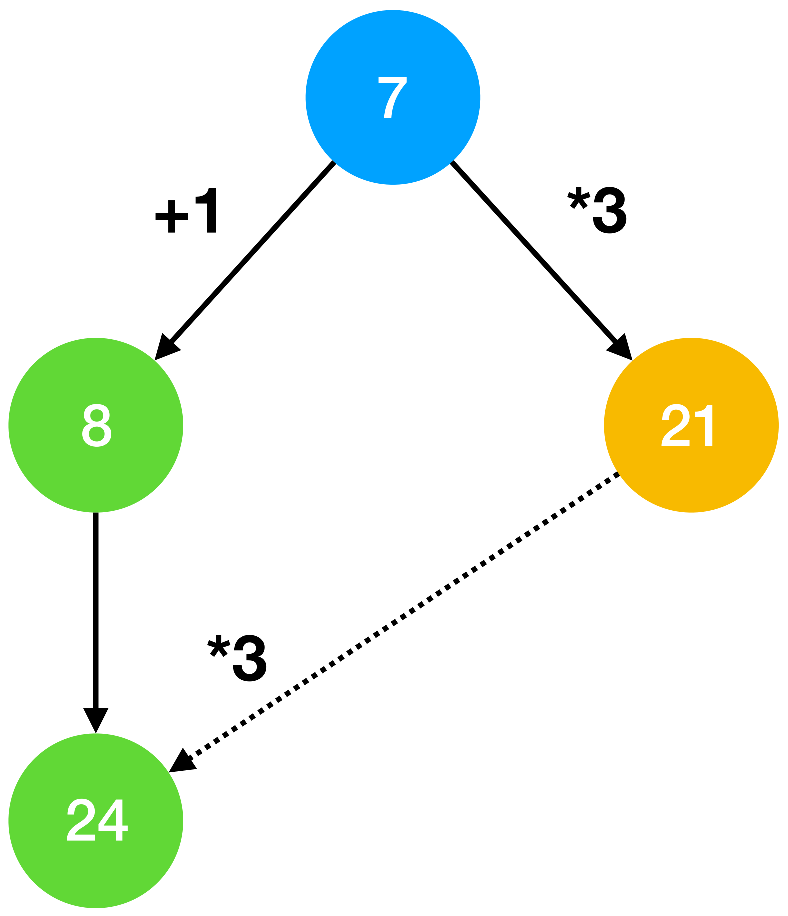

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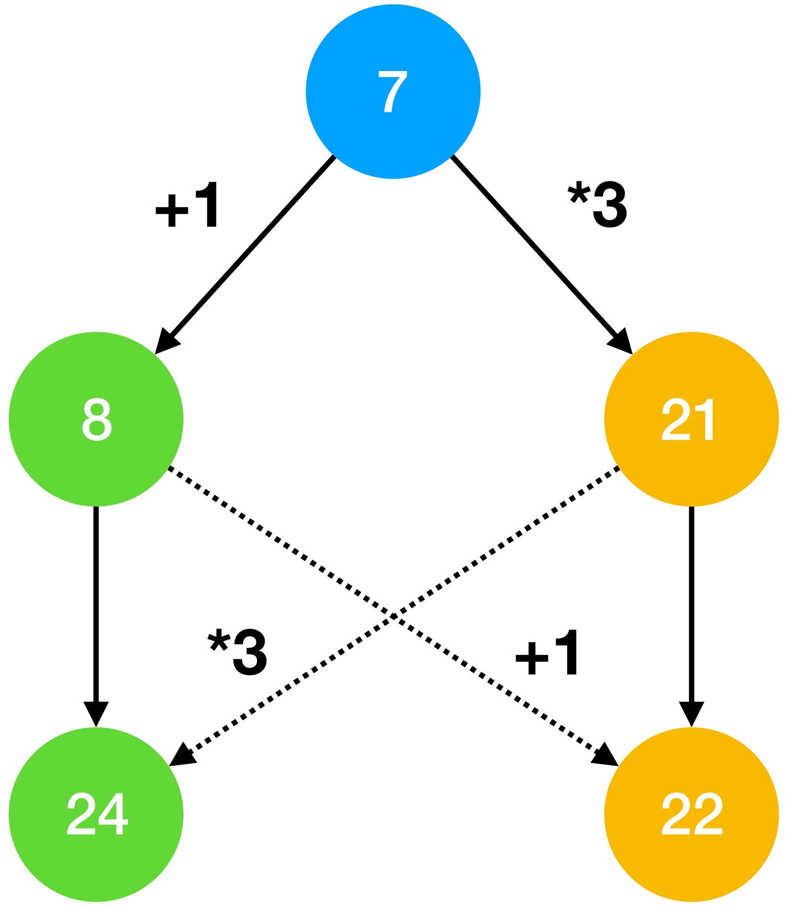

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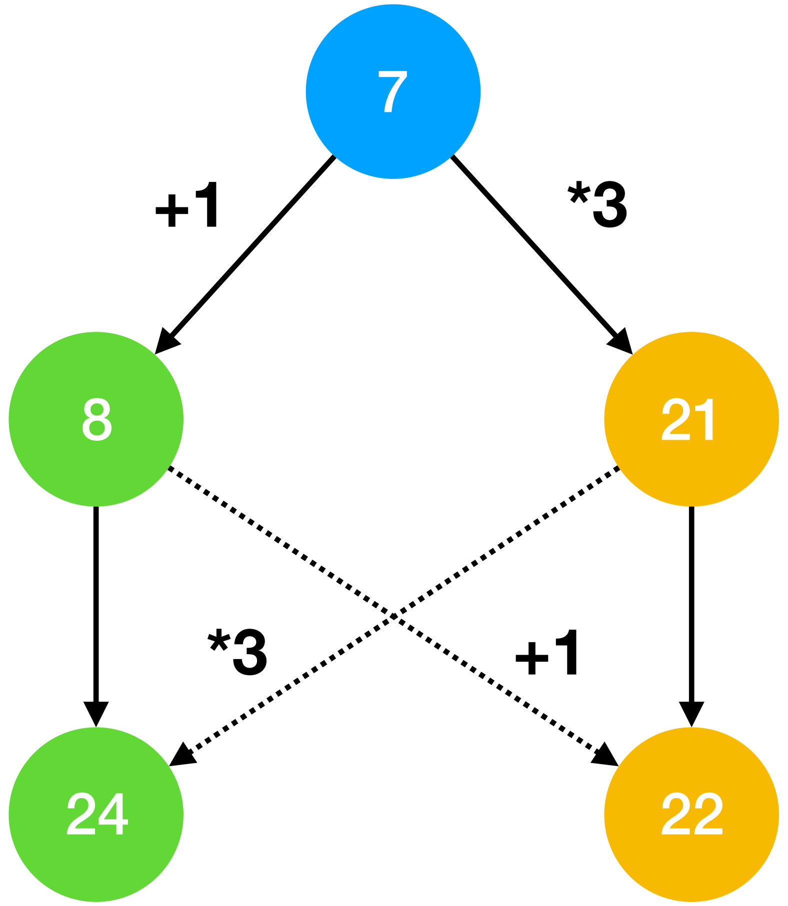

Diverges

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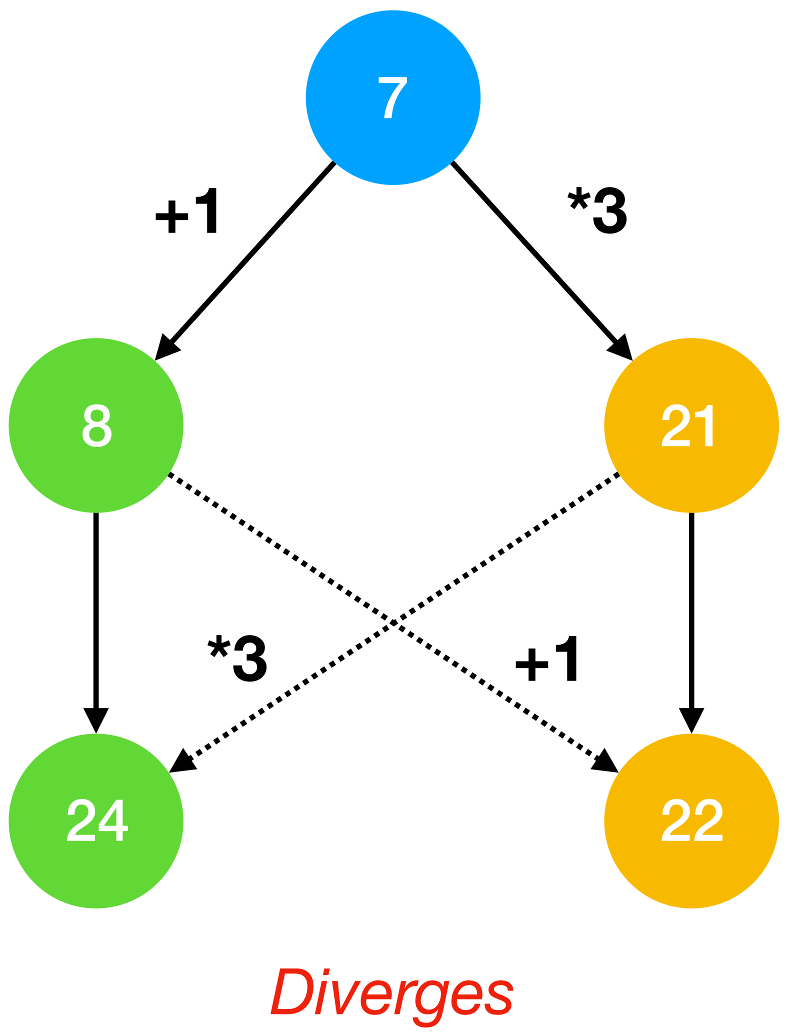

Addition and multiplication do not commute

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- Capture the effect of multiplication through the commutative addition operation

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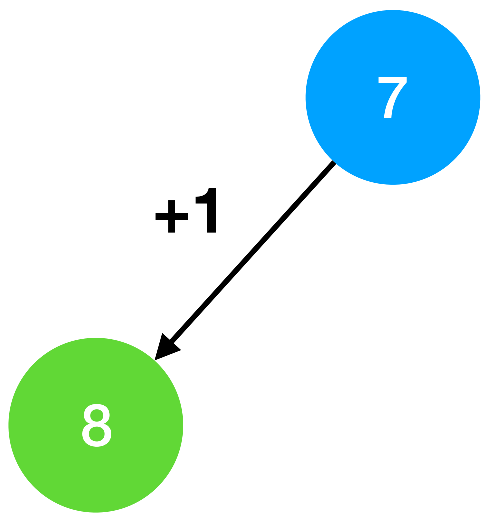

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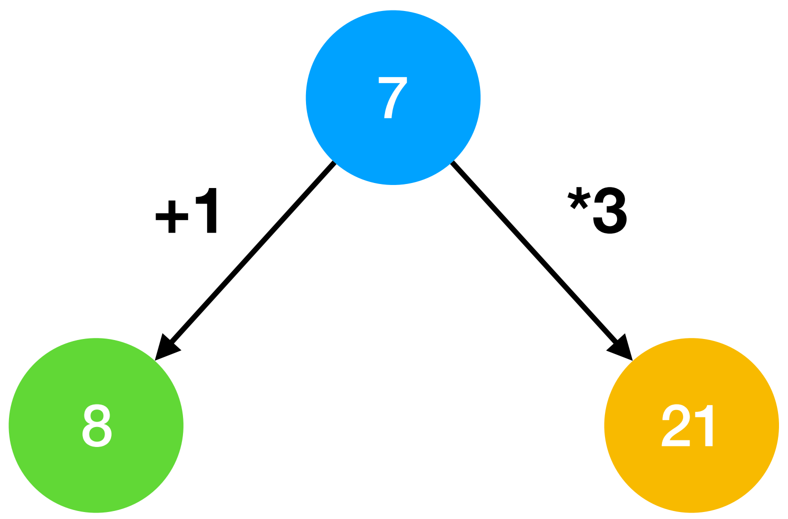

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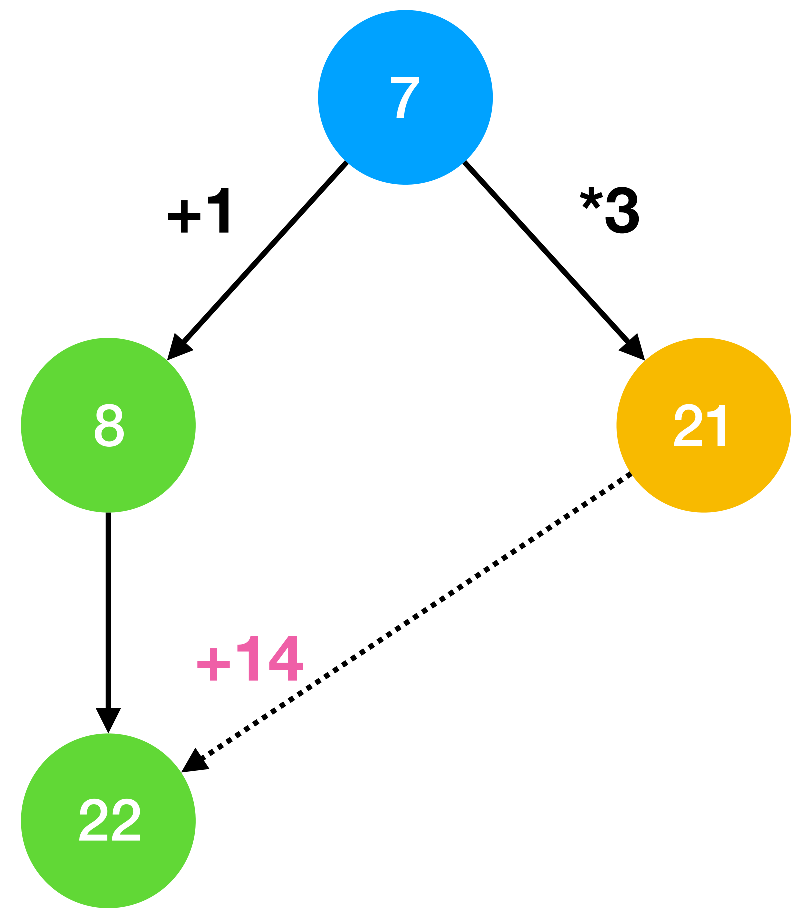

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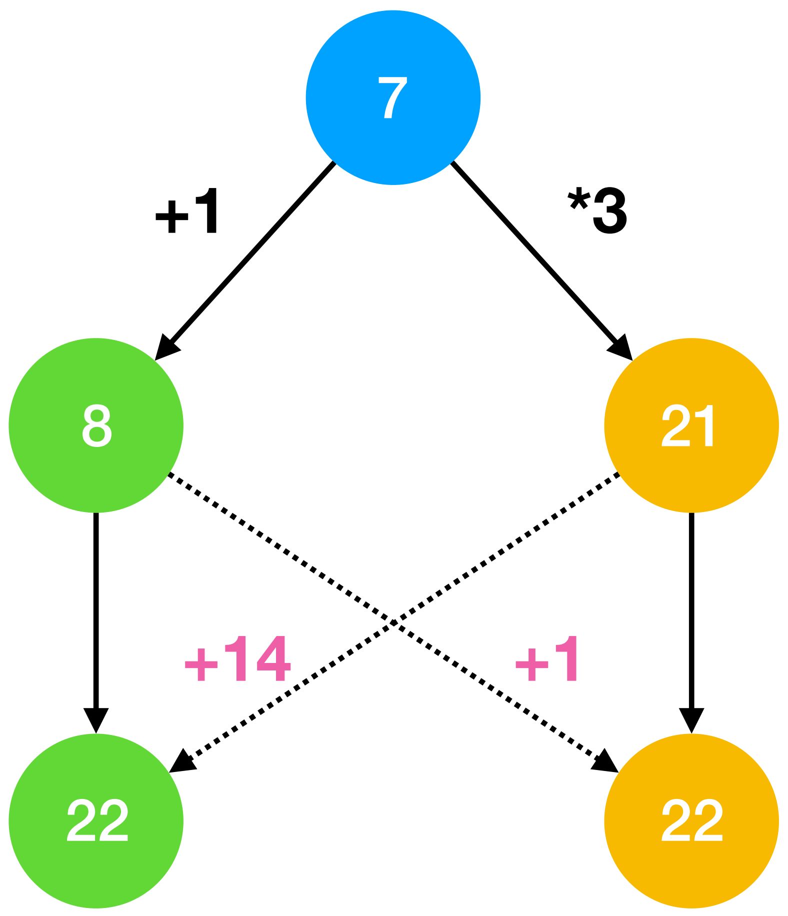

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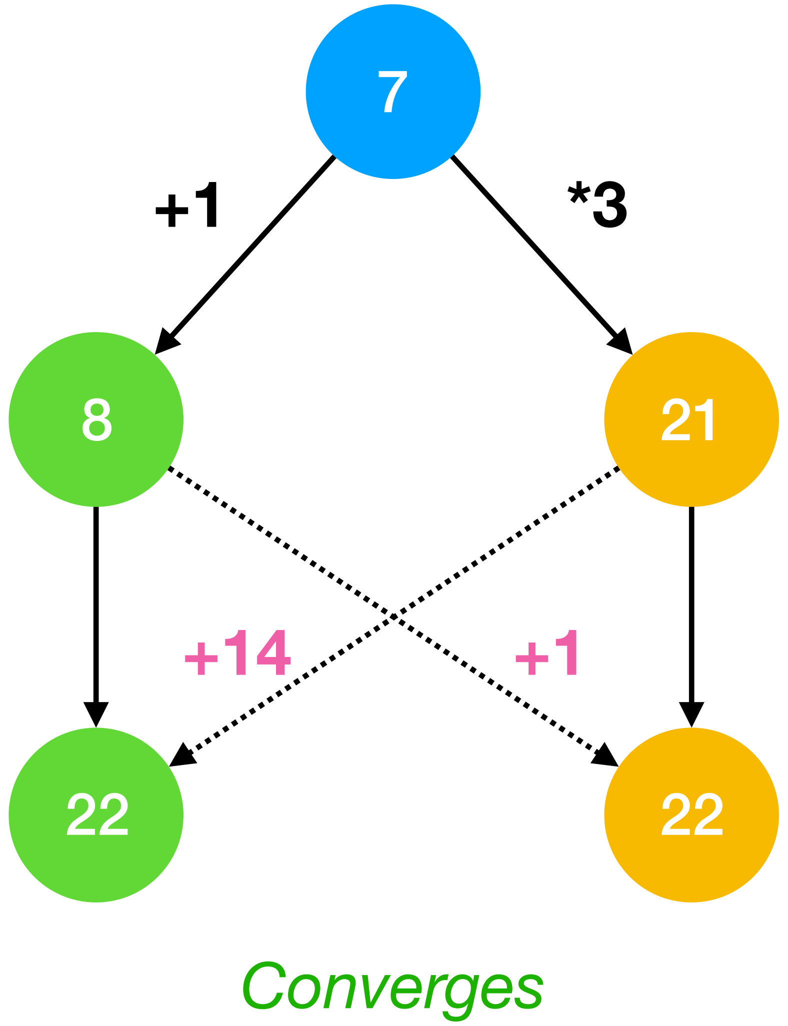

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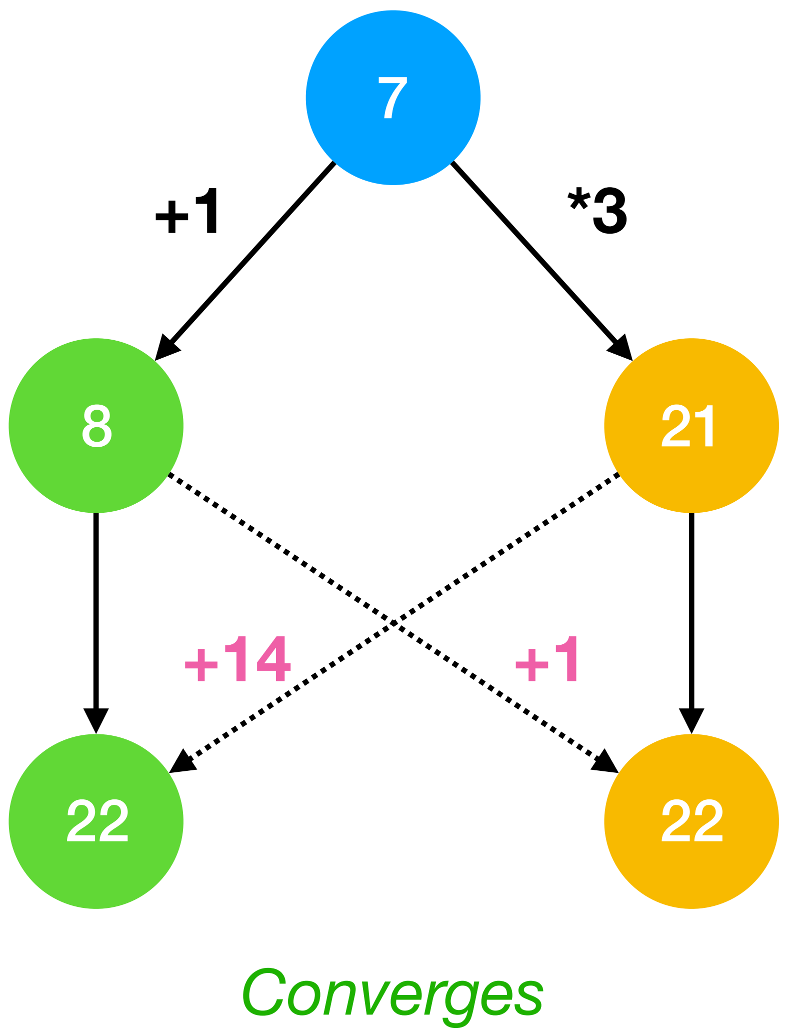

- Capture the effect of multiplication through the commutative addition operation
- *CRDTs*

Conflict-free Replicated Data Types (CRDT)

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- CRDT is guaranteed to ensure *strong eventual consistency (SEC)*
  - ★ G-counters, PN-counters, OR-Sets, Graphs, Ropes, docs, sheets
  - ★ Simple interface for the clients of CRDTs

Conflict-free Replicated Data Types (CRDT)

- CRDT is guaranteed to ensure *strong eventual consistency (SEC)*
  - ★ G-counters, PN-counters, OR-Sets, Graphs, Ropes, docs, sheets
  - ★ Simple interface for the clients of CRDTs
- Need to reengineer every datatype to ensure SEC (commutativity)
  - ★ Do not mirror sequential counter parts => implementation & proof burden
  - ★ Do not compose!
    - ✦ `counter set` is not a composition of `counter` and `set` CRDTs

Can we *program & reason about* replicated data types as an extension of their sequential counterparts?

```ocaml
module Counter : sig
  type t
  val read : t -> int
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  val mult : t -> int -> t
  val merge : lca:t -> v1:t -> v2:t -> t
end = struct
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  let merge ~lca ~v1 ~v2 =
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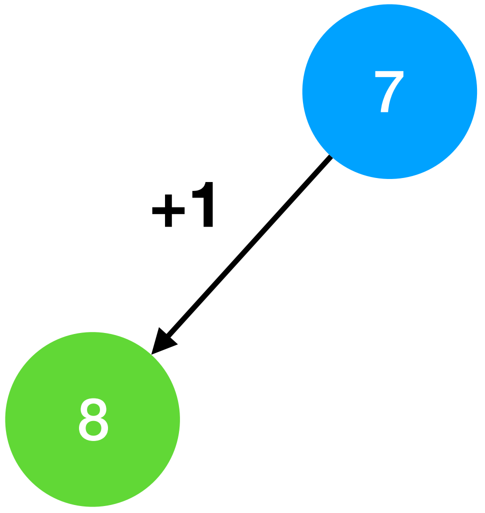

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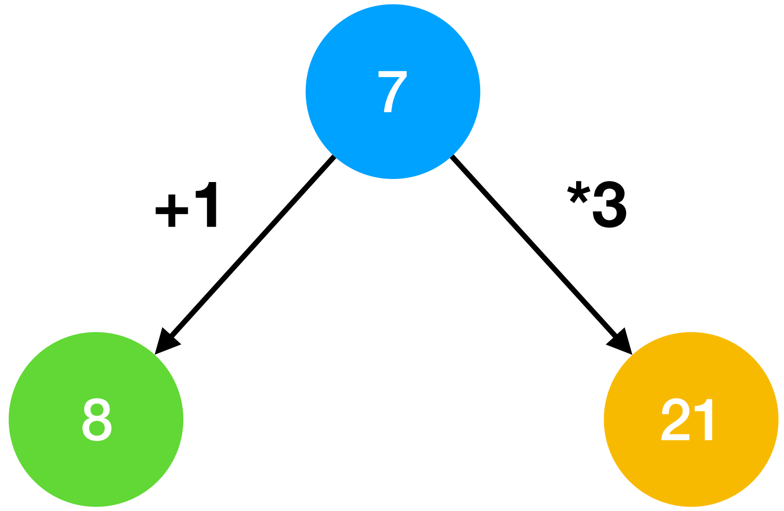

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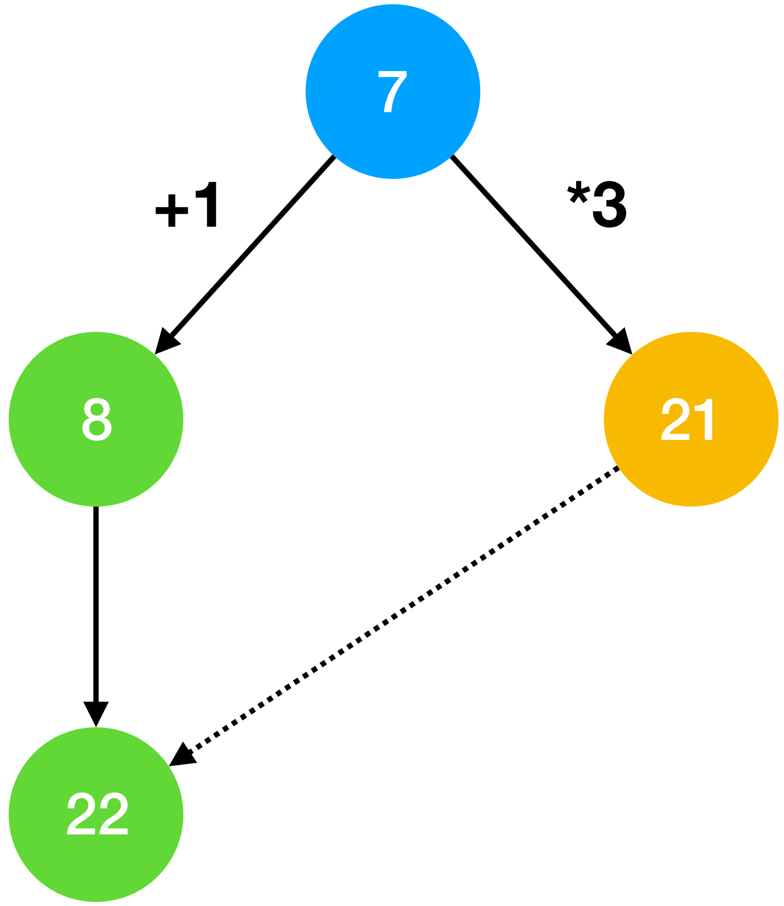

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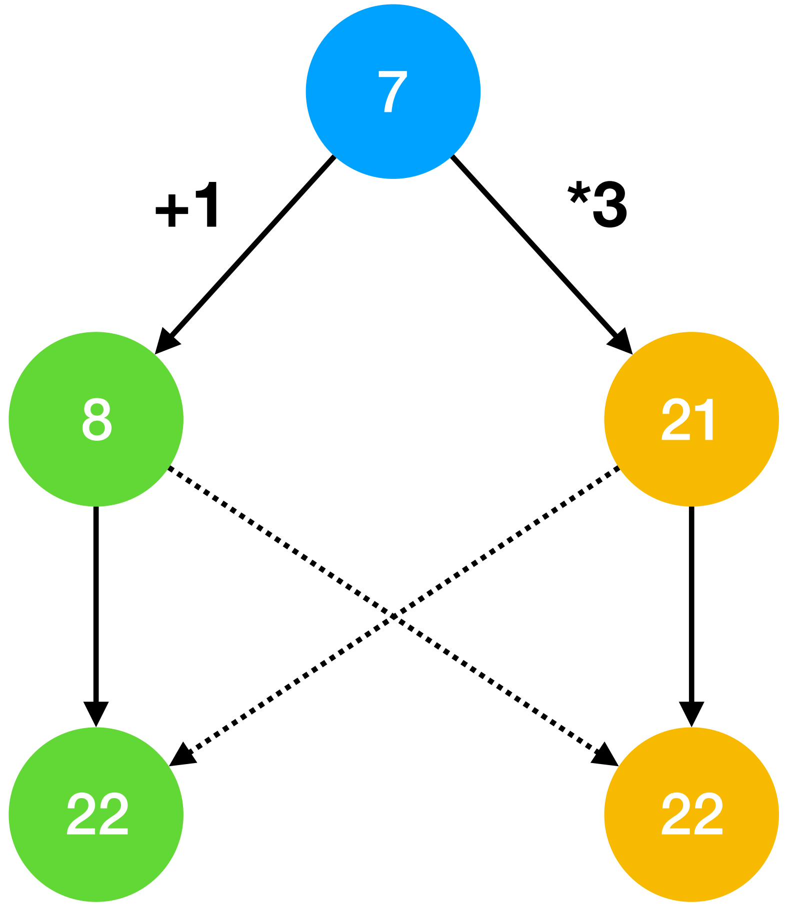

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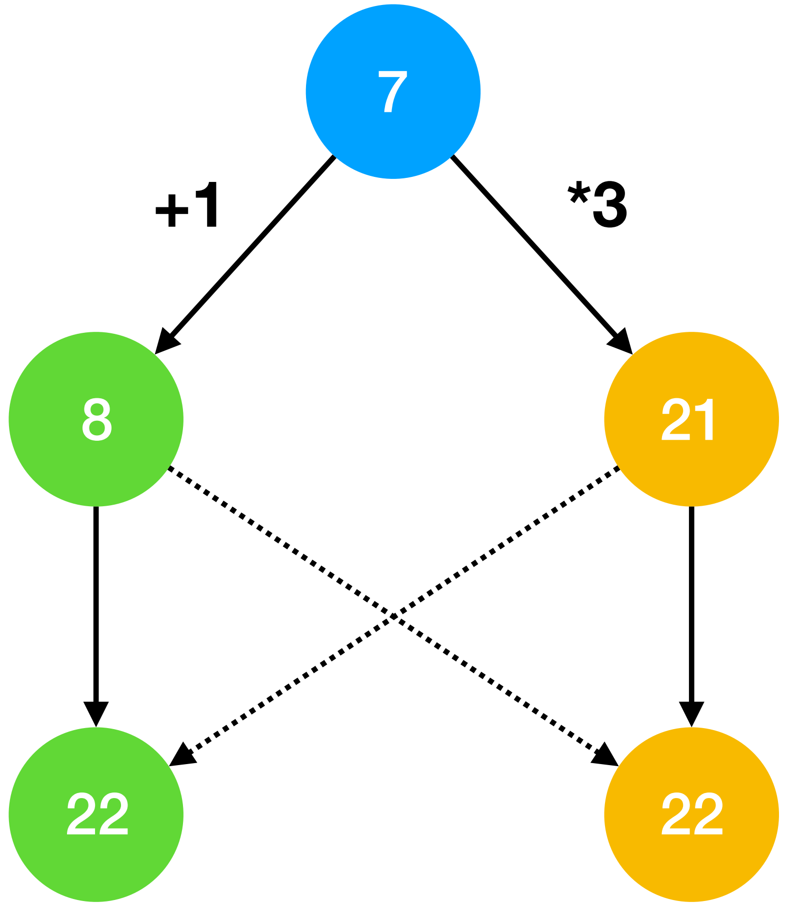

22 = 7 + (8-1) + (21 -7)

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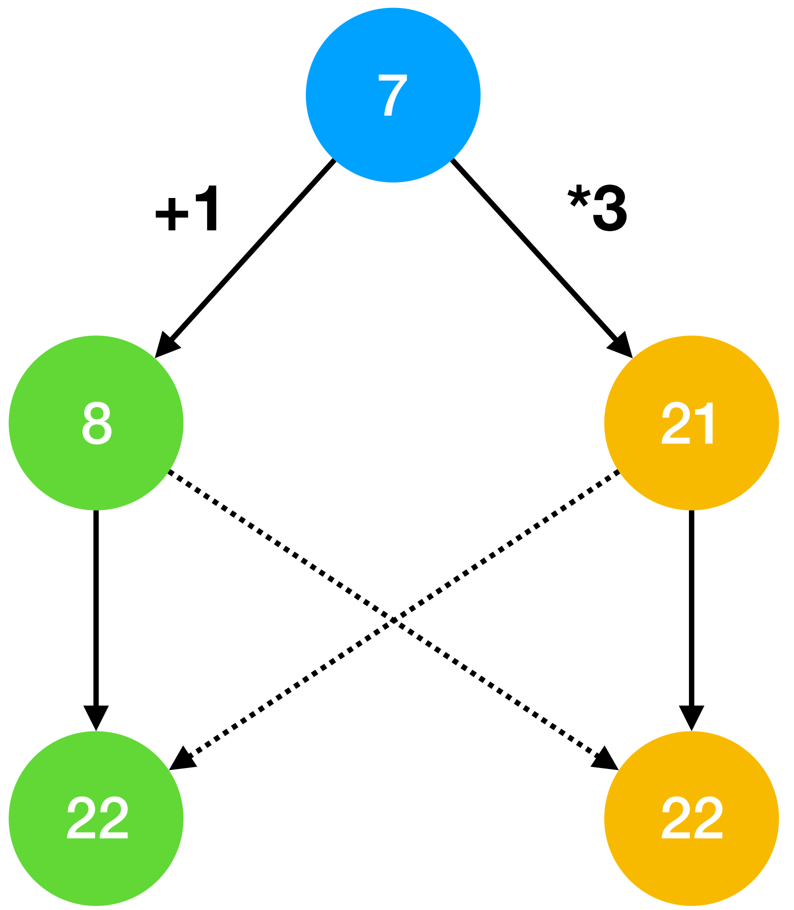

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- 3-way merge function makes the counter suitable for distribution

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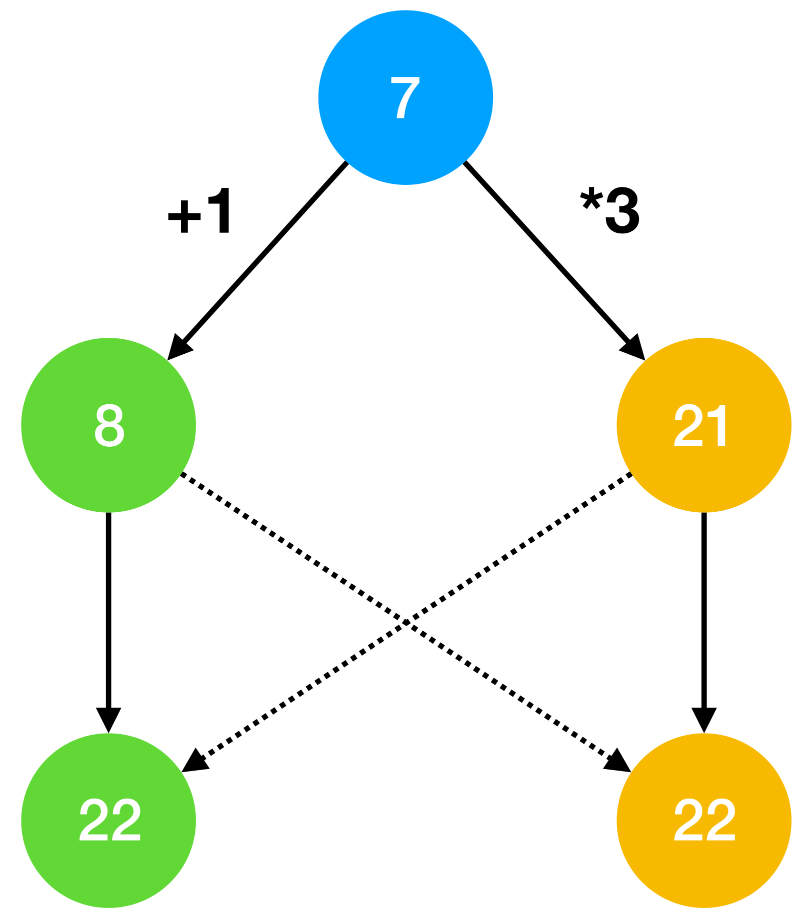
end
```



22 = 7 + (8-1) + (21 -7)

- 3-way merge function makes the counter suitable for distribution
- Does not appeal to individual operations => independently extend data-type

Systems → PL

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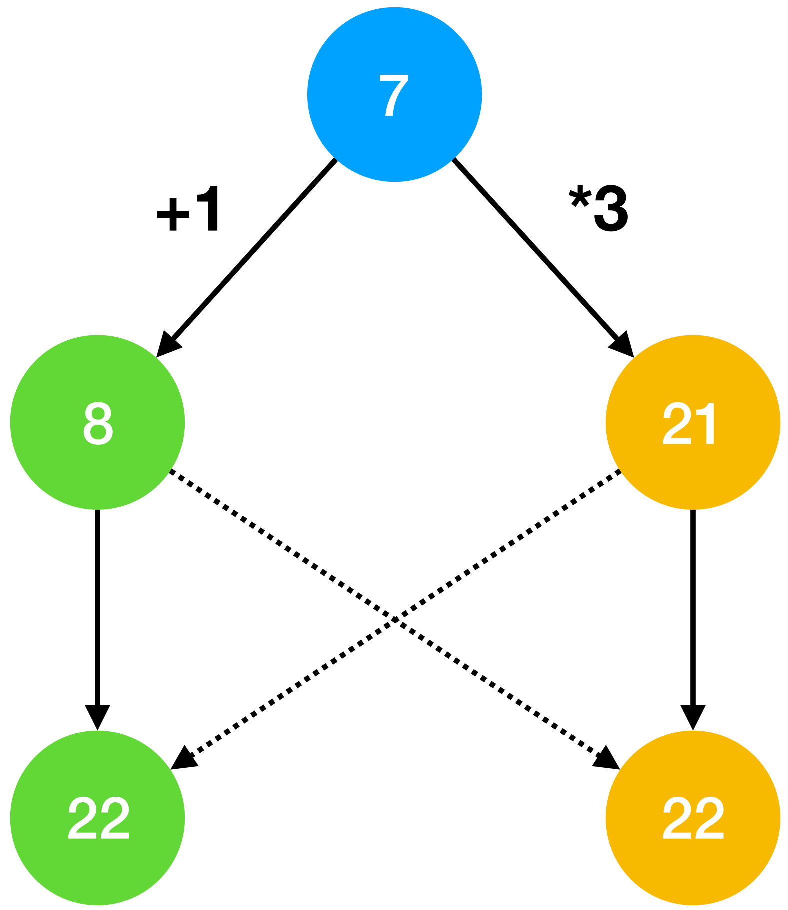

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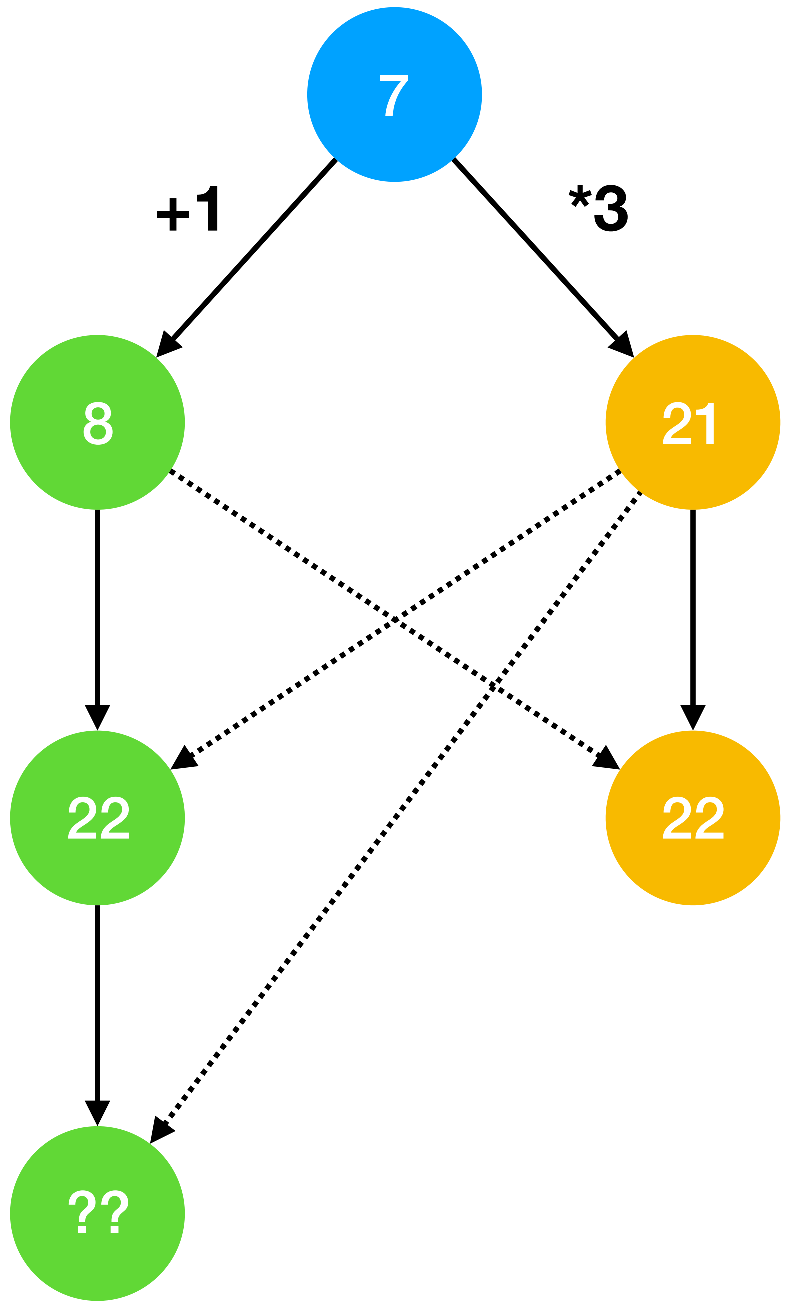

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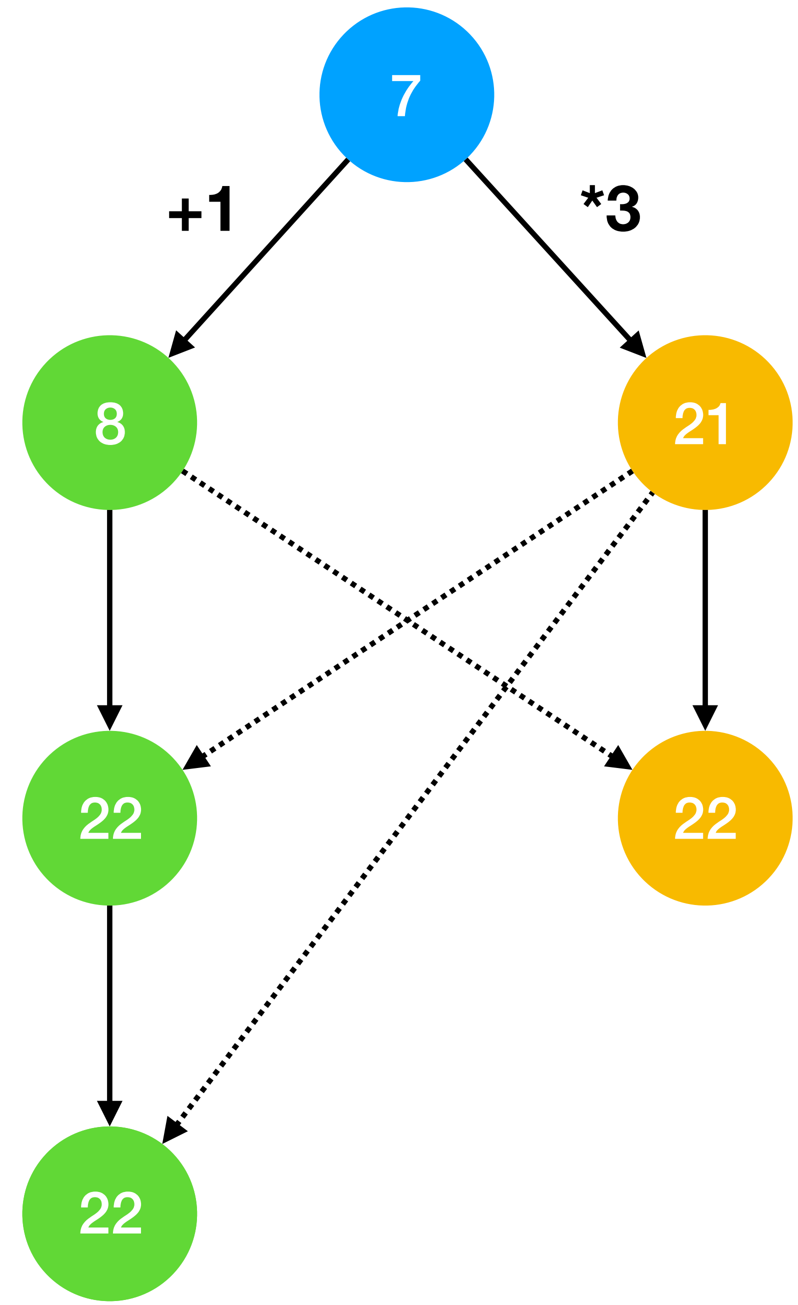

22 = 21 + (21-21) + (22 -21)

Does the 3-way merge idea generalise?

```
module type Queue = sig
  type 'a t
  val push : 'a t -> 'a -> 'a t
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- Try replicating queues by asynchronously transmitting operations

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[1,2]

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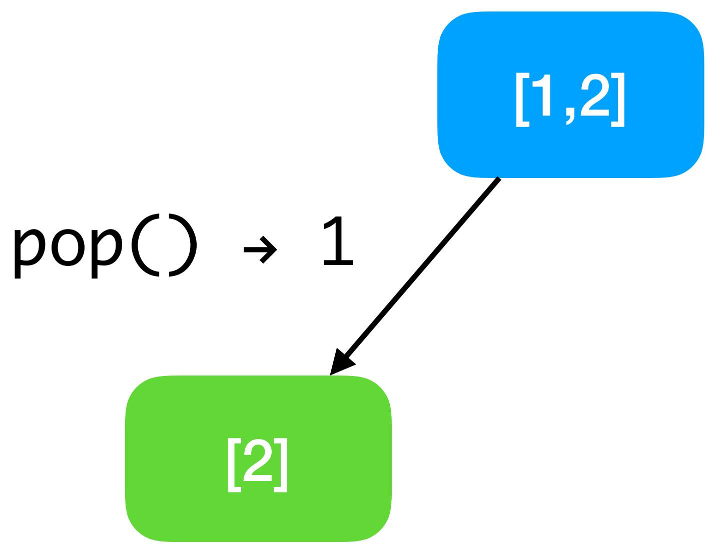

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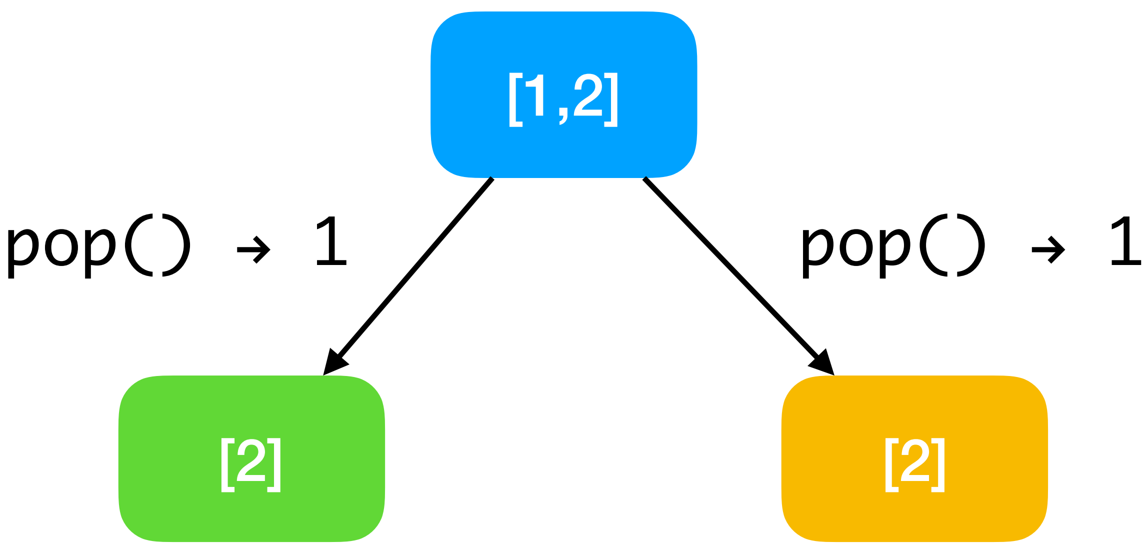

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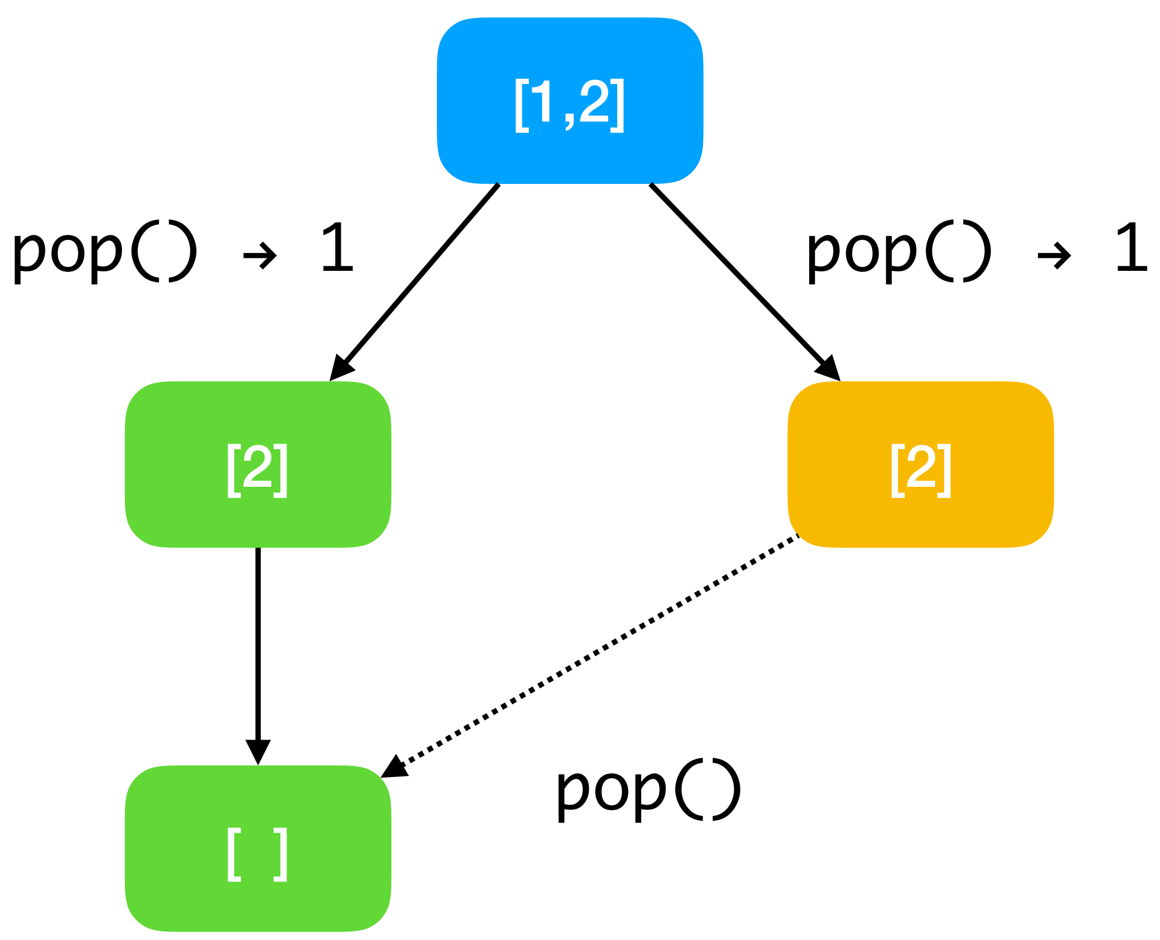

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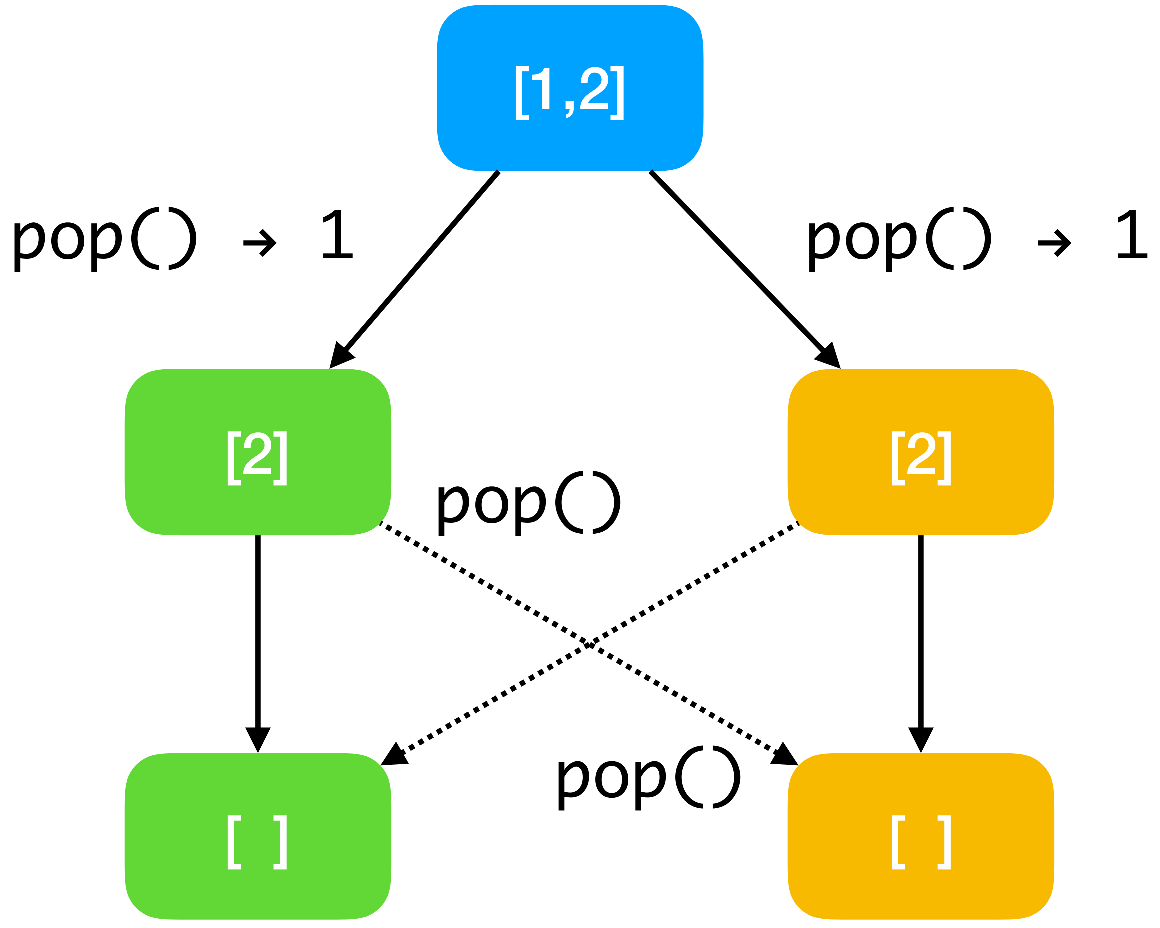

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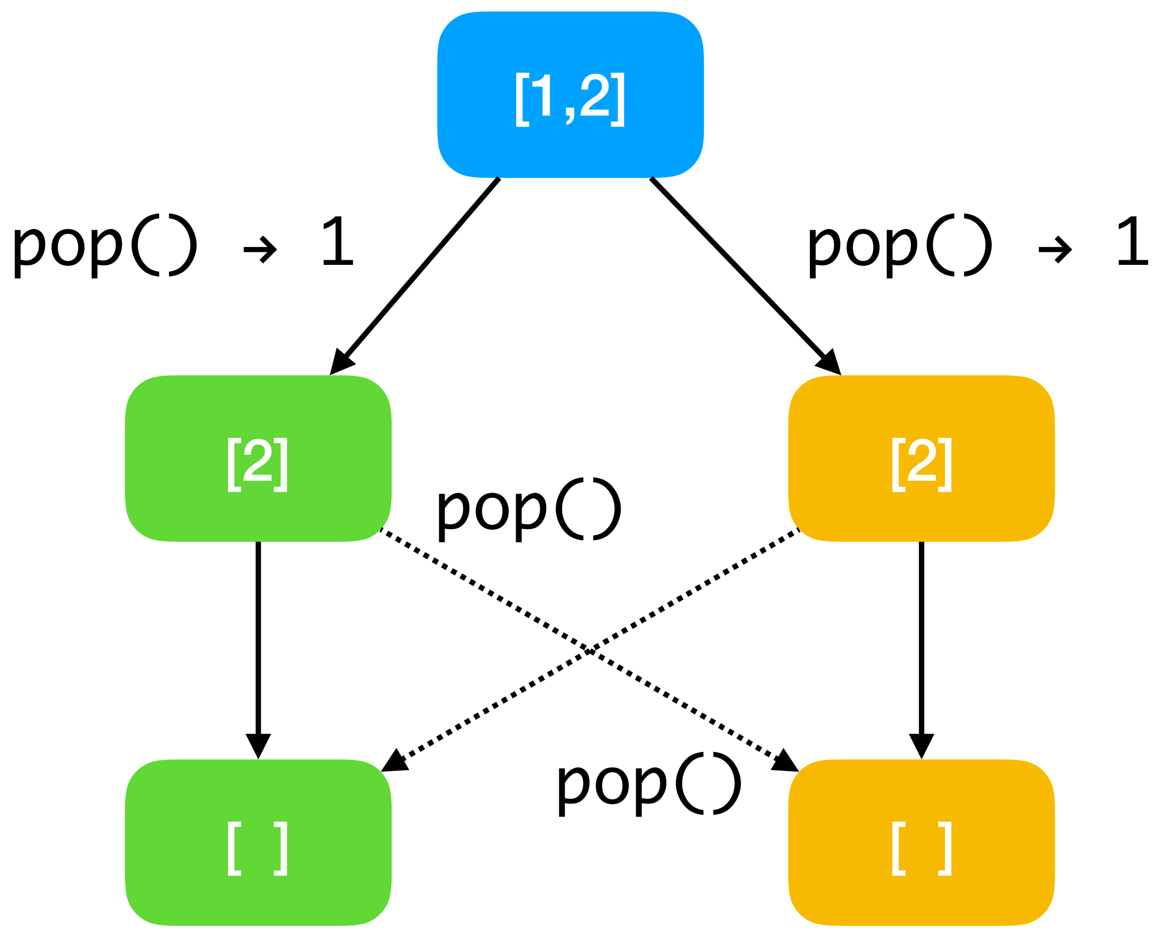

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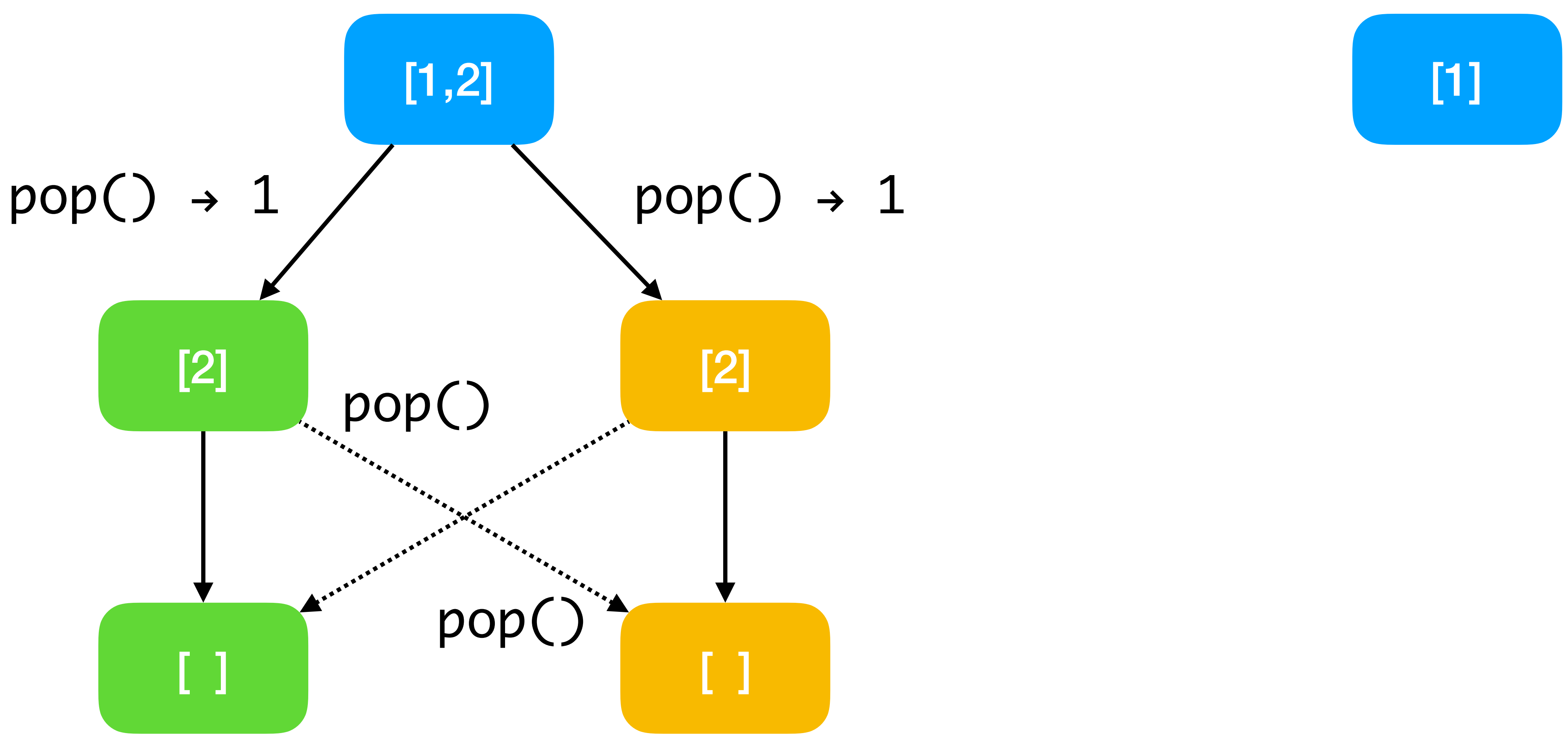

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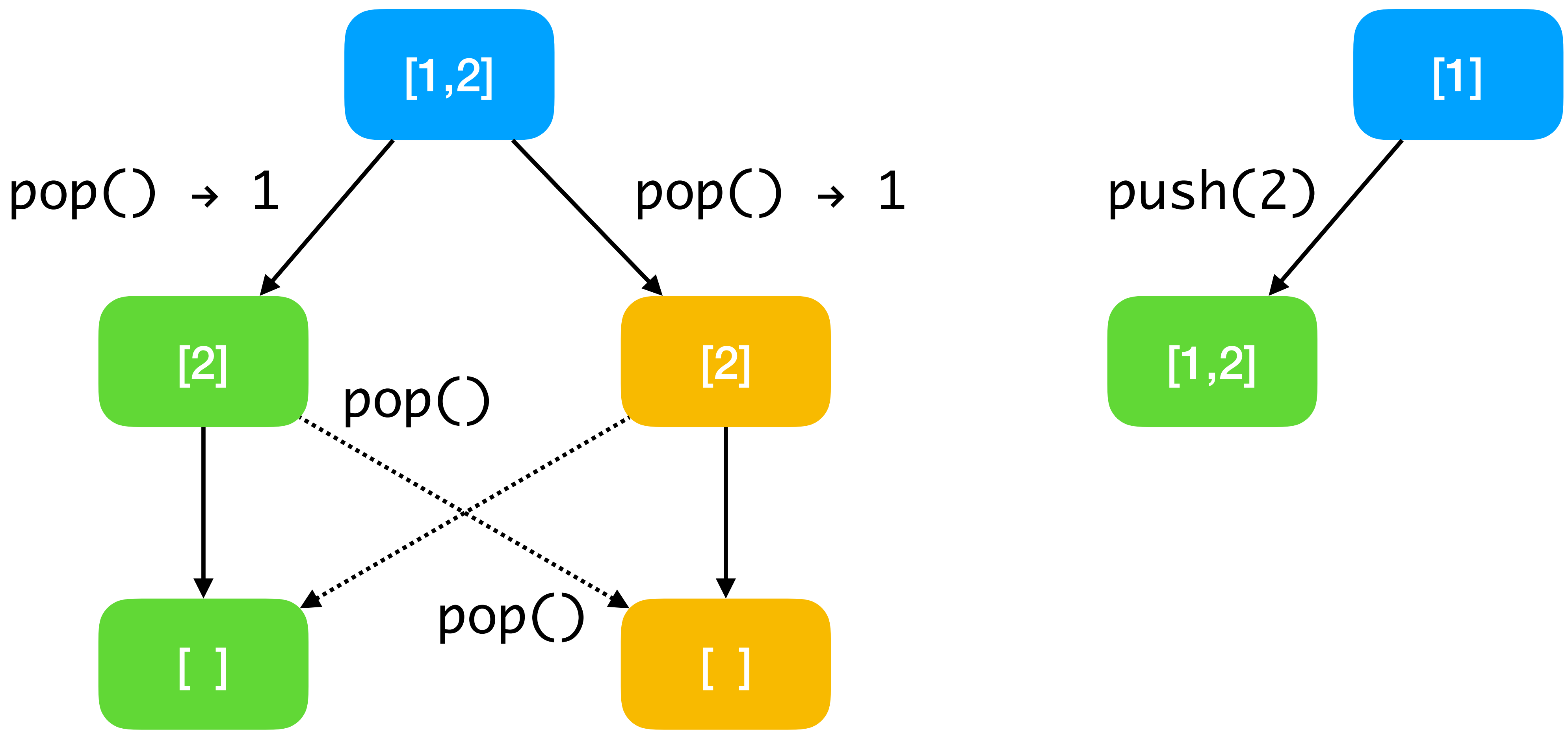

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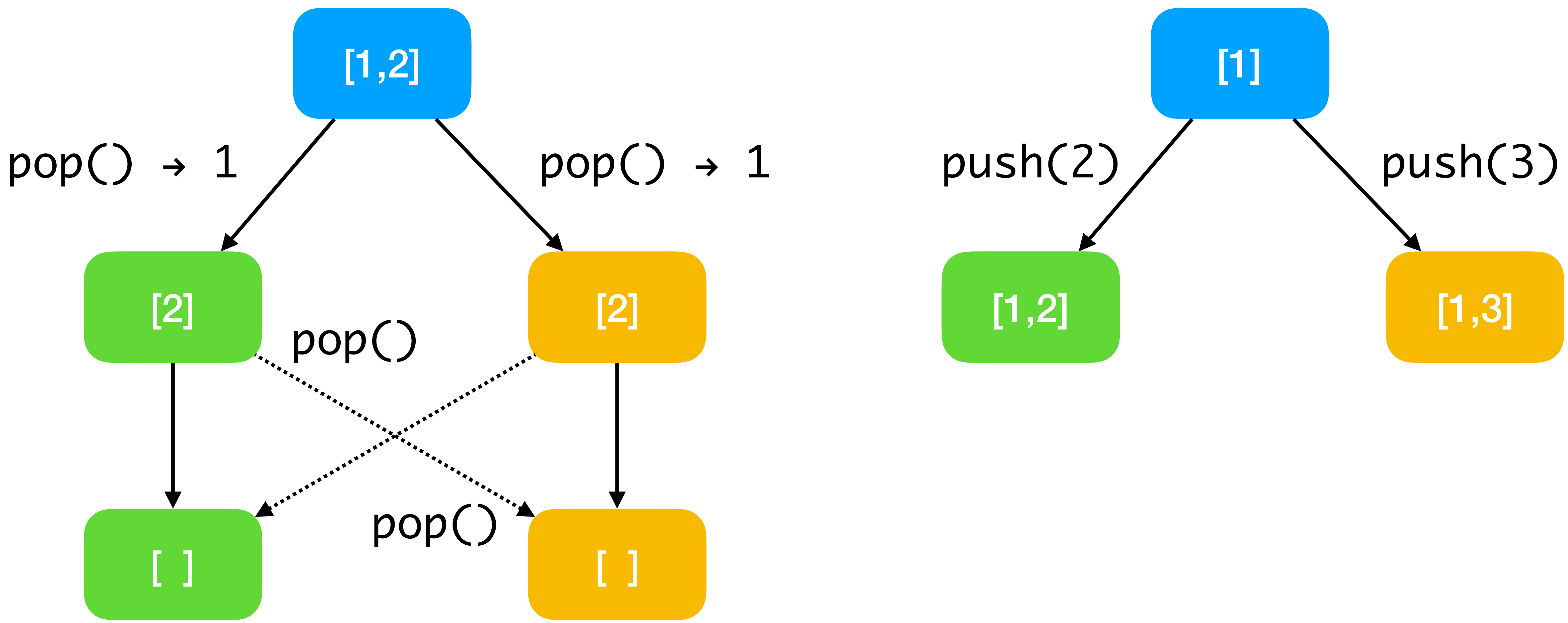

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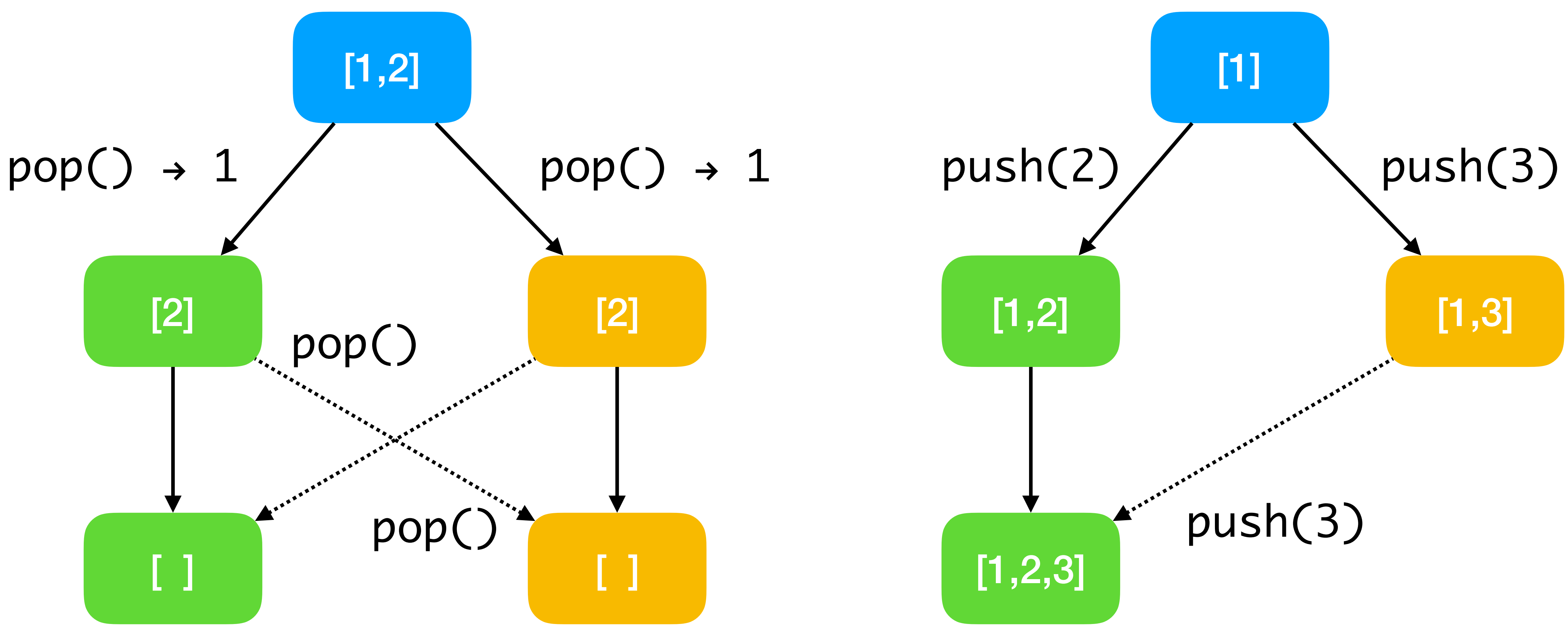

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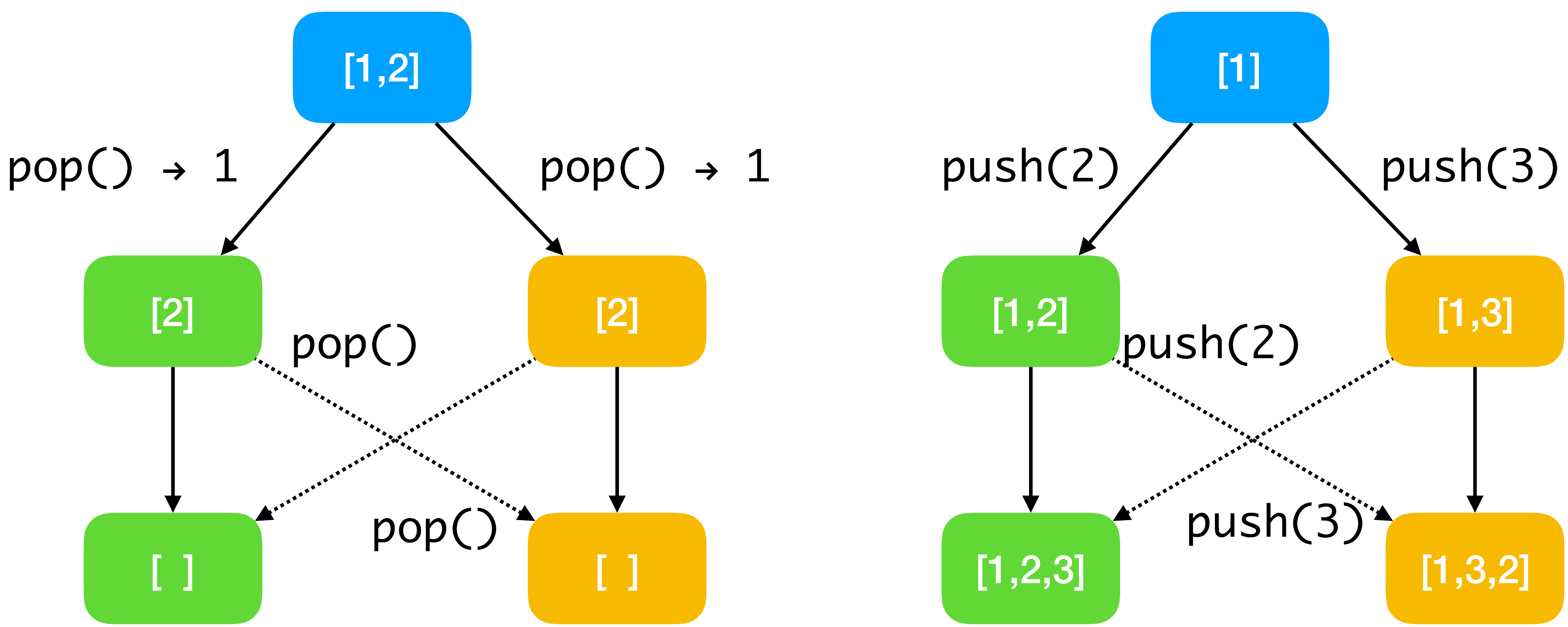

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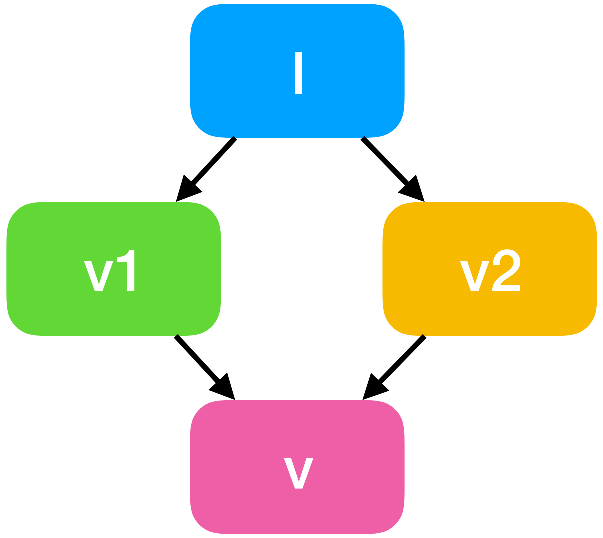

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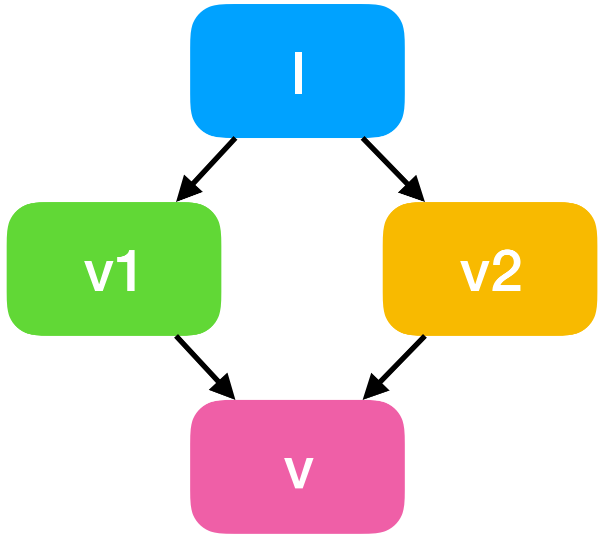

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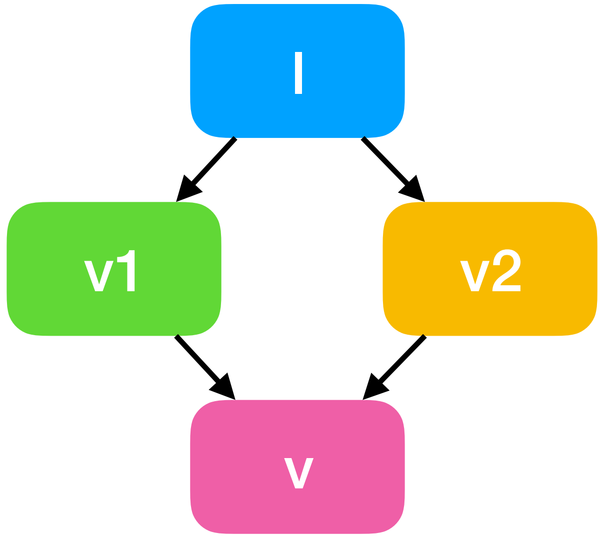

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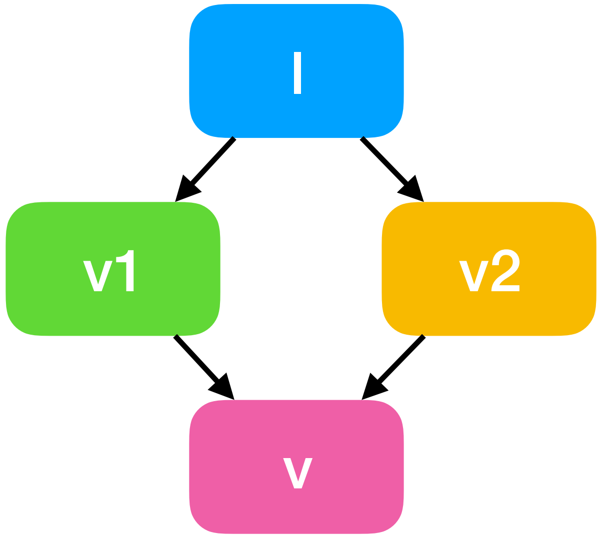

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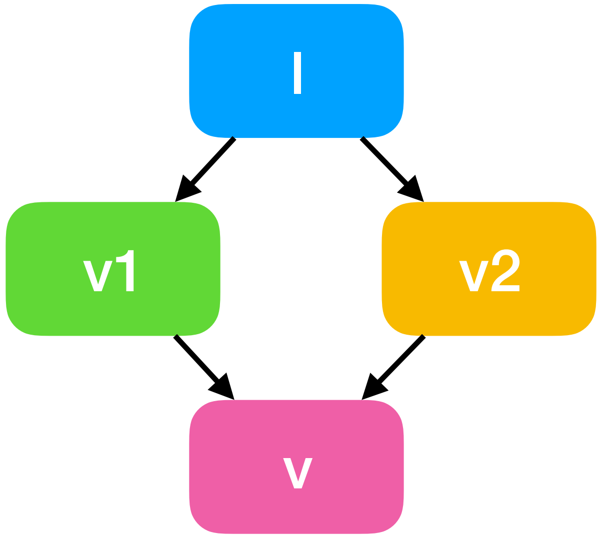

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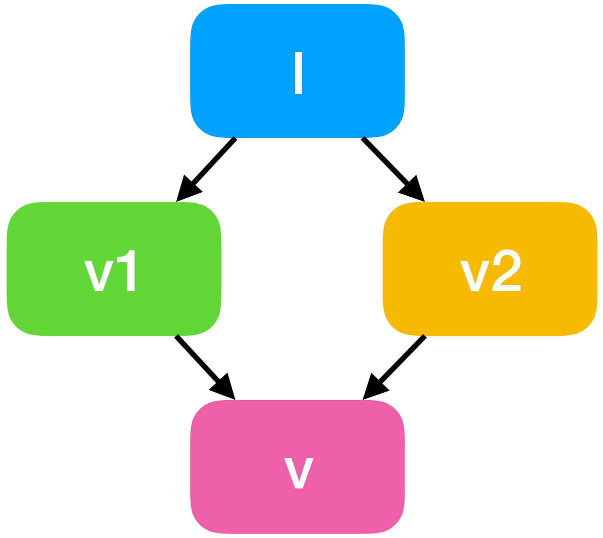
4. Order of pairs of elements in l, v1, v2 must be preserved in m, if those elements are present in v.



Relational Specification

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- Let's define relations R_{mem} and R_{ob} to capture membership and ordering
  - ★ R_{mem} [1,2,3] = {1,2,3}
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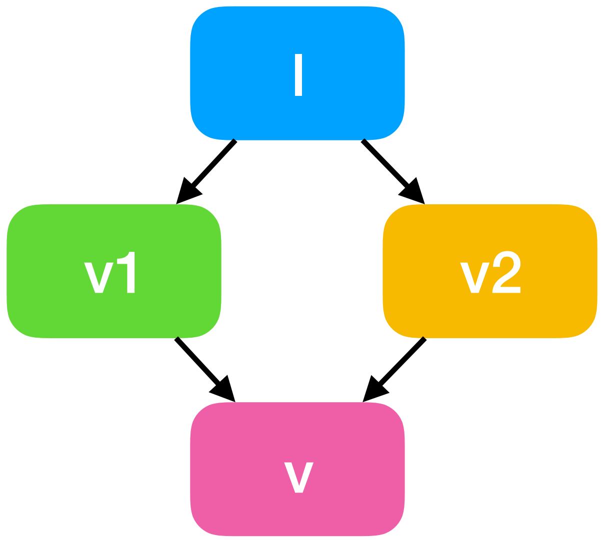

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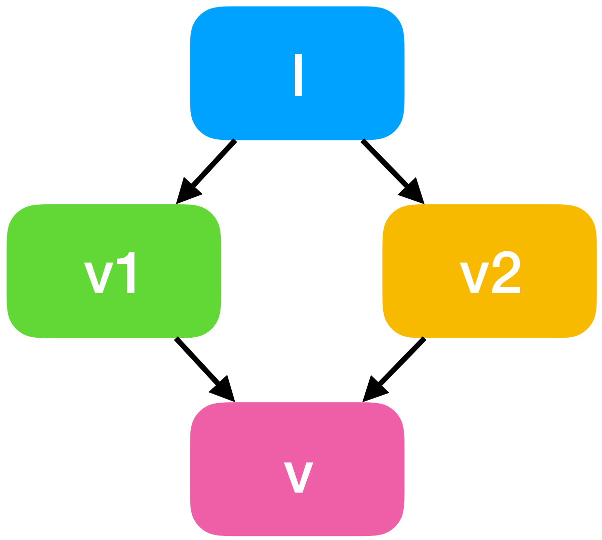

$$\begin{aligned} R_{mem}(v) \quad &= \quad R_{mem}(l) \cap R_{mem}(v_1) \cap R_{mem}(v_2) \\ &\qquad \cup \quad R_{mem}(v_1) - R_{mem}(l) \quad \cup \quad R_{mem}(v_2) - R_{mem}(l) \end{aligned}$$

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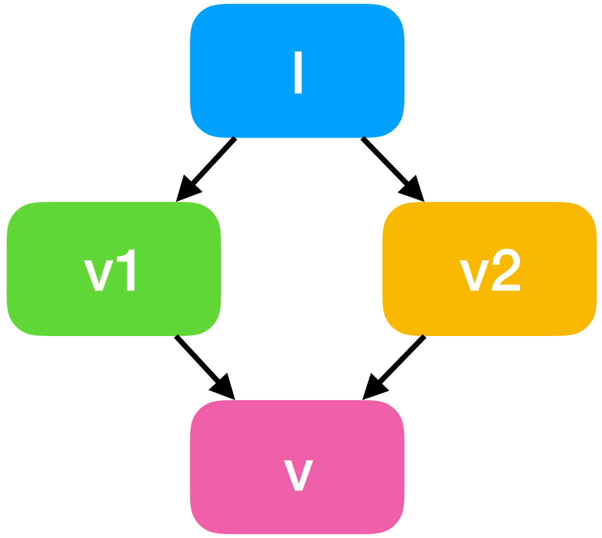

$$R_{mem}(v) = \boxed{R_{mem}(l) \cap R_{mem}(v_1) \cap R_{mem}(v_2)} \cup R_{mem}(v_1) - R_{mem}(l) \cup R_{mem}(v_2) - R_{mem}(l)$$

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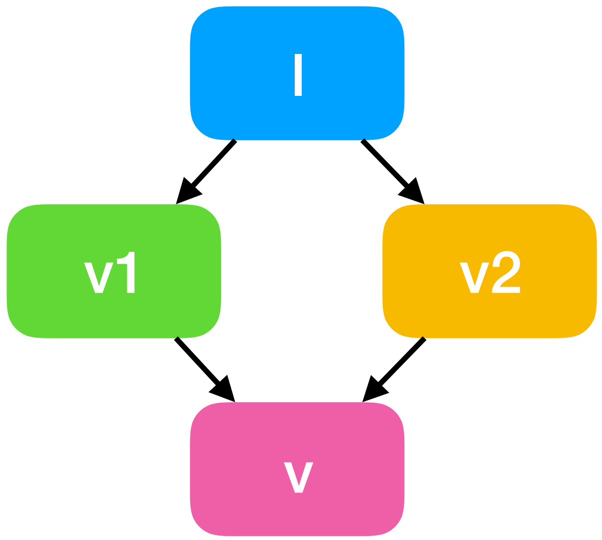

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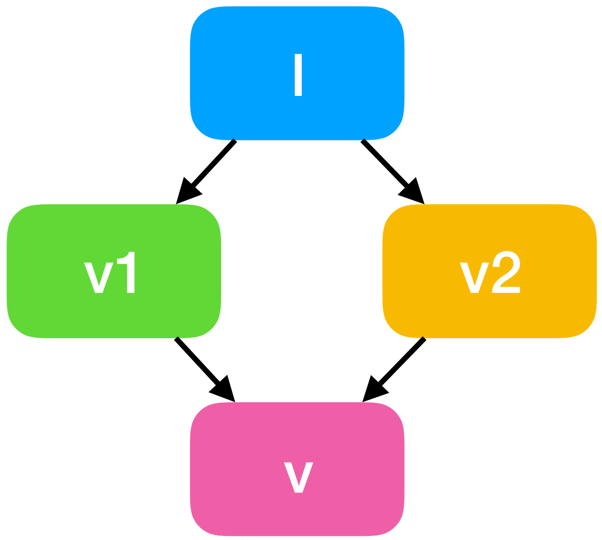

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R_{ob}(v) \quad \supseteq \quad & (R_{ob}(l) \cap R_{ob}(v_1) \cap R_{ob}(v_2) \\
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\cap \quad & (R_{mem}(v) \times R_{mem}(v))
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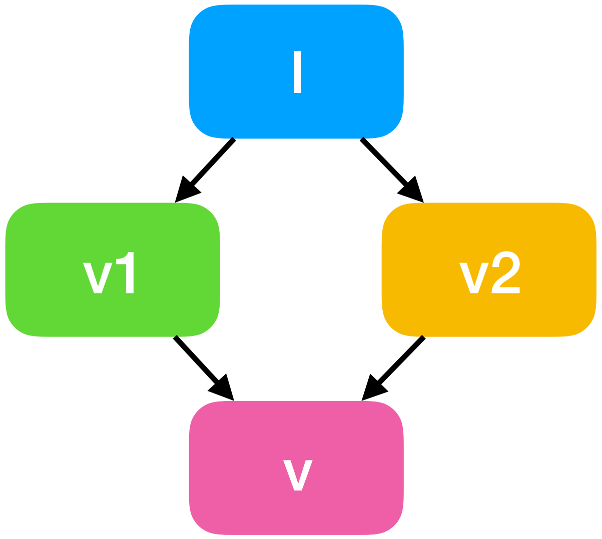

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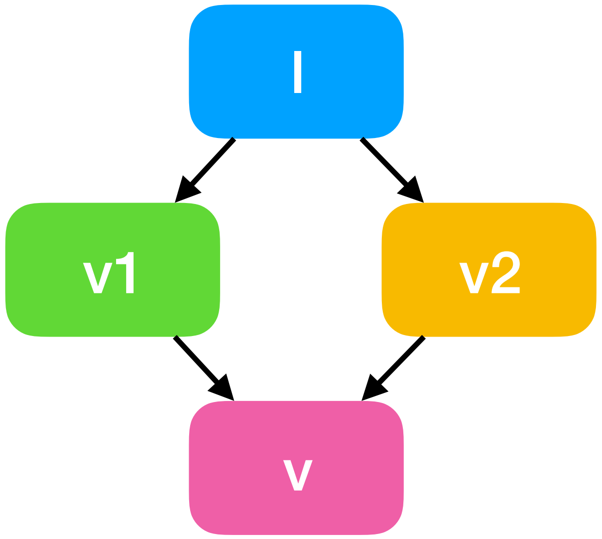

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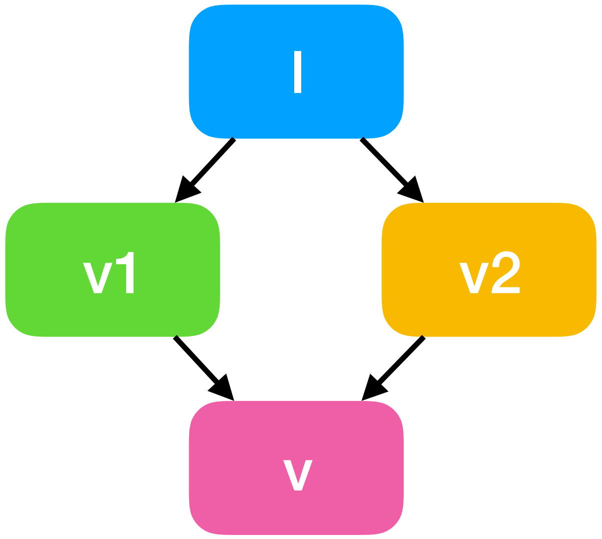

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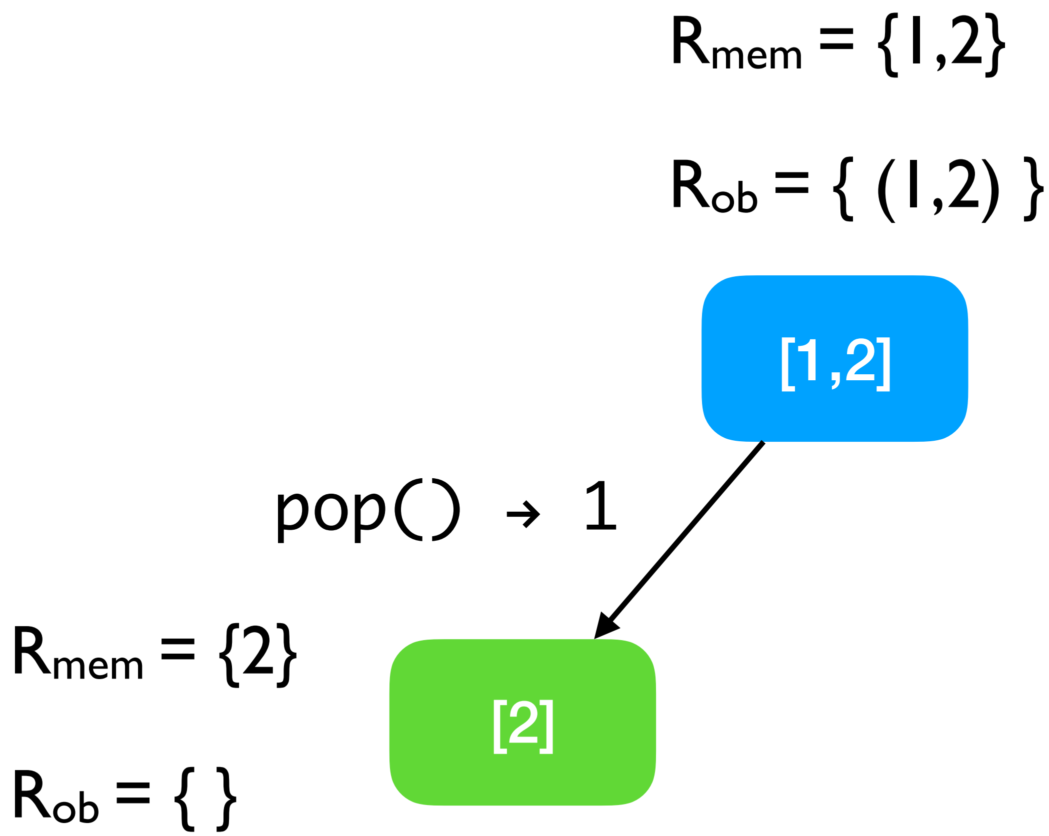
  - ★ Consider l = [0], v1= [0,1], v2 = [ ], v = [1]
- RHS is an underspecification since orders between concurrent insertions will only be present in $R_{ob}(v)$
  - ★ Consider l = [ ], v1= [0], v2 = [1], v = [0,1]

$R_{mem} = \{1,2\}$

$R_{ob} = \{ (1,2) \}$

[1,2]


$R_{mem} = \{1,2\}$
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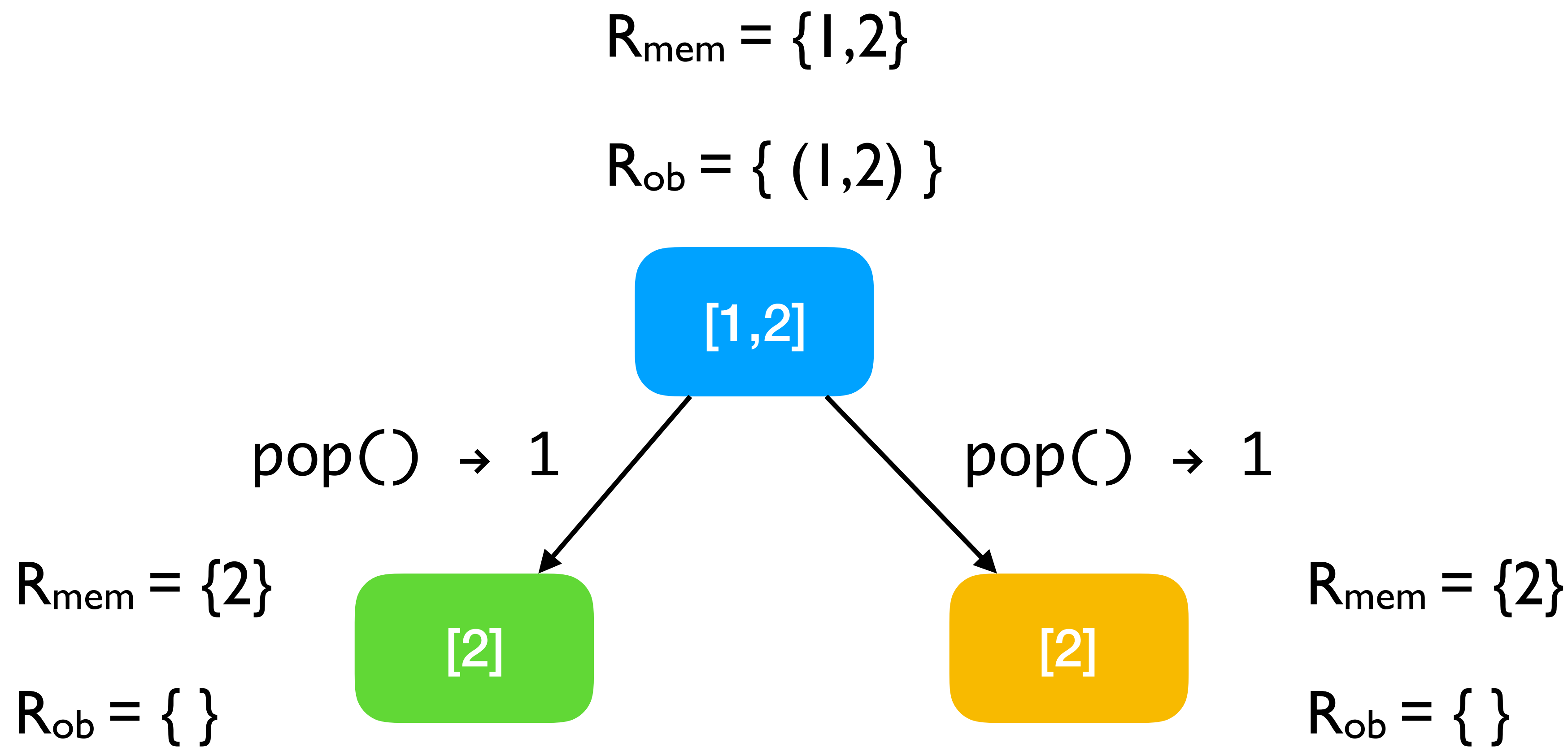
[1,2]
pop() → 1
$R_{mem} = \{2\}$
$R_{ob} = \{ \}$
[2]



R_{mem} = {1,2}
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[2]
[2]
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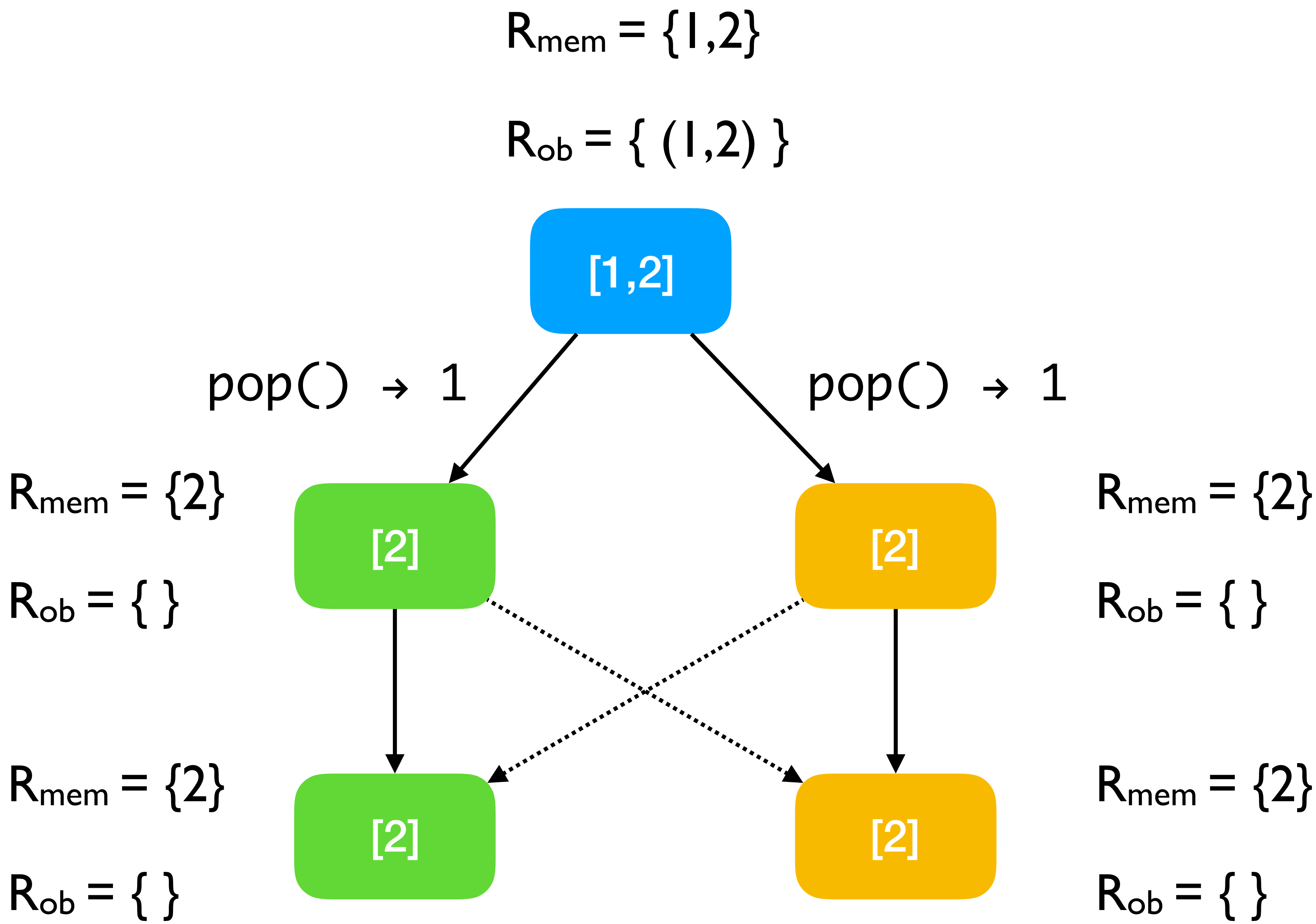

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R_{mem} = {2}
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[2]
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[2]
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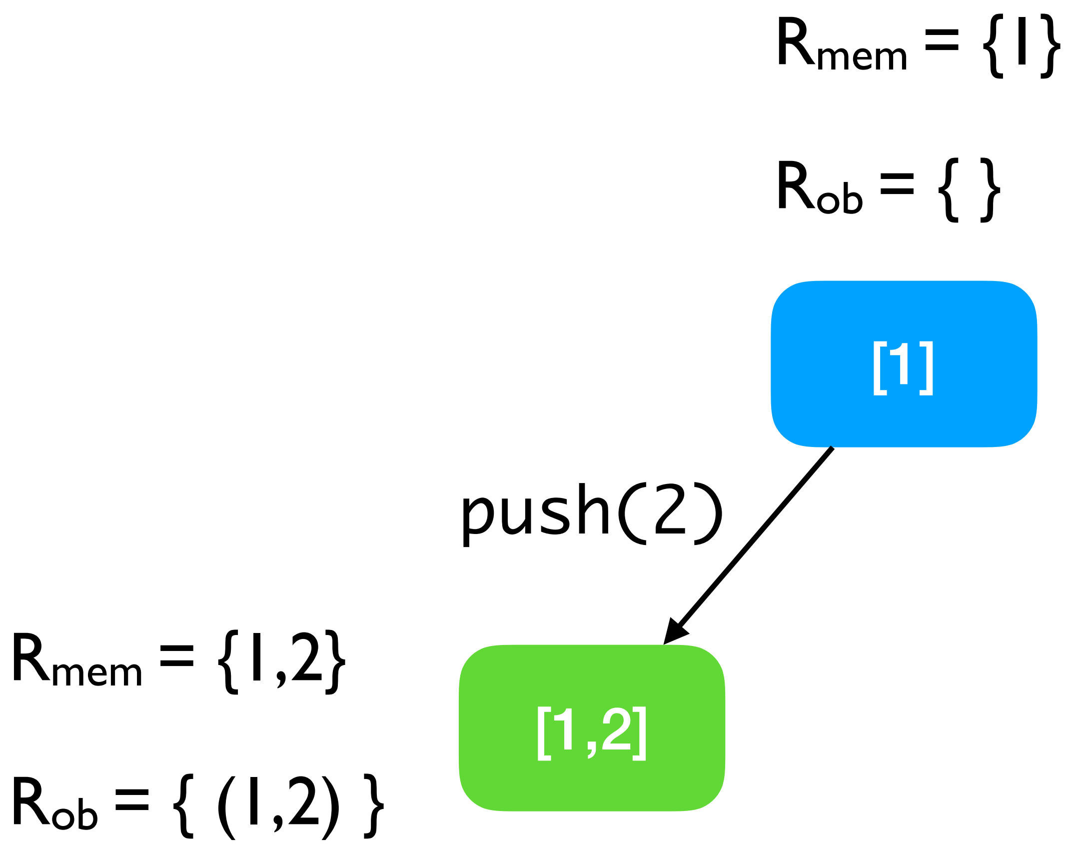
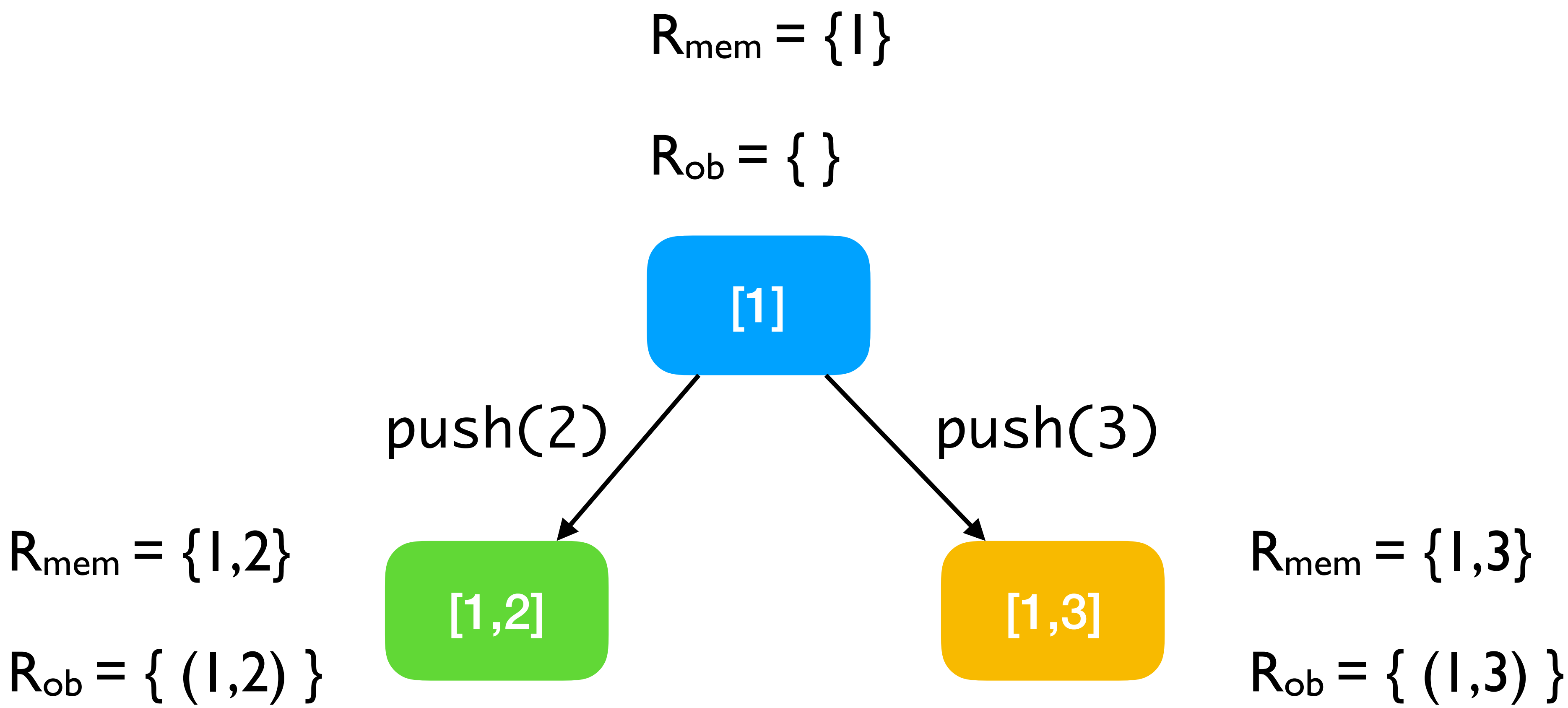
$R_{mem} = \{1\}$

$R_{ob} = \{\ \}$

[1]



R_mem = {1}
R_ob = { }
[1]
push(2)
R_mem = {1,2}
R_ob = { (1,2) }
[1,2]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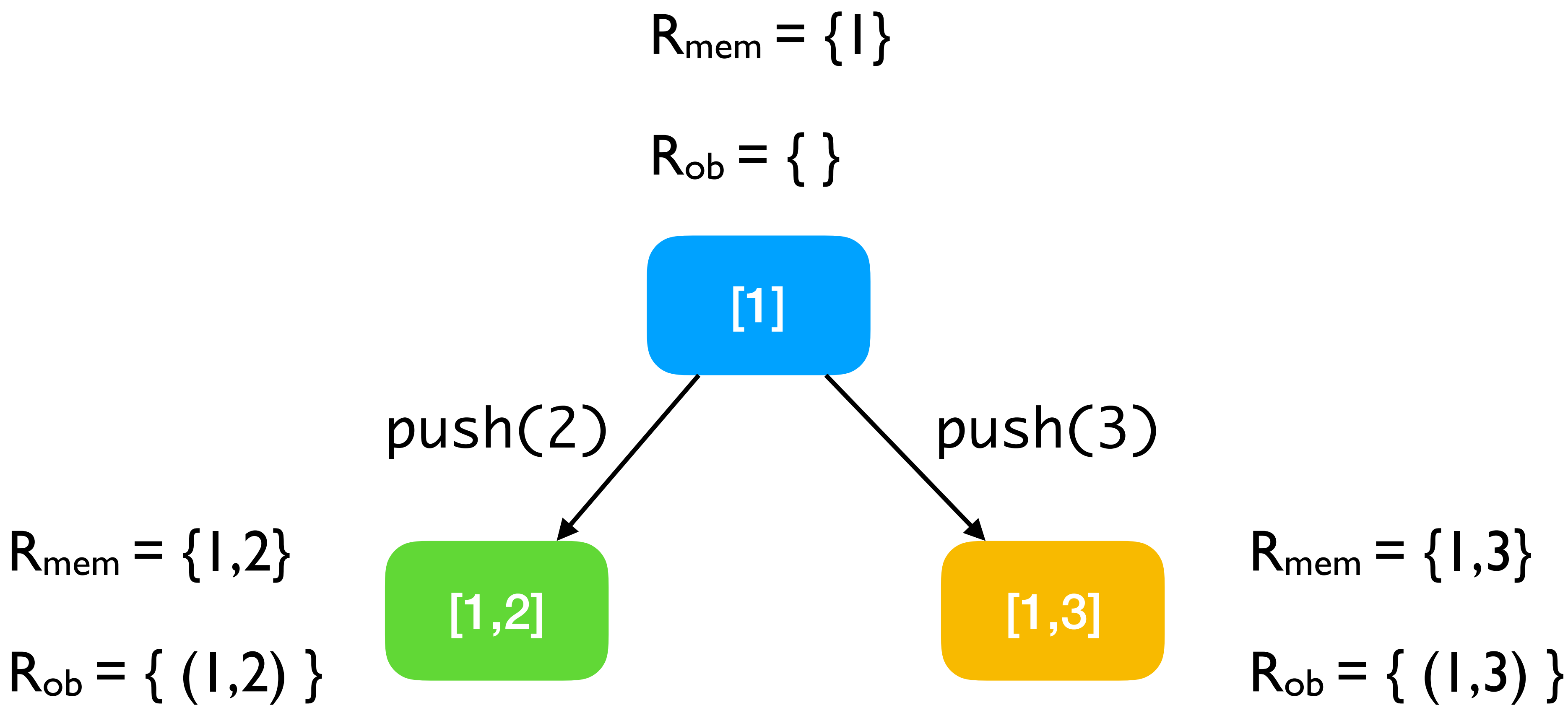
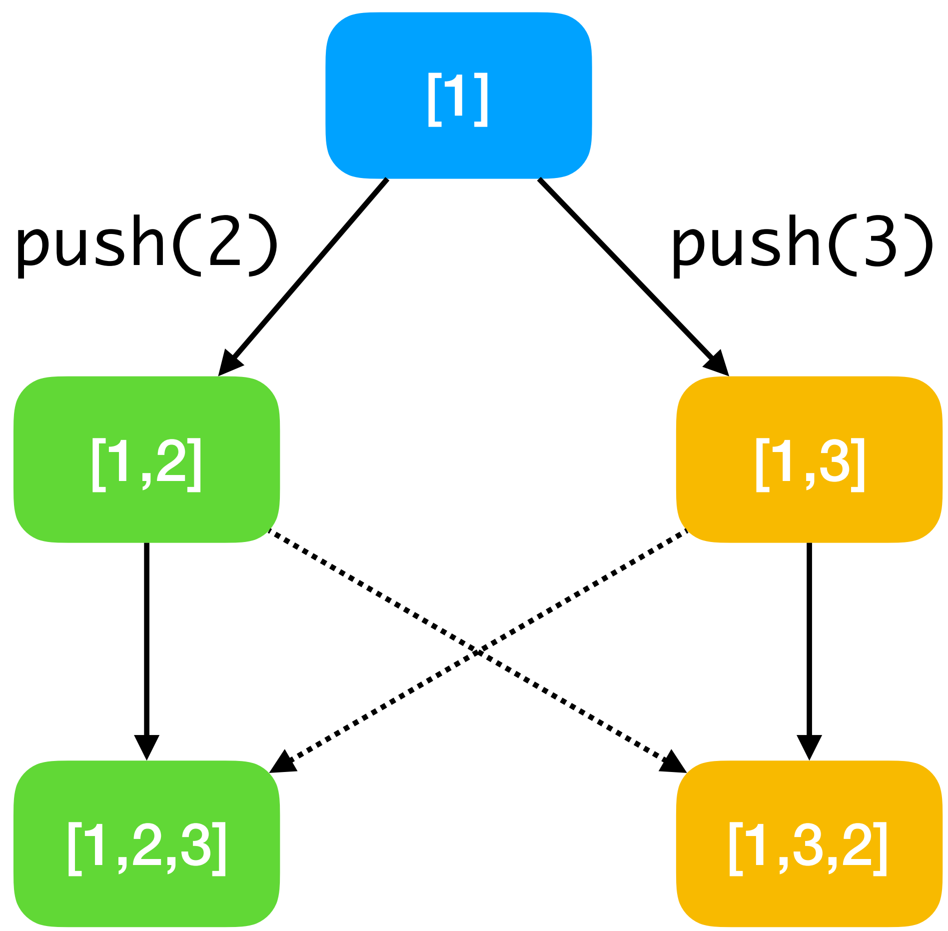
[1]
push(2)
push(3)
$R_{mem} = \{1,2\}$
$R_{ob} = \{\ (1,2)\ \}$
[1,2]
[1,3]
$R_{mem} = \{1,3\}$
$R_{ob} = \{\ (1,3)\ \}$



Use < as an arbitration function between concurrent insertions

$R_{mem} = \{1\}$

$R_{ob} = \{ \}$



$R_{mem} = \{1,2\}$

$R_{ob} = \{ (1,2) \}$

$R_{mem} = \{1,3\}$

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$R_{mem} = \{1,2,3\}$

$R_{ob} = \{ (1,2), (1,3), (2,3) \}$

$R_{mem} = \{1,2,3\}$

$R_{ob} = \{ (1,2), (1,3). (2,3) \}$

Use $<$ as an arbitration function between concurrent insertions

Characteristic Relations

A sequence of relations $\overline{R}_T$ is called a characteristic relation of a data type T, if for every $x : T$ and $y : T$, $\overline{R}_T(x) = \overline{R}_T(y)$ iff x and y are extensionally equal as interpreted under T.

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- R_{mem} and R_{ob} are the characteristic relations of queue
- Appeals only to the sequential properties of the data type
  - ★ Ignore distribution when defining characteristic relations.

Synthesizing Merge

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- Semantics of merge in relational domain is quite standard across data types

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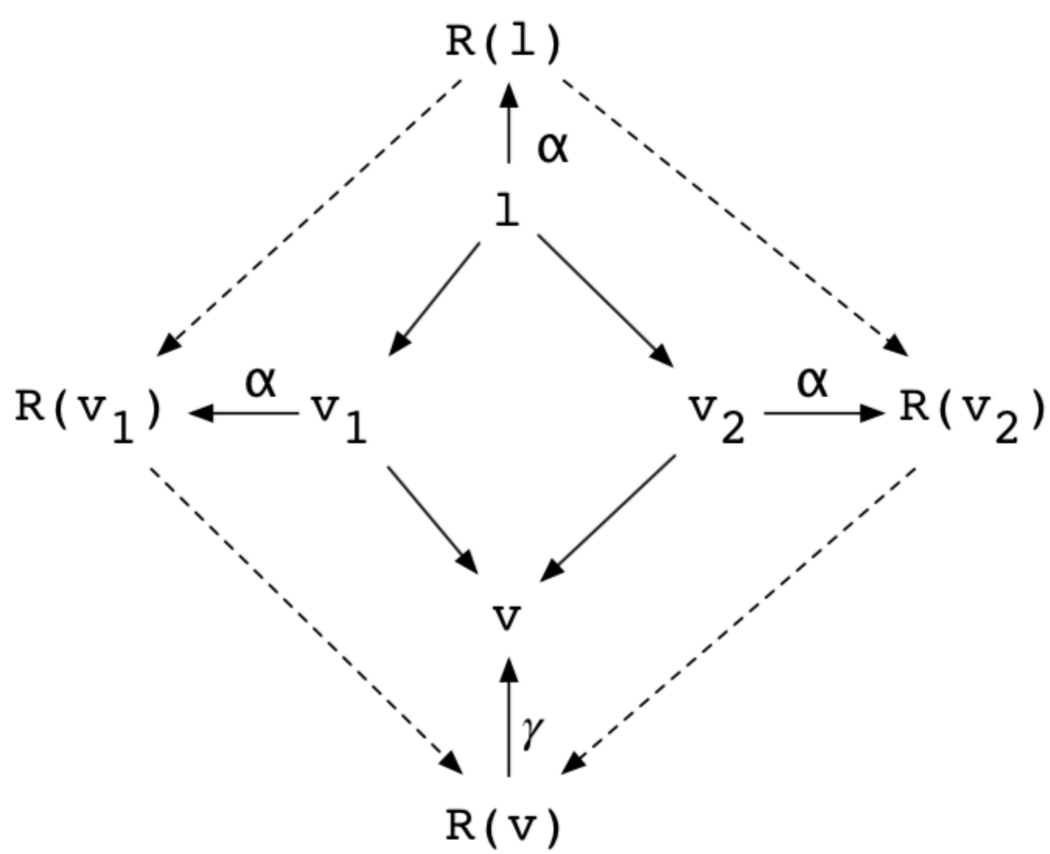

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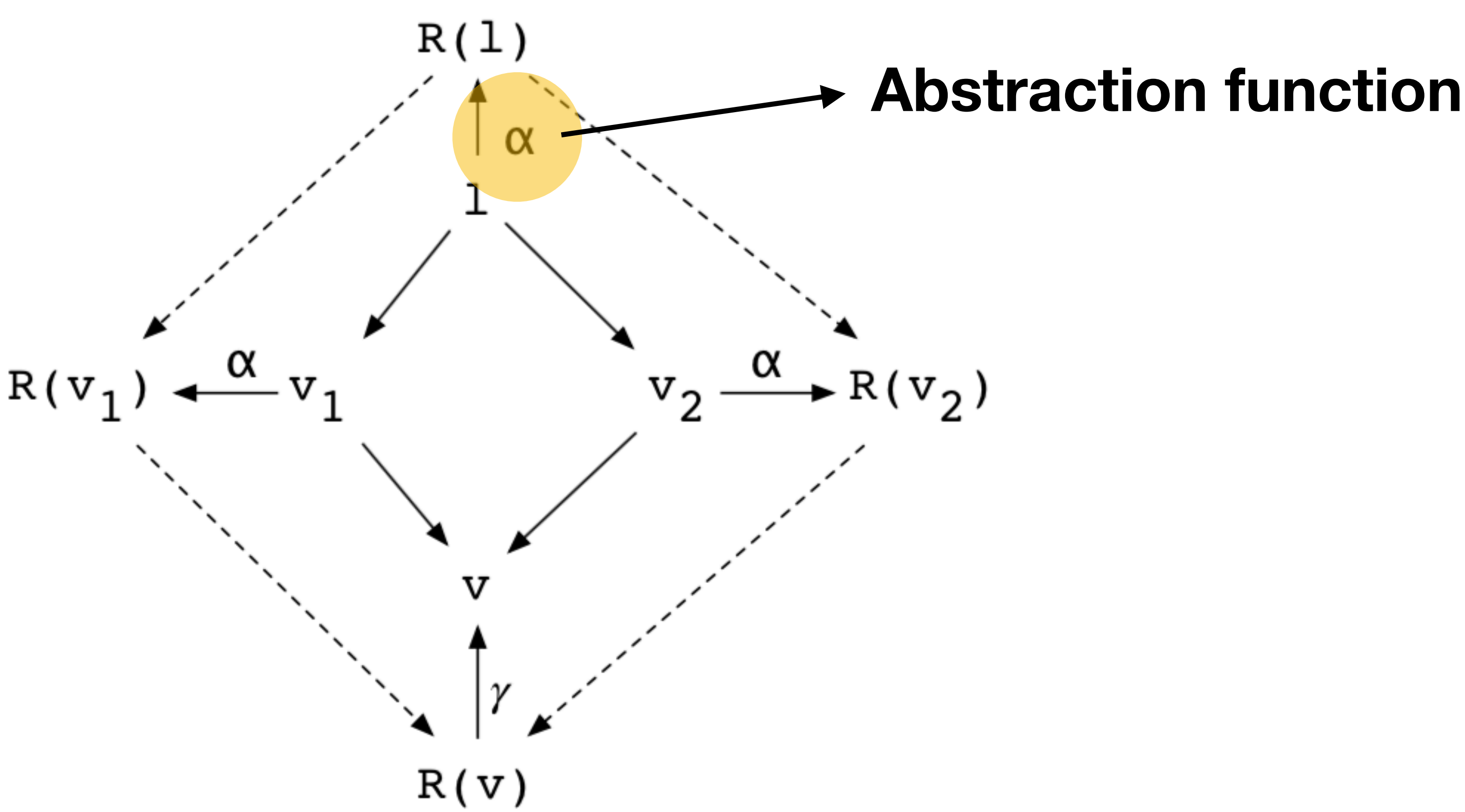

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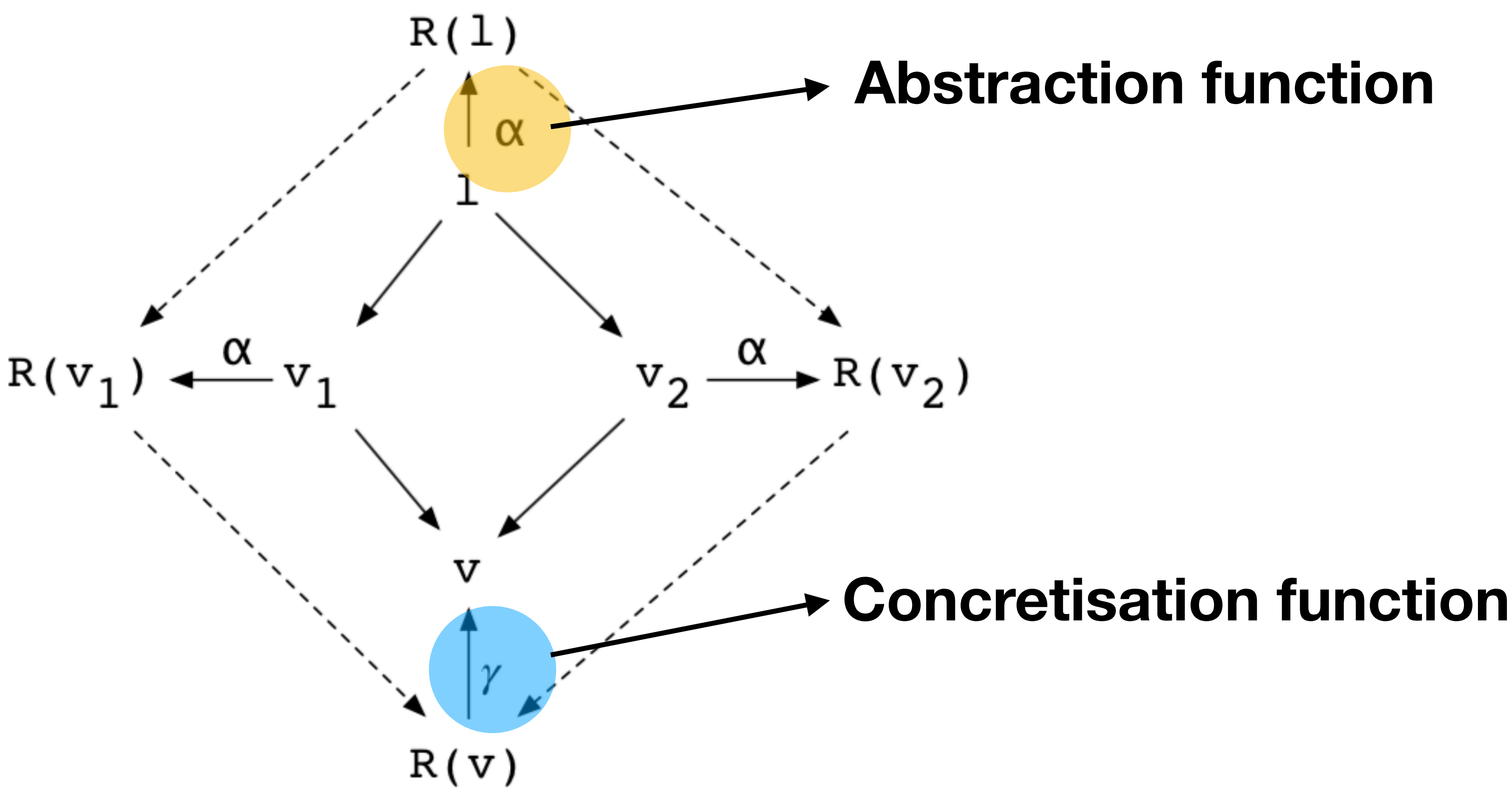

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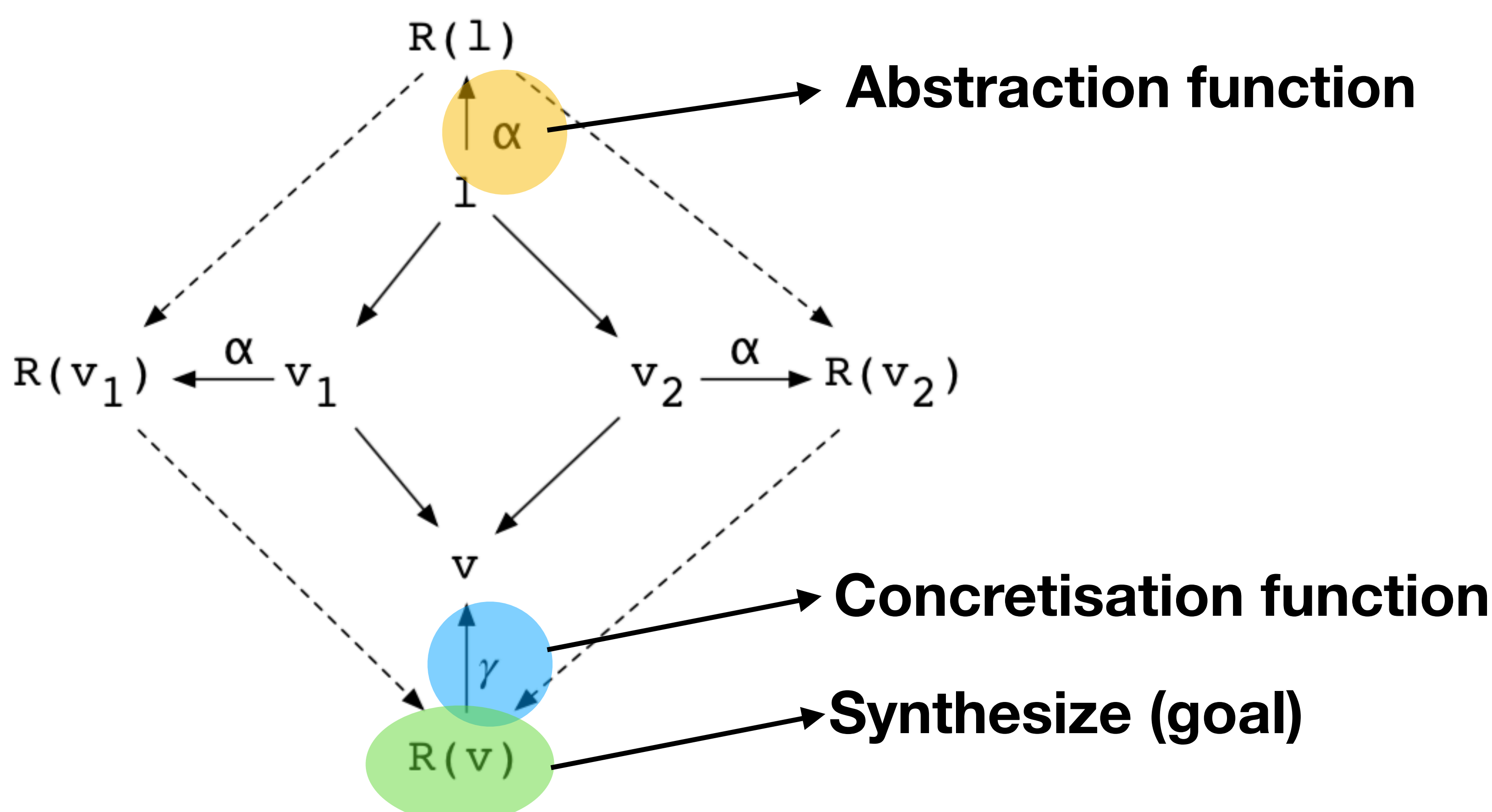

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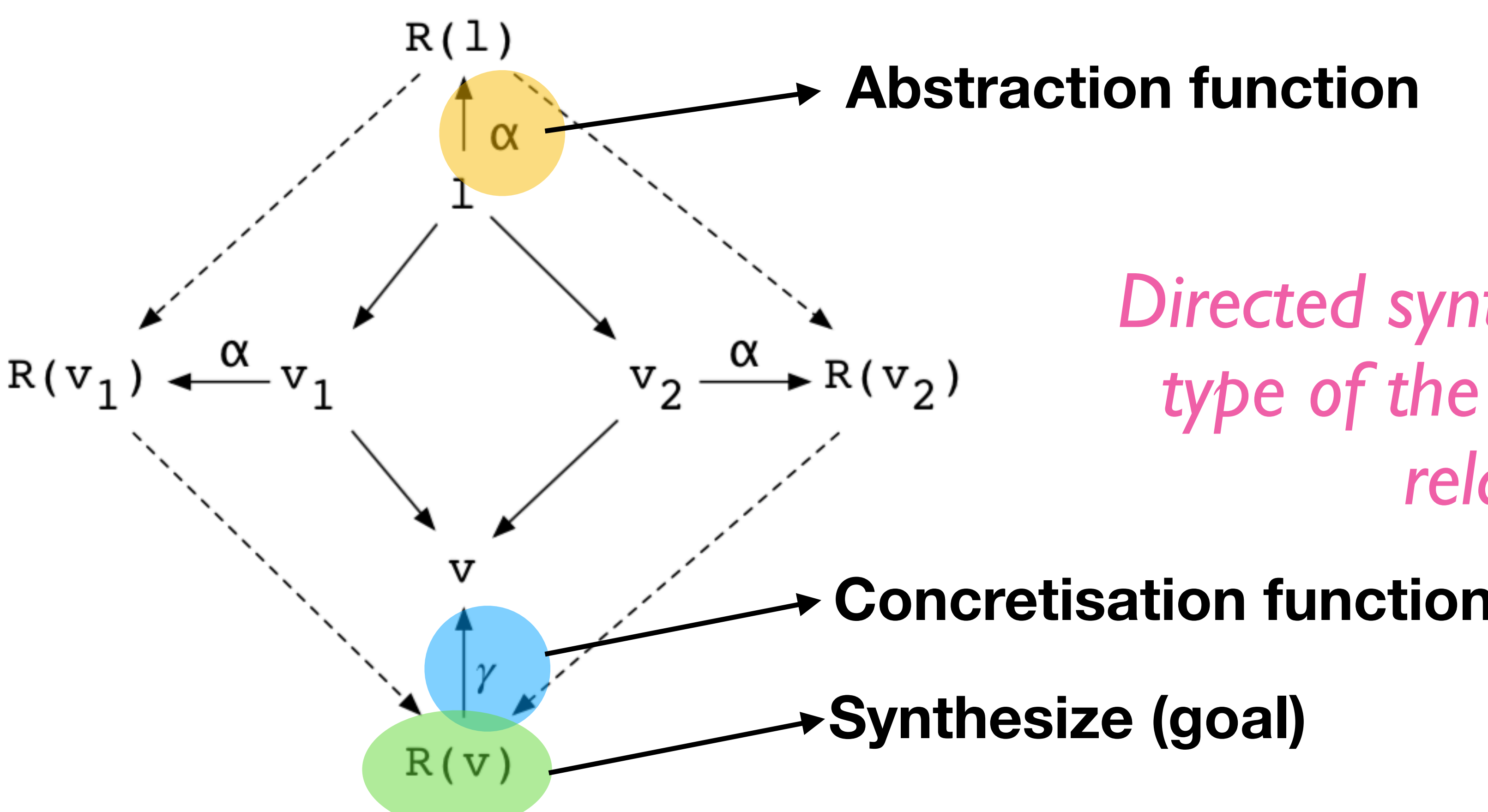

Directed synthesis using the type of the characteristic relations

Abstraction Function: Queue

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let rec R_mem = function
    | [] -> ∅
    | x::xs -> {x} ∪ R_mem(xs)
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```
let rec R_ob = function
  | [] -> ∅
  | x::xs -> ({x} × R_mem(xs)) ∪ R_ob(xs)
```

Abstraction Function: Queue

$$
\begin{array}{l}
\texttt{let rec } R_{mem} \texttt{ = function} \\
\quad\quad \texttt{| [] -> } \emptyset \\
\quad\quad \texttt{| x::xs -> \{x\} } \cup R_{mem}\texttt{(xs)}
\end{array}
$$

$$R_{mem} : \{\nu : \texttt{int list}\} \rightarrow \mathcal{P}(\texttt{int})$$

$$
\begin{array}{l}
\texttt{let rec } R_{ob} \texttt{ = function} \\
\quad \texttt{| [] -> } \emptyset \\
\quad \texttt{| x::xs -> (\{x\} } \times R_{mem}\texttt{(xs))} \cup R_{ob}\texttt{(xs)}
\end{array}
$$

$$R_{ob} : \{\nu : \texttt{int list}\} \rightarrow \mathcal{P}(R_{mem}(\nu) \times R_{mem}(\nu))$$

Abstraction Function: Binary Tree

```
type 'a tree = | E
               | N of 'a tree * 'a * 'a tree
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let rec R_mem = function
  | E -> ∅
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```

$$\texttt{let rec } R_{mem} = \texttt{function}$$
$$\texttt{| E -> } \emptyset$$
$$\texttt{| N(l,x,r) -> } R_{mem}\texttt{(l)} \cup \{\texttt{x}\} \cup R_{mem}\texttt{(r)}$$

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```

```
type label = L | R
let rec R_to = function
  | E -> ∅
  | N(l,x,r) ->
    let l_des = {x} × {L} × R_mem(l) in
    let r_des = {x} × {R} × R_mem(r) in
    R_to(l) ∪ l_des ∪ r_des ∪ R_to(r)
```

Abstraction Function: Binary Heap

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```

```
let rec R_mem = function
  | E -> ∅
  | N(l,x,r) -> R_mem(l) ∪ {x} ∪ R_mem(r)
```

```
let rec R_ans = function
  | E -> ∅
  | N(l,x,r) ->
    let des_x = R_mem(l) ∪ R_mem(r) in
    let r_ans = {x} × des_x in
    R_ans(l) ∪ r_ans ∪ R_ans(r)
```

Data Type	Characteristic Relations
Binary Heap	Membership (R_{mem}), Ancestor ($R_{ans} \subseteq R_{mem} \times R_{mem}$)
Priority Queue	Membership (R_{mem})
Set	Membership (R_{mem})
Graph	Vertex (R_V), Edge (R_E)
Functional Map	Key-Value (R_{kv})
List	Membership (R_{mem}), Order (R_{ob})
Binary Tree	Membership (R_{mem}), Tree-order ($R_{to} \subseteq R_{mem} \times \texttt{label} \times R_{mem}$)
Binary Search Tree	Membership (R_{mem})

Table 1. Characteristic relations for various data types

Compositionality: Pair

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- Then, pair merge spec is:

$$
\begin{aligned}
\phi_{c\times c}(l, v_1, v_2, v) \quad \Leftrightarrow \quad & \forall x, y, z, s.\ x \in R_{fst}(l) \ \wedge\ y \in R_{fst}(v_1) \ \wedge\ z \in R_{fst}(v_2) \\
& \qquad \wedge\ \phi_c(x, y, z, s) \Rightarrow s \in R_{fst}(v) \\
& \wedge\ \forall s.\ s \in R_{fst}(v) \Rightarrow \exists x, y, z.\ x \in R_{fst}(l) \ \wedge\ y \in R_{fst}(v_1) \\
& \qquad \wedge\ z \in R_{fst}(v_2) \ \wedge\ \phi_c(x, y, z, s) \\
& \wedge\ \ldots(\texttt{respectively for } R_{snd})
\end{aligned}
$$

Generalising Pairs to Ordinates

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$$\begin{aligned}
\phi_{c\times c} \quad = \quad & \forall (k : \texttt{int}).\forall (x, y, z, s : \texttt{counter}).\ (k, x) \in R_{pair}(l) \ \wedge\ (k, y) \in R_{pair}(v_1) \\
& \qquad \wedge\ (k, z) \in R_{pair}(v_2) \ \wedge\ \phi_c(x, y, z, s) \Rightarrow (k, s) \in R(v) \\
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- Given appropriate characteristic relation for a n-tuple ($R_{n\text{-tuple}}$), the same merge specification can be used.

Generalising Pairs to Ordinates

- Similar encoding can be given to maps with non-mergeable types as keys and mergeable types as values

$$
\begin{aligned}
R_k &: \{\nu : (\texttt{string}, \texttt{int})\ \texttt{map}\} \rightarrow \mathcal{P}(\texttt{string}), \\
R_{kv} &: \{\nu : (\texttt{string}, \texttt{int})\ \texttt{map}\} \rightarrow \mathcal{P}(R_k(\nu) \times \mathsf{counter})
\end{aligned}
$$

Types of Characteristic Relations

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$$R : \{v : T\} \to \mathcal{P}\left(\overline{T}\right), \text{ where } T \text{ is a non-mergeable type}$$

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  - ★ Ordering (R_{ob}, R_{ans}, R_{to})

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    - ✦ where ρ is a sequence of non-mergeable types and other relations, which flattens to a sequence of non-mergeable types
  - ★ Ordinates (R_{pair}, R_{kv})

    $$R : \{v : T\} \to \mathcal{P}\left(\overline{T} \times \overline{\tau}\right), \text{ where } \tau \text{ is a mergeable type}$$

Deriving merge spec

$$\frac{R : \{\nu : T\} \to \mathcal{P}\left(\overline{T}\right)}{\phi_T(l, v_1, v_2, v) \supseteq \forall(\overline{x} : \overline{T}).\ \overline{x} \in (R(l) \diamond R(v_1) \diamond R(v_2)) \Leftrightarrow \overline{x} \in R(v)} \quad \text{[Set-Merge]}$$

$$\frac{R : \{\nu : T\} \to \mathcal{P}\,(\rho) \qquad \lfloor \rho \rfloor = \overline{T}}{\phi_T(l, v_1, v_2, v) \supseteq \forall(\overline{x} : \overline{T}).\ \overline{x} \in (R(l) \diamond R(v_1) \diamond R(v_2)\ \cap\ \rho) \Rightarrow \overline{x} \in R(v)} \quad \text{[Order-Merge-1]}$$

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- Not complete, but practical
- Can derive merge spec for
  - ★ Data structures: Set, Heap, Graph, Queue, TreeDoc
  - ★ Larger apps: TPC-C, TPC-E, Twissandra, Rubis

Distributed Implementation

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- For making this programming model practical, we need to:
  - ★ Quickly compute LCA
  - ★ Optimise storage through sharing
  - ★ Optimise network transmissions (state based merge)

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  - ★ Optimise network transmissions (state based merge)
- Irmin
  - ★ A reimplementation of Git in pure OCaml
  - ★ Arbitrary OCaml objects, not just files + User-defined 3-way merges
  - ★ Only transmit diffs over the network
  - ★ Multiple storage backends including in-memory, filesystems, log-structured-merge database, distributed databases

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- Setup
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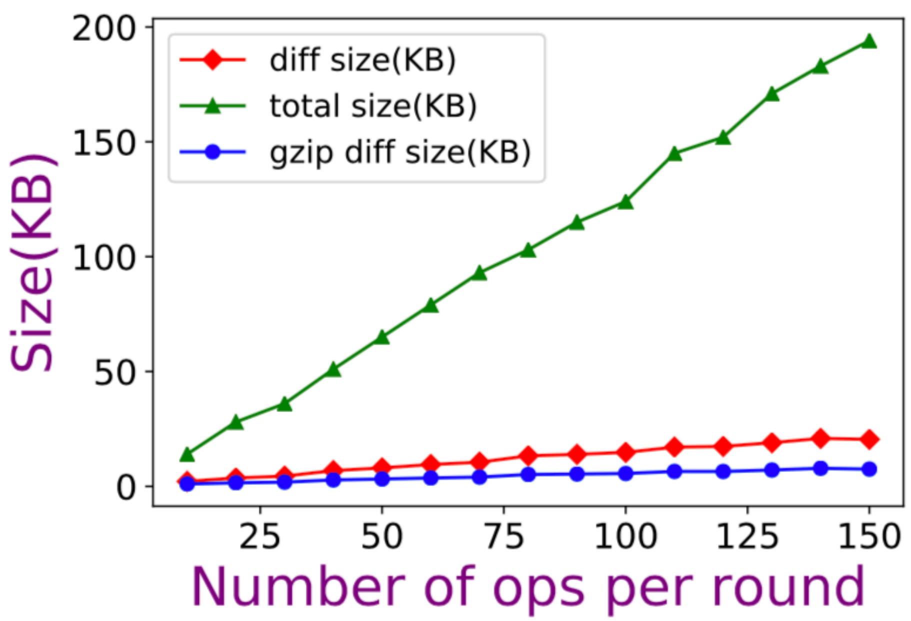
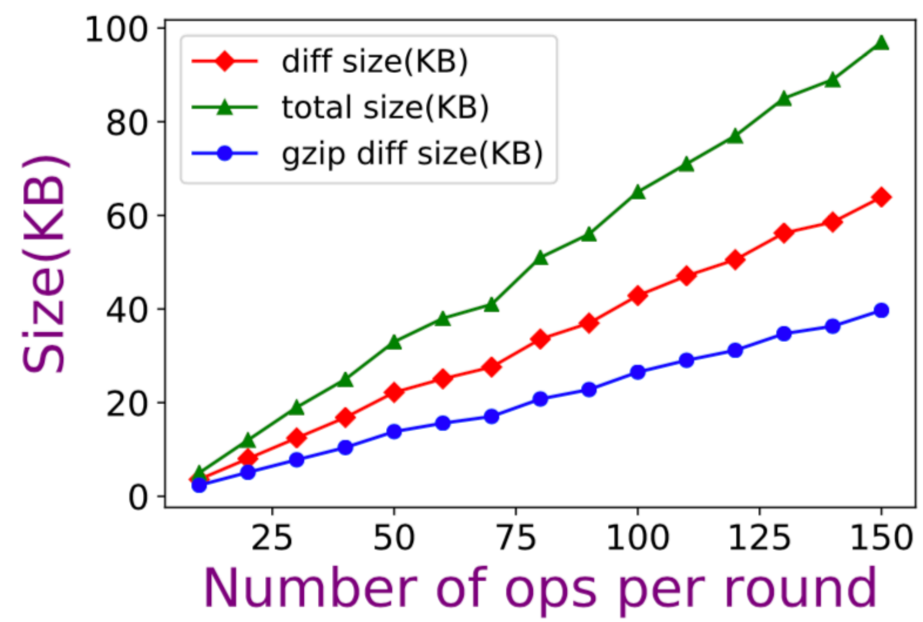


Binary Heap



Growable Array

Thanks for listening!